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STRONG BOOTSTRAP CONDITIONS

Budker INP 2000-64
UNICAL-TH 00/6

Novosibirsk
2000

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Abstract

We reformulate the so-called “strong bootstrap” conditions for the gluon Reggeization in the next-to-leading approximation (NLA), firstly suggested by Braun and Vacca, using a different approach, which is not based on properties of eigenstates of the NLA octet BFKL kernel. We write the second strong bootstrap condition for the NLA octet impact factors in a form which makes clear their dependence on the process. According to this condition, the NLA octet impact factors must be given by the product of the corresponding Reggeon interaction vertices with a universal coefficient function. This function can be used also in the formulation of the first strong bootstrap condition for the NLA BFKL kernel in the octet state.

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* Work supported in part by the Ministero italiano dell'Università e della Ricerca Scientifica e Tecnologica, in part by INTAS and in part by the Russian Fund of Basic Researches.

†¹ On leave of absence from the Budker Institute for Nuclear Physics, Novosibirsk, Russia.

Introduction

The gluon Reggeization is one of the remarkable properties of QCD. It plays an essential role in the derivation of the BFKL equation [1], which is very important for the description of high energy semi-hard processes in perturbative QCD. A rigorous proof of the gluon Reggeization has been constructed in the leading logarithmic approximation (LLA), which means summation of the terms $(\alpha_s \ln s)^n$ [2]. In the next-to-leading approximation (NLA) it has been observed in the first three orders of perturbation theory [3], but its validity to all orders of perturbation theory was only assumed in the derivation of the NLA BFKL equation. It is therefore very important to submit this assumption to careful tests.

A very stringent test of the gluon Reggeization is the requirement of its compatibility with the s -channel unitarity for the NLA elastic processes. This compatibility has been considered in Ref. [4], where two “bootstrap” conditions for the NLA octet BFKL kernel and impact factors were obtained. Recently, Braun and Vacca [5, 6] suggested a stronger requirement, namely that the gluon Reggeization should be realized also in the (unphysical) particle-Reggeon scattering amplitude with colour octet in the t -channel and negative signature. This requirement leads to “strong bootstrap” conditions for the NLA octet BFKL kernel and impact factors.

In this paper we reformulate this strong bootstrap in a different framework. Although strictly speaking all the results of this work can be derived from those of Refs. [5, 6], we think that this reformulation can be useful for at least two reasons. First, it does not rely on proper-

are neither evident nor easy to prove. Second, in this approach the second strong bootstrap condition can be presented in a form which makes manifest the dependence of the NLA octet impact factors on the particular process. According to this condition, the NLA octet impact factors must take a very simple form, since they can be written as the corresponding Reggeon effective vertex times a universal coefficient function. This function can be used also in the formulation of the first strong bootstrap condition for the NLA BFKL kernel in the octet state. We determine this coefficient function and check the fulfillment of the strong bootstrap conditions related to the quark and gluon octet impact factors.

The paper is organized as follows: in the next Section we introduce the relevant quantities and notations and briefly review the bootstrap for the NLA elastic processes; in Section 3 we consider the strong bootstrap by Braun and Vacca and write the related conditions in term of a universal function R ; then in Section 4 we determine this function R and check the strong bootstrap related to the quark and gluon octet impact factors.

1 Bootstrap conditions from NLA elastic processes

In the BFKL approach [1] elastic (or quasi-elastic) scattering amplitudes $(\mathcal{A})_{AB}^{A'B'}$ for the processes $AB \rightarrow A'B'$ in the Regge kinematics are presented in the following form [7]:

$$\begin{aligned}
 (\mathcal{A})_{AB}^{A'B'} &= \frac{is}{(2\pi)^{D-1}} \sum_{\mathcal{R},\nu} \langle \Phi_{B'B}^{(\mathcal{R},\nu)} | \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{\sin(\pi\omega)} \\
 &\times \left[\left(\frac{-s}{s_0} \right)^\omega - \tau_{\mathcal{R}} \left(\frac{s}{s_0} \right)^\omega \right] \hat{G}_\omega^{\mathcal{R}} | \Phi_{A'A}^{(\mathcal{R},\nu)} \rangle, \quad (1.1)
 \end{aligned}$$

where s is the squared c.m.s. energy of the colliding particles, which is assumed to be tending to infinity, the sum is taken over representations

and over states ν from full sets of states in these representations, $\tau_{\mathcal{R}}$ is the t -channel signature which is equal $+1$ (-1) for symmetric (antisymmetric) representation \mathcal{R} . The parameter s_0 was introduced in Ref. [7] to give a convenient definition of the t -channel partial wave and is artificial in the sense that the amplitude does not depend on it in the NLA; $D = 4 + 2\epsilon$ is the space-time dimension in the dimensional regularization. Finally, Φ 's are the impact factors of the colliding particles and G_ω is the Mellin transform of the Green function for the scattering of two Reggeized gluons,

$$\hat{G}_\omega^{\mathcal{R}} = \frac{1}{\omega - \hat{\mathcal{K}}^{\mathcal{R}}}, \quad (1.2)$$

$\hat{\mathcal{K}}$ being the kernel of the BFKL equation. Let us consider the transverse momentum representation, where ‘‘transverse’’ is related to the plane of initial particle momenta. In this representation, defined by

$$\begin{aligned} \hat{q}|\vec{q}_i\rangle &= \vec{q}_i|\vec{q}_i\rangle, & \langle\vec{q}_1|\vec{q}_2\rangle &= \vec{q}_1^2\vec{q}_1'^2\delta^{(D-2)}(\vec{q}_1 - \vec{q}_2), \\ \vec{q}_i' &= \vec{q}_i - \vec{q}, & \langle A|B\rangle &= \langle A|\vec{k}\rangle\langle\vec{k}|B\rangle = \int \frac{d^{D-2}k}{\vec{k}^2\vec{k}'^2}A(\vec{k})B(\vec{k}), \end{aligned} \quad (1.3)$$

the exact definitions of the impact factors and of the BFKL kernel in the NLA,

$$\Phi_{A'A}^{(\mathcal{R},\nu)}(\vec{q}_1, \vec{q}, s_0) = \langle\vec{q}_1|\Phi_{A'A}^{(\mathcal{R},\nu)}\rangle, \quad \mathcal{K}^{\mathcal{R}}(\vec{q}_2, \vec{q}_1, \vec{q}) = \langle\vec{q}_2|\hat{\mathcal{K}}^{\mathcal{R}}|\vec{q}_1\rangle, \quad (1.4)$$

were given in Ref. [4]. In the above equalities \vec{q} is the transverse momentum transfer for the amplitude (1.1), so that we have

$$t = q^2 \approx q_\perp^2 = -\vec{q}^2. \quad (1.5)$$

Note that throughout this paper \vec{q} is considered as a fixed parameter and has nothing in common with the operator \hat{q} . In the derivation of the representation (1.1) a certain Reggeized form for the production amplitudes was assumed; in particular, for the elastic amplitude with

needs to have

$$(\mathcal{A}_{8^-})_{AB}^{A'B'} = \Gamma_{B'B}^a(s_0) \frac{s}{t} \left[\left(\frac{s}{s_0} \right)^{\omega(t)} + \left(\frac{-s}{s_0} \right)^{\omega(t)} \right] \Gamma_{A'A}^a(s_0), \quad (1.6)$$

where Γ 's are the particle-Reggeon effective interaction vertices and $\omega(t)$ is the Reggeized gluon trajectory. The dependence of the vertices on s_0 was indicated explicitly to stress that it cancels the analogous dependence of the Regge factor in Eq. (1.6) (the dependence on other possible arguments not being shown). From the comparison of the term with $\mathcal{R} = 8^-$ in Eq. (1.1) and Eq. (1.6), two self-consistency relations (the so-called ‘‘bootstrap conditions’’) have been obtained in Ref. [4]:

$$\int \frac{d^{D-2}q_1}{\bar{q}_1^2 \bar{q}'_1{}^2} \left[\int \frac{d^{D-2}q_2}{\bar{q}_2^2 \bar{q}'_2{}^2} \langle \bar{q}_1 | \hat{\mathcal{K}}^{(1)} | \bar{q}_2 \rangle - \omega^{(2)}(t) \right] = 0, \quad (1.7)$$

$$-\frac{2ig\sqrt{N}\bar{q}^2}{(2\pi)^{D-1}} \int \frac{d^{D-2}q_1}{\bar{q}_1^2 \bar{q}'_1{}^2} \langle \bar{q}_1 | \Phi_{A'A}^{a(1)} \rangle = 2\omega^{(1)}(t) \Gamma_{A'A}^{a(1)}(s_0) + \omega^{(2)}(t) \Gamma_{A'A}^{a(0)}. \quad (1.8)$$

In the above relations $\Gamma^{(0)}$ and $\omega^{(1)}$ denote the leading logarithm approximation (LLA) contributions to the vertex and the gluon trajectory, respectively. Analogously, $\Gamma^{(1)}$, $\omega^{(2)}$, $\mathcal{K}^{(1)}$ and $\Phi^{(1)}$ mean the NLA corrections to the corresponding LLA quantities. We omitted the superscript (or subscript) $\mathcal{R} = 8^-$, assuming that here and everywhere below all the quantities belong to this representation. The conditions (1.7) and (1.8) are the bootstrap requirement in the NLA in the elastic sector. It is not yet clear whether they are sufficient to prove in the NLA the Regge form also of the inelastic amplitudes¹, which is used in the BFKL approach. Nevertheless, these conditions provide a strong confirmation of the self-consistency of the NLA BFKL approach. In fact, no doubts in the validity of the BFKL approach in the NLA must remain after the verification that the bootstrap conditions (1.7) and (1.8) are fulfilled.

¹In the leading order such form was proved [2].

in [8] and was found to be satisfied at arbitrary space-time dimension D . Recently, the gluon part of the colour octet kernel was calculated [9] in the NLA for $D \rightarrow 4$ and the fulfillment of the bootstrap condition for this part was proved in this limit [10]. The calculation of the kernel at arbitrary D is in progress now [11].

The second bootstrap condition (1.8) was checked for the gluon and quark impact factors [12, 13] and was proved to be satisfied at arbitrary space-time dimension both for the helicity conserving and non-conserving parts of the impact factors. Note, that the first bootstrap condition is universal, i.e. process independent, and even if it is a quite nontrivial problem to demonstrate its fulfillment, one has to do it just once. In this respect the second bootstrap condition looks much worse, because it is process dependent, i.e. any amplitude with gluon quantum numbers in the t -channel has its own bootstrap condition. If one uses such amplitude in the unitarity relation for a given process, one should be sure that this bootstrap condition is satisfied. For example, the NLA correction to the forward colour singlet BFKL kernel is already available [14, 15] and it would be very interesting to apply it for the description of a real experiment; for this purpose we need to know the forward impact factors of colourless particles. The calculation of the virtual photon NLA impact factor is in progress now [16]. Huge mathematical difficulties must be overcome in this calculation, so that before going on with it, it is desirable to be sure of the self-consistency of the NLA BFKL approach and, in particular, of the fulfillment of the bootstrap condition (1.8) for the impact factor describing the $\gamma \rightarrow q\bar{q}$ transition in the photon-Reggeon scattering process, which is involved in the calculation. Unfortunately, a direct check of this bootstrap condition is a not less complicated problem than the calculation of the photon impact factor itself. Therefore one should investigate as much as possible the bootstrap conditions in general form, without specifying the scattering process, in order to understand the reasons for the relations (1.7) and (1.8).

Braun and Vacca [5, 6] suggested stronger bootstrap conditions than those obtained in [4]. Their suggestion is based on the requirement of Reggeization of the unphysical particle-Reggeon scattering amplitude with colour octet in the t -channel and negative signature

$$(\mathcal{A})_{AR}^{A'R'} = \frac{i\kappa}{2\pi} \langle \vec{q}_1 | \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{\sin(\pi\omega)} \left[\left(\frac{-\kappa}{s_0} \right)^\omega + \left(\frac{\kappa}{s_0} \right)^\omega \right] \hat{G}_\omega | \Phi_{A'A}^a \rangle, \quad (2.1)$$

where κ is the particle-Reggeon squared invariant mass, in the sense that the amplitude should not contain admixture of eigenstates of the kernel different from the one corresponding to the gluon trajectory. In our reformulation we will not use the language of eigenvalues and eigenstates, because, as already discussed, this would require the justification of some properties of the kernel, such as completeness of the eigenstates, which are neither evident nor easy to prove. In the following we will show briefly how to obtain the strong bootstrap conditions not assuming these properties. The meaning of Reggeization was discussed in details in Ref. [4] and in the case under consideration it implies

$$(\mathcal{A})_{AR}^{A'R'} = \tilde{R}(\vec{q}_1, \vec{q}, s_0) \frac{\kappa}{t} \left[\left(\frac{\kappa}{s_0} \right)^{\omega(t)} + \left(\frac{-\kappa}{s_0} \right)^{\omega(t)} \right] \Gamma_{A'A}^a(s_0), \quad (2.2)$$

where \tilde{R} can be called the Reggeon scattering vertex. Comparing the κ -channel discontinuities of Eqs. (2.1) and (2.2) we get

$$\langle \vec{q}_1 | \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{\kappa}{s_0} \right)^\omega \hat{G}_\omega | \Phi_{A'A}^a \rangle = \left(\frac{\kappa}{s_0} \right)^{\omega(t)} \Gamma_{A'A}^a(s_0) \langle \vec{q}_1 | R \rangle, \quad (2.3)$$

where

$$\langle \vec{q}_1 | R \rangle \equiv R(\vec{q}_1, \vec{q}, s_0) \equiv \frac{\sin(\pi\omega(t))}{-t} \tilde{R}(\vec{q}_1, \vec{q}, s_0). \quad (2.4)$$

Using the relation

$$\left(\hat{\mathcal{K}}^{(0)} - \omega^{(1)}(t) \right) | \Phi_{A'A}^{a(0)} \rangle = 0, \quad (2.5)$$

$\Phi(\vec{q}_1, \vec{q})$ from \vec{q}_1 and from the form of the Born octet BFKL kernel (see, for example [7]), we have in the NLA

$$\begin{aligned} \hat{G}_\omega |\Phi_{A'A}^a\rangle &= (\omega - \hat{\mathcal{K}}^{(0)})^{-1} |\Phi_{A'A}^{a(1)}\rangle \\ &+ (\omega^{(1)}(t) - \hat{\mathcal{K}}^{(0)})^{-1} \left((\omega - \omega^{(1)}(t))^{-1} - (\omega - \hat{\mathcal{K}}^{(0)})^{-1} \right) \hat{\mathcal{K}}^{(1)} |\Phi_{A'A}^{a(0)}\rangle. \end{aligned} \quad (2.6)$$

Therefore we obtain for the L.H.S. of Eq. (2.3) with NLA accuracy (in simplified notations)

$$\begin{aligned} &\left(\frac{\kappa}{s_0}\right)^{\omega^{(1)}} \langle \vec{q}_1 | \{ |\Phi^{(0)}\rangle + |\Phi^{(1)}\rangle \} \\ &+ \ln\left(\frac{\kappa}{s_0}\right) \left[\hat{\mathcal{K}}^{(1)} |\Phi^{(0)}\rangle + (\hat{\mathcal{K}}^{(0)} - \omega^{(1)}) |\Phi^{(1)}\rangle \right] \\ &+ \sum_{n=0}^{\infty} \frac{\ln^{n+2}(\kappa/s_0)}{(n+2)!} (\hat{\mathcal{K}}^{(0)} - \omega^{(1)})^{n+1} \\ &\times \left[\hat{\mathcal{K}}^{(1)} |\Phi^{(0)}\rangle + (\hat{\mathcal{K}}^{(0)} - \omega^{(1)}) |\Phi^{(1)}\rangle \right] \}. \end{aligned} \quad (2.7)$$

Analogously, the R.H.S. of Eq. (2.3) takes the form

$$\left(\frac{\kappa}{s_0}\right)^{\omega^{(1)}} \langle \vec{q}_1 | \left\{ |R^{(0)}\rangle \left[\Gamma^{(0)} \left(1 + \ln\left(\frac{\kappa}{s_0}\right) \omega^{(2)} \right) + \Gamma^{(1)} \right] + |R^{(1)}\rangle \Gamma^{(0)} \right\}, \quad (2.8)$$

and therefore the compatibility relations are

$$\begin{aligned} |\Phi^{(0)}\rangle + |\Phi^{(1)}\rangle &= |R^{(0)}\rangle (\Gamma^{(0)} + \Gamma^{(1)}) + |R^{(1)}\rangle \Gamma^{(0)}, \\ (\hat{\mathcal{K}}^{(1)} - \omega^{(2)}) |\Phi^{(0)}\rangle + (\hat{\mathcal{K}}^{(0)} - \omega^{(1)}) |\Phi^{(1)}\rangle &= 0, \\ (\hat{\mathcal{K}}^{(0)} - \omega^{(1)}) \left[\hat{\mathcal{K}}^{(1)} |\Phi^{(0)}\rangle + (\hat{\mathcal{K}}^{(0)} - \omega^{(1)}) |\Phi^{(1)}\rangle \right] &= 0. \end{aligned} \quad (2.9)$$

We notice now that the third of these relations is not independent: it follows directly from the second one, taking into account also Eq. (2.5).

the compatibility relations in NLA can be written in the following form

$$\left(\hat{\mathcal{K}} - \omega(t)\right) |R\rangle = 0, \quad |\Phi_{A'A}^a\rangle = \Gamma_{A'A}^a |R\rangle, \quad (2.10)$$

where $\langle \vec{q}_1 | R \rangle = R(\vec{q}_1, \vec{q}, s_0)$ is the universal function defined by the relations (2.2) and (2.4). Finally, there is one more condition, which is nothing but the generalization to all orders of the second bootstrap condition (1.8) for the particle-particle scattering amplitudes. Indeed, the amplitude (1.1) for the case of antisymmetric colour octet state in the t -channel can be represented as the convolution of the impact factor for the particle B and the particle-Reggeon scattering amplitude (2.1). Using for the latter amplitude the Regge form (2.2) and then Eqs. (2.10), the comparison with the Regge form (1.6) leads to the relation

$$\frac{\vec{q}^2}{(2\pi)^{D-2}} \int \frac{d^{D-2} q_1}{\vec{q}_1^2 \vec{q}'_1{}^2} \langle \vec{q}_1 | R \rangle^2 = \sin(\pi\omega(t)) . \quad (2.11)$$

In the next section we will see that in the NLA, with account of the second from the relations (2.10), this equation coincides with the second bootstrap condition (1.8).

The set of strong bootstrap conditions, defined by Eqs. (2.10) and (2.11) is completely equivalent to the one formulated in Refs. [5, 6], but here it is rewritten in a form which fixes explicitly the process dependence of the impact factors: they turn out to be proportional to the corresponding effective vertices with the same universal coefficient function R . This function is used also in the formulation of the strong bootstrap condition for the kernel. To be precise, the Eqs. (2.10) and (2.11) can be obtained combining the Eq. (23) of Ref. [5] and Eqs. (7) and (8) of Ref. [6]. It was also stressed in [5, 6] that the necessity to assume the Reggeization of unphysical particle-Reggeon scattering amplitudes in order to formulate the strong bootstrap conditions can be avoided if one puts aside perturbation theory. In our approach this can be done as well, again without involving the completeness of eigenstates of the kernel. Indeed, comparing the s -channel

notations)

$$\langle \phi_B | \left(\frac{s}{s_0} \right)^{\hat{\mathcal{K}} - \omega} | \phi_A \rangle = 1 , \quad (2.12)$$

where

$$| \phi_A \rangle = | \Phi_A \rangle \sqrt{\frac{-t}{(2\pi)^{D-2} \sin(\pi\omega) \Gamma_A}} . \quad (2.13)$$

An expansion in $\ln(s/s_0)$ powers gives

$$1 = \langle \phi_B | \phi_A \rangle + \sum_{n=1}^{\infty} \frac{\ln^n(s/s_0)}{n!} \langle \phi_B | (\hat{\mathcal{K}} - \omega)^n | \phi_A \rangle . \quad (2.14)$$

Therefore, for arbitrary A and B (including the case $A = B$) we have

$$\langle \phi_B | \phi_A \rangle = 1 , \quad (2.15)$$

which means that the relation

$$| \phi_A \rangle = | \phi \rangle \quad (2.16)$$

is valid for an arbitrary A (since two unit vectors having unity as overlap coincide). Now, to remove the higher powers of energy logarithms from Eq. (2.14), we need only to put

$$(\hat{\mathcal{K}} - \omega) | \phi \rangle = 0 , \quad (2.17)$$

which gives, together with the previous equality, exactly the strong bootstrap. Of course, we cannot claim for a non-perturbative consideration, at least in the BFKL approach. The accurate perturbative analysis of the NLA elastic bootstrap [4] shows that in perturbation theory the only necessary conditions are the soft ones (1.7) and (1.8).

3 Determination of the function R

To determine the function R introduced in the previous section, we use the results of Refs. [13] (see also [6]) and [17] for the quark NLA

respondingly. For simplicity, we restrict ourselves to the case of completely massless QCD. Then the quark impact factor takes the form

$$\begin{aligned}
\Phi_{Q'Q}^a(\vec{q}_1, \vec{q}, s_0) &= \Phi_{Q'Q}^{a(0)} \left(1 + \frac{\omega^{(1)}(t)}{2} \left[\tilde{K}_1 + \left(\left(\frac{\vec{q}_1^2}{\vec{q}^2} \right)^\epsilon + \left(\frac{\vec{q}_1'^2}{\vec{q}^2} \right)^\epsilon \right) \right. \right. \\
&\quad \times \left. \left. \left\{ \frac{1}{2\epsilon} + \psi(1+2\epsilon) - \psi(1+\epsilon) \right. \right. \right. \\
&\quad \left. \left. \left. + \frac{11+7\epsilon}{2(1+2\epsilon)(3+2\epsilon)} - \frac{n_f}{N} \frac{(1+\epsilon)}{(1+2\epsilon)(3+2\epsilon)} \right\} \right] \right), \\
&\quad + \ln \left(\frac{s_0}{\vec{q}^2} \right) + 2\psi(1) - 2\psi(1+2\epsilon) - \frac{3}{2(1+2\epsilon)} - \frac{1}{N^2} \left(\frac{1}{\epsilon} - \frac{(3-2\epsilon)}{2(1+2\epsilon)} \right) \Bigg], \tag{3.1}
\end{aligned}$$

where n_f is the number of quark flavours and the function \tilde{K}_1 has the following integral representation:

$$\tilde{K}_1 = \frac{(4\pi)^{2+\epsilon} \Gamma(1+2\epsilon) \epsilon (\vec{q}^2)^{-\epsilon}}{4\Gamma(1-\epsilon) \Gamma^2(1+\epsilon)} \int \frac{d^{D-2}k}{(2\pi)^{D-1}} \ln \left(\frac{\vec{q}^2}{\vec{k}^2} \right) \frac{\vec{q}^2}{(\vec{k} - \vec{q}_1)^2 (\vec{k} - \vec{q}_1')^2}. \tag{3.2}$$

The result of the integration of Eq. (3.2), in form of expansion in ϵ , can be found in Ref. [13]. The corresponding effective vertex can be presented as follows:

$$\begin{aligned}
\Gamma_{Q'Q}^a(s_0) &= \Gamma_{Q'Q}^{a(0)} \left(1 + \frac{\omega^{(1)}(t)}{2} \left[\ln \left(\frac{s_0}{\vec{q}^2} \right) + \frac{1}{\epsilon} + \psi(1-\epsilon) + \psi(1) - 2\psi(1+\epsilon) \right. \right. \\
&\quad \left. \left. + \frac{2+\epsilon}{2(1+2\epsilon)(3+2\epsilon)} - \frac{1}{N^2} \left(\frac{1}{\epsilon} - \frac{(3-2\epsilon)}{2(1+2\epsilon)} \right) - \frac{n_f}{N} \frac{(1+\epsilon)}{(1+2\epsilon)(3+2\epsilon)} \right] \right). \tag{3.3}
\end{aligned}$$

Using also the well known expressions for $\Gamma^{(0)}$ and $\Phi^{(0)}$ (see, for example, [7]) we obtain from (2.10)

$$\langle \vec{q}_1 | R \rangle \equiv R(\vec{q}_1, \vec{q}) = R^{(0)} \left(1 + \frac{\omega^{(1)}(t)}{2} \left[\tilde{K}_1 + \left(\left(\frac{\vec{q}_1^2}{\vec{q}^2} \right)^\epsilon + \left(\frac{\vec{q}_1'^2}{\vec{q}^2} \right)^\epsilon - 1 \right) \right] \right)$$

$$\times \left\{ \frac{1}{2\epsilon} + \psi(1+2\epsilon) - \psi(1+\epsilon) + \frac{1}{2(1+2\epsilon)(3+2\epsilon)} - \frac{1}{N(1+2\epsilon)(3+2\epsilon)} \right. \\ \left. - \frac{1}{2\epsilon} + \psi(1) + \psi(1+\epsilon) - \psi(1-\epsilon) - \psi(1+2\epsilon) \right\}, \quad R^{(0)} = \frac{-ig\sqrt{N}}{2}. \quad (3.4)$$

Let us now discuss the condition (2.11). Using the expression (3.4) for $R^{(0)}$ and the well known integral representation for $\omega^{(1)}$ (see, for example, [7]), it can be rewritten in the NLA as follows (in simplified notations):

$$2\omega^{(1)} + \omega^{(2)} = 4R^{(0)} \int \frac{d^{D-2}q_1}{(2\pi)^{D-1}} \frac{\vec{q}^2}{\vec{q}_1^2 \vec{q}'^2} R. \quad (3.5)$$

Then, multiplying both parts with the vertex Γ , one gets

$$2\omega^{(1)}\Gamma^{(0)} + 2\omega^{(1)}\Gamma^{(1)} + \omega^{(2)}\Gamma^{(0)} = 4 \left(R^{(0)} \right)^2 \Gamma^{(0)} \int \frac{d^{D-2}q_1}{(2\pi)^{D-1}} \frac{\vec{q}^2}{\vec{q}_1^2 \vec{q}'^2} \\ + 4R^{(0)} \int \frac{d^{D-2}q_1}{(2\pi)^{D-1}} \frac{\vec{q}^2}{\vec{q}_1^2 \vec{q}'^2} \Phi^{(1)}. \quad (3.6)$$

The LLA part of the above relation is evidently satisfied and the NLA one gives exactly the second bootstrap condition (1.8). It means that this condition is fulfilled automatically for any process if the conditions (2.10) and (2.11) are satisfied, that justifying the term “strong bootstrap conditions”. Let us finally note that it is not necessary to check the fulfillment of the condition (2.11) for the function (3.4); in fact, it was already done when the bootstrap condition (1.8) for the quark impact factor was checked in Ref. [13]. Therefore, the above discussion completes our formulation of the strong bootstrap conditions. They can be presented by the relations (2.10) and (3.4). The relation (2.11) is not necessary to include anymore, because the R -function (3.4) does have the correct normalization.

We have now the chance to check the second of the strong bootstrap conditions defined by Eqs. (2.10) and (3.4) for the case of gluon scattering, because the corresponding impact factor and effective vertex are already known in the NLA and can be found in Refs. [12] (see

$$\begin{aligned}
& \frac{\Phi_{G'G}^a(\vec{q}_1, \vec{q}, s_0)}{gT_{G'G}^a R^{(0)}} = \delta_+ \left\{ 1 + \frac{\omega^{(1)}(t)}{2} \left[\tilde{K}_1 + \left(\left(\frac{\vec{q}_1^2}{\vec{q}^2} \right)^\epsilon + \left(\frac{\vec{q}_1'^2}{\vec{q}^2} \right)^\epsilon - 1 \right) \right. \right. \\
& \times \left(\frac{1}{2\epsilon} + \psi(1+2\epsilon) - \psi(1+\epsilon) + \frac{11+7\epsilon}{2(1+2\epsilon)(3+2\epsilon)} - \frac{n_f}{N} \frac{(1+\epsilon)}{(1+2\epsilon)(3+2\epsilon)} \right) \\
& \quad + \ln \left(\frac{s_0}{\vec{q}^2} \right) + \frac{3}{2\epsilon} + 2\psi(1) - \psi(1+\epsilon) - \psi(1+2\epsilon) \\
& \quad \left. \left. - \frac{9(1+\epsilon)^2 + 2}{2(1+\epsilon)(1+2\epsilon)(3+2\epsilon)} + \frac{n_f}{N} \frac{(1+\epsilon)^3 + \epsilon^2}{(1+\epsilon)^2(1+2\epsilon)(3+2\epsilon)} \right] \right\} \\
& \quad - \frac{\delta_- \epsilon \omega^{(1)}(t)}{2(1+\epsilon)^2(1+2\epsilon)(3+2\epsilon)} \left(1 + \epsilon - \frac{n_f}{N} \right). \quad (3.7)
\end{aligned}$$

Here the notations

$$\delta_+ = - \left(e_{\perp}^{\prime *} e_{\perp} \right), \quad \delta_- = -e_{\perp \mu}^{\prime *} e_{\perp \nu} \left(g_{\perp \perp}^{\mu\nu} - (D-2) \frac{q_{\perp}^{\mu} q_{\perp}^{\nu}}{q_{\perp}^2} \right) \quad (3.8)$$

were used, with e and e' the polarizations of the incoming and scattered gluons, respectively. Eq. (3.7), together with the Eq. (3.4), gives the expression

$$\begin{aligned}
& \frac{\Phi_{G'G}^a(\vec{q}_1, \vec{q}, s_0)}{gT_{G'G}^a R(\vec{q}_1, \vec{q})} = \delta_+ \left\{ 1 + \frac{\omega^{(1)}(t)}{2} \left[\ln \left(\frac{s_0}{\vec{q}^2} \right) + \frac{2}{\epsilon} + \psi(1) + \psi(1-\epsilon) - 2\psi(1+\epsilon) \right. \right. \\
& \quad \left. \left. - \frac{9(1+\epsilon)^2 + 2}{2(1+\epsilon)(1+2\epsilon)(3+2\epsilon)} + \frac{n_f}{N} \frac{(1+\epsilon)^3 + \epsilon^2}{(1+\epsilon)^2(1+2\epsilon)(3+2\epsilon)} \right] \right\} \\
& \quad - \frac{\delta_- \epsilon \omega^{(1)}(t)}{2(1+\epsilon)^2(1+2\epsilon)(3+2\epsilon)} \left(1 + \epsilon - \frac{n_f}{N} \right) \quad (3.9)
\end{aligned}$$

which must coincide with $\Gamma_{G'G}^a(s_0)/(gT_{G'G}^a)$ if the strong bootstrap for impact factors is fulfilled. Simple inspection of the result [18] for the vertex confirms its fulfillment. As for the first strong bootstrap condition (the first of Eqs. (2.10)), its fulfillment in the part concerning the quark contribution follows from the results of Refs. [5, 19]. Of course, nothing can be stated as a fact without a check of the first strong bootstrap condition for the gluon part.

In this paper we have considered the strong bootstrap conditions, introduced in Refs. [5] and [6], which appear as consequences of the hypothesis that the unphysical particle-Reggeon scattering amplitudes have also Regge form. Our main aims were: first, to show that strong bootstrap conditions do not need the assumption of completeness of eigenstates of the octet BFKL kernel; second, to rewrite them in a form which fixes explicitly their dependence on process under consideration. The strong bootstrap conditions rewritten in such form (see Eqs. (2.10) and (2.11)) are completely equivalent to the analogous ones of Refs. [5, 6], as it was explained in the Section 3. We have then determined the universal function R using the results for the quark NLA octet impact factor [13] and for the corresponding effective interaction vertex [17] and have shown that the complete set of strong bootstrap conditions can be presented by Eqs. (2.10) and (3.4) (Eq. (2.11) is not necessary to include, having the function R given by Eq. (3.4) the proper normalization). Finally, we have checked the strong bootstrap condition for the gluon NLA impact factor and found that it is fulfilled.

The strong bootstrap conditions are satisfied in the LLA [4] and are likely to be satisfied also in the NLA, as it may be concluded from the previous section and from the results of Refs. [5, 19]. This fact is rather intriguing, because the strong bootstrap is evidently not necessary for the NLA Reggeization of the usual particle scattering amplitudes, being the soft one (expressed by Eqs. (1.7) and (1.8)), together with the LLA strong bootstrap, sufficient for it [4]. The possibility that the strong bootstrap conditions are necessary for the self-consistency of NLA Reggeization in inelastic sector was mentioned in Ref. [6], but, in our opinion, this does not look to be the case. The assumption of the NLA Reggeization for the unphysical particle-Reggeon scattering amplitude itself cannot be considered a good explanation, since it is not yet clear why this requirement should be necessary. In any case, there is interesting physics under the strong bootstrap (if satisfied) which deserves further clarification.

more important than strong ones. However, since the soft ones are automatically verified if the strong ones are fulfilled, also the strong bootstrap can be practically used, especially because it involves one integration less and is therefore easier to check. It is also clear that the set of checks of the bootstrap conditions for all the possible processes can be replaced by the search of general arguments for the NLA Reggeization of the particle-Reggeon scattering amplitudes.

Acknowledgment: Two of us (V.S.F. and M.I.K.) thank the Dipartimento di Fisica della Università della Calabria for the warm hospitality while this work was done. V.S.F. acknowledges the financial support of the Istituto Nazionale di Fisica Nucleare.

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Strong bootstrap conditions

Budker INP 2000-64

Ответственный за выпуск А.М. Кудрявцев

Работа поступила 10.08.2000 г.

Сдано в набор 5.10.2000 г.

Подписано в печать 5.10.2000 г.

Формат бумаги 60×90 1/16 Объем 1.0 печ.л., 0.8 уч.-изд.л.

Тираж 100 экз. Бесплатно. Заказ № 64

Обработано на IBM PC и отпечатано на
ротапринте ИЯФ им. Г.И. Будкера СО РАН

Новосибирск, 630090, пр. академика Лаврентьева, 11.