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STRUCTURE OF THE QUANTUM STATES OF HYDROGEN ATOM IN MAGNETIC FIELD NEAR THE IONIZATION LIMIT

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НОВОСИБИРСК

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ABSTRACT

Continues and discrete spectrum states near the ionization limit are considered. The origin of the continuous spectrum narrow resonances is elucidated. It is shown that the wave functions structure is intermediate between the regular structure and chaotic one.

In the work [1] at the photoionization of the Lithium atom in the magnetic field the continues spectrum narrow resonances were observed. The energy of the resonances was comparable with the cyclotron frequency ω. The magnetic field was 6.1T and corresponding ω= 5.7cm^{-1}. The excitation was carried out from the 3s-state by the laser beam linearly polarized along the magnetic field. Therefore the projection of the orbital angular momentum on the magnetic field L_z=0. In the present work we will use the atomic units. In these units the magnetic length is equal to $a = 1/\sqrt{\omega} \approx 196$. The Coulomb turning point is $r_Q \sim 1/E \sim 1/\omega \sim 196^2$. It is obvious that at such a size the difference of the Lithium atom from thé Hydrogen atom is negligible and therefore we will discuss the Hydrogen atom.

At L_z=0 the Hamiltonian in the cylindrical coordinats is of the form

$$H= \frac{1}{2} (p_z^2 + p_\rho^2) -(\rho^2+z^2)^{-1/2} + \frac{1}{8} \omega^2 \rho^2. \qquad (1)$$

The ionization limit corresponds to the energy of the free electron at the lowest Landau level $\varepsilon_{IP} = \frac{1}{2}\omega$. Since $r_Q \gg a$ the states we are interested in are strongly streched along the magnetic field. Therefore we can expand the Hamiltonian (1) at small ρ^2/z^2.

$$H \approx \frac{1}{2}(p_z^2 + p_\rho^2) - 1/z + \frac{1}{8}(\omega^2 + 4/z^3)\rho^2. \qquad (2)$$

At $z \gg \omega^{-2/3} = a^{4/3}$ the eigenstates of the Hamiltonian (2) are of the form

$$\psi_{n,\varepsilon}(z,\rho) = A \frac{1}{\sqrt{p_z}} \exp\left(\pm i\int_0^z p_z dz\right) \phi_n(\rho), \qquad (3)$$

where

$$\phi_n(\rho) = \frac{1}{a} e^{-\rho^2/4a^2} F(-n,\ l,\ \rho^2)/2a^2) \qquad (4)$$

is the wave function of the electron transverse motion in the magnetic field [2]. (F is the confluent hypergeometric function), p_z is the longitudinal momentum: $p_z = \sqrt{2(1/z - \varepsilon_n + \varepsilon)}$, $\varepsilon_n = \omega(n+1/2)$.

The first impression is that the explanation of the continuous spectrum resonances is very simple: These are the one dimensional Coulomb levels which are built on the states (3): $\varepsilon = \varepsilon_n - \delta\varepsilon$. The value of $\delta\varepsilon$ as well as the normalization constant A in the state (3) can be found using the Bohr quantisation rule.

$$\delta\varepsilon = 1/2n_Q^2\ ,\ A = 1/\sqrt{\pi n_Q^3}. \qquad (5)$$

Let us stress that the WKB phase as well as the normalization of the state is determined by the region $z \gg a^{4/3}$ where the approximation (3) is applicable. The further scenario

could look as follows. For example Coulomb state built on the Landau level with n=1 (but with the total energy $\varepsilon > \varepsilon_{IP}$) acquires the width due to the small mixing with the n=0. The problem is that the mixing of the asymptotical states (3) which happens at $z \leq a^{4/3}$ is not small. The stationary state is a rather complicated combination of the basis states (3). Narrowness of the resonances is due to the special structure of the mixing matrix.

In the region $a^{4/3} \ll z \ll r_Q \sim n_Q^2$ the problem of the states (3) mixing can be reduced to the scattering problem. Let from the right side (z>0) the wave incident on the origin.

$$|n\rangle_{in} = \frac{1}{\sqrt{p_z}} \exp\left(-i\int_0^z p_z dz\right) \phi_n(\rho). \qquad (6)$$

It is reflected and we should calculate the scattering matrix to the states

$$|m\rangle_{out} = \frac{1}{\sqrt{p_z}} \exp\left(i\int_0^z p_z dz\right) \phi_m(\rho). \qquad (7)$$

The S-matrix in the quasi-classical approximation is calculated in the Appendix (A22). We denote it by $S^{(0)}$ to stress that this is the scattering at the Coulomb center. It is shown in Appendix that the transmitted wave is small. Just therefore it is ommited in the basis of the |out> states. Due to this reason all the stationary states of the opposite parity (reflection $z \to -z$) are degenerate (cf. with Ref. [3]).

Let us consider first of all the states of the discrete spectrum: The energy level lies below the ionization limit.

The matrix $S^{(0)}$ defines the boundary condition at small distance. To solve the eigenvalue problem one should add the boundary condition at the large distance. It is convenient to introduce this condition using the formal scattering matrix $S^{(\infty)}$ from the state (7) to the state (6) on to the Coulomb turning point. Due to the standard quasi-classical formulae

$$S^{(\infty)}_{nm}(\varepsilon)= \delta_{nm}\exp\left(2i\int_0^{z_n} p_z(\varepsilon,\varepsilon_n)dz-i\pi/2\right)=\delta_{nm}\exp\left(\frac{i\pi\sqrt{2}}{\sqrt{\varepsilon_n-\varepsilon}}-i\pi/2\right) \quad (8)$$

Here $z_n=1/(\varepsilon_n-\varepsilon)$ is the Coulomb turning point. One can represent the stationary state wave function in the form $\Psi=\psi+\bar{\psi}$, where the ψ is combination of the states (6), and $\bar{\psi}= S^{(0)}\psi$ is combination of the states (7). Then the equation for the stationary states looks as follows

$$S^{(0)}S^{(\infty)}(\varepsilon)\psi = \psi. \quad (9)$$

Equation for the energy levels is

$$|S^{(0)}S^{(\infty)}(\varepsilon) - 1|=0. \quad (10)$$

According to Eq.(A22) $S^{(0)}$ strongly mixes different Landau levels, and besides that $S^{(\infty)}$ has rather complicated dependence on ε (8). Therefore it is very natural to suppose that the solution of the Eqs.(9),(10) is of the form

$$\psi = \sum_{n=0}^{N_{max}} \alpha_n|n>_{in}, \quad (11)$$

where all the α_n are of the same order of magnitude $|\alpha_n|\sim\alpha$. What is N_{max}? To formulate the scattering problem we need the condition $a^{4/3} \ll r_Q\sim n_Q^2$ for Coulomb turning point r_Q be

fulfilled (see above). If the total energy $\varepsilon\sim\omega$ the turning point for the state (3), (4) built on the n-th Landau level is equal $r_Q\sim(\omega n)^{-1}$. Thus the condition we need is violated at $n>a^{2/3}$. There is other argument*). At $n>a^{2/3}$ the splitting between the Coulomb levels built on the n-th Landau level is larger then the cyclotron frequency: $1/n_Q^3>\omega$ (remind that $\varepsilon\sim 0$), and due to the usual perturbation theory arguments this is the case of regular dynamics. Thus the upper limit in (11) is $N_{max}\sim a^{2/3}$. The solution (11) is written down in terms of unnormalized states (6). In the basis of normalized states (3) it looks as follows

$$\psi = \frac{1}{\sqrt{C}}\sum_{n=0}^{N_{max}} \beta_n\psi_{n,\varepsilon},$$

where

$$\beta_n=\alpha_n[\omega(n+1/2) -\varepsilon]^{-3/4}, \qquad C= \sum_0^{N_{max}} |\beta_n|^2. \quad (12)$$

We remind the reader that $\varepsilon< \frac{1}{2}\omega$. Thus the contribution of the higher Landau levels in (12) is suppressed. At $\frac{1}{2}\omega-\varepsilon \ll \omega$ the state (12) is saturated by the zero Landau level, and we have the case of regular dynamics. The spectrum of these states is Coulomb one: $\frac{1}{2}\omega-\varepsilon =1/2n_Q^2$, $n_Q \gg a$. As far as we understand such the levels were observed in the Ref. [4].

At $|\varepsilon| \gg \omega$ many Landau levels ($\sim \varepsilon/\omega$) are mixed in the state and we can say that this is chaotic state. However it is known that there are the regions of regular dynamics even

*) This explanation was suggested by D.L. Shepelyansky.

at $|\varepsilon| \gg \omega$ [3].

At $\frac{1}{2}\omega-\varepsilon \sim \omega$ we have intermediate case. For example the contributions of the different Landau levels into normalization drop as $n^{-3/2}$. So the main contribution comes from the lowest levels. The opposite situation is with the value $<\rho^2>$ which was measured in the work [1]. Due to the Eq.(12) it equals.

$$<\psi|\rho^2|\psi>=\frac{1}{C}\left\{\sum_{n=0}^{N_{max}}|\beta_n|^2<\phi_n|\rho^2|\phi_n>+\sum_{\substack{n\\ m=n\pm2}}^{N_{max}}\beta_n^*\beta_m<\phi_n|\rho^2|\phi_m>\right\} \qquad (13)$$

To estimate the mean value of $<\rho^2>$ we should omit the interference term. If $\delta\varepsilon = \frac{1}{2}\omega-\varepsilon$ is not very small we get from (13)

$$<<\psi|\rho^2|\psi>>\sim 2a^2\sum_{n=0}(n+1/2)(n+1)^{-3/2}\sim 4a^2\sqrt{N_{max}} \sim 4a^2a^{1/3}. \qquad (14)$$

Double brakets mean the double averaging: over the quantum state and over the states. The value (14) reasonably agrees with the data from Ref. [1]. If $\delta\varepsilon = \frac{1}{2}\omega-\varepsilon \ll \omega N_{max}^{-1/3}\sim\omega a^{-2/9}$ the sum in the Eq.(13) is saturated by the term with $n=0$, and therefore $\sqrt{<\rho^2>} = \sqrt{2}a$.

Now we consider the states of the continues spectrum. For example $3/2\omega >\varepsilon >1/2\omega$. Let us switch off by hands the coupling with the continues spectrum. Say we change $S_{00}^{(\infty)}$ from Eq.(8) to any $e^{i\phi}$. Then the state becomes the stationary one and the wave function has the same form (12). However the physical sense have only the components with

$n\geq1$. Similar to the discrete spectrum case at $\delta\varepsilon = \frac{3}{2}\omega-\varepsilon \ll \omega$ the state is saturated by the first Landau level (n=1), and the spectrum is Coulomb like: $\delta\varepsilon=1/2n_Q^2$, $n_Q\gg a$. At $\delta\varepsilon \sim \omega$ there are many Landau levels mixed in the state with the averaged weight $\sim n^{-3/2}$.

Let us switch on the coupling with the continues spectrum and calculate the widths of the levels. The most simple to do it for the levels with $\delta\varepsilon \ll \omega$.

$$\Gamma= \nu|S_{10}^{(0)}|^2= \frac{\Omega}{2\pi}\,0.188 \qquad (15)$$

Here $\nu=1/T=\Omega/2\pi$ is the frequency of classical motion; $\Omega=\Delta\varepsilon= =1/n_Q^3$ is the splitting between the resonances. The value $|S_{10}|^2$=0.188 is calculated using (A22). Thus

$$\Gamma/\Delta\varepsilon = \frac{0.188}{2\pi} \qquad (16)$$

It is evident that at $\delta\varepsilon \sim \omega$ the ratio $\Gamma/\Delta\varepsilon$ is of the same order of magnitude.

Let now $(n_0+1/2)\omega >\varepsilon >(n_0-1/2)\omega$. Similar to previous consideration we switch off for a moment the coupling with the continues spectrum. (Change of $S_{nn}^{(\infty)}$ from Eq.(8) to any $e^{i\phi n}$ at $n< n_0$). The wave function of the obtained in such a way stationary state has decomposition (12). Physical sense have only the components with $n \geq n_0$. At $\delta\varepsilon=(n_0+1/2)\omega-\varepsilon\ll \omega$ the wave function (12) is saturated by the level $|n_0>$ and the spectrum is Coulomb like. The width arises due to the coupling with the channels with $n\leq n_0$.

$$\Gamma = \nu \sum_{m<n_0} |S^{(0)}_{mn_0}|^2 \approx \frac{\Omega}{2\pi}\, 0.16. \qquad (17)$$

We take into account that due to the (A22) $\sum_{m<n_0} |S^{(0)}_{mn_0}|^2 \approx 0.16$ at $n_0 \gg 1$. Thus $\Gamma/\Delta\varepsilon \approx 0.16/2\pi$. At $\delta\varepsilon \sim \omega$ our arguments are not applicable, but it is obvious that the magnitude of the ratio should be the same. We would like to stress that narrowness of the resonances is due to the peculiarity of $S^{(0)}$-matrix: The probability of downwards transition is relatively small.

One can estimate the numerical value of the width for the conditions of the experiment [1]. If $\delta\varepsilon \sim 1/2n_Q^2 \sim \frac{1}{2}\,\omega$ then $n_Q \sim a$ and

$$\Gamma \sim \frac{0.16}{2\pi}\,\Delta\varepsilon \sim \frac{0.16}{2\pi}\,\omega/n_Q \sim 10^{-3}\,\mathrm{cm}^{-1}. \qquad (18)$$

This reasonably agree with the experimental values [1].

In conclusion we would like to remark that the results of the present work can be easily generalized to the case $L_z \neq 0$.

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APPENDIX

Here we calculate in quasi-classical approximation the scattering matrix between the states (6),(7). In this approximation the wave function is of the form.

$$\psi(\rho,z) = B(\rho,z)\, e^{i\sigma(\rho,z)}, \qquad (A1)$$

where σ is classical action. At $t \to -\infty$ the ψ coincides with the state (6) and

$$\sigma(\rho,z) = -\int_0^z p_z dz + \int_0^\rho p_\rho d\rho$$

$$p_z = \sqrt{2(1/z - \varepsilon_n + \varepsilon)}, \qquad p_\rho = \sqrt{2(\varepsilon_n - \frac{1}{8}\,\omega^2\rho^2)} \qquad (A2)$$

$$\varepsilon_n = \omega(n+1/2), \quad B(\rho,\ z) = \frac{1}{\sqrt{p_z}}\,\frac{C}{\sqrt{p_\rho}}.$$

Here $C = \sqrt{\omega/\pi}$ is the normalization constant for the transversal motion. To calculate the variation of the action at motion we find the classical trajectories of the particle in the field (1), (2). In the region of validity of expansion (2) the equations of motion look as follows.

$$\ddot{z} = -1/z^2 + \frac{3}{2}\,\rho^2/z^4 \qquad (A3a)$$

$$\ddot{\rho} = -\frac{1}{4}\,(\omega^2 + 4/z^3)\rho. \qquad (A3b)$$

Neglecting the terms of the order of ρ^2/z^2 as well as the energy of the longitudinal motion we find the solution of Eq (A3a) in zero approximation

$$z_0(t)=(9/2)^{1/3}|t|^{2/3} \tag{A4}$$

After substitution of z_0 to the Eq.(A3b) we find $\rho(t)$

$$\rho(t)= \sqrt{|t|}\left[bJ_{1/6}(\omega|t|/2) + dJ_{-1/6}(\omega|t|/2)\right], \tag{A5}$$

where J_ν is the Bessel function. At $z \gg a^{4/3}$ $(\omega|t|\gg 1)$

$$\rho(t) \approx \frac{2}{\sqrt{\pi\omega}}\left[b\cos(\omega|t|/2-\pi/3) + d\cos(\omega|t|/2-\pi/6)\right]. \tag{A6}$$

It is evident from this equation that $b{\sim}d{\sim}\sqrt{n}$. At $z \ll a^{4/3}$ $(\omega|t|\ll 1)$

$$\rho \approx d\,\frac{(2/9)^{1/6}}{\Gamma(5/6)}(\omega/4)^{-1/6}\sqrt{z} + b\,\frac{(2/9)^{1/3}}{\Gamma(7/6)}(\omega/4)^{1/6}z. \tag{A7}$$

This expansion is valid at $na^{2/3} \ll z \ll a^{4/3}$. At $z \lesssim na^{2/3}$ the condition $\rho^2 \ll z^2$ is violated. To go through the region $z \lesssim na^{2/3}$ one should observe that at $z \ll a^{4/3}$ magnetic field in the Hamiltonian (1) can be neglected, and therefore the problem is reduced to the pure Coulomb one. Actually the Eq. (A7) is the equation of parabola slightly rotated with respect to z-axis. This exactly corresponds to the motion with zero energy in the Coulomb field. After the flying around the nucleus an electron moves at other branch of parabola. This corresponds to the reflection $d \to -d$. Thus after the scattering on the center the solution (A5) transfers into itself with substitution $d \to -d$. Just due to the compleet reflection of the trajectories we omit the transmitted wave in the basis of |out> states for S- matrix

(Eq.(7)).

Let us fix the initial time $t_0= -T$, so that $\omega T \gg 1$, $\omega T/2-\pi/6=2\pi l+\pi/2$. Let at this moment $\rho(t_0)=\rho_0$, $p(t_0)=p_0$. Using (A6) one can easy verify that at the moment $t_1=+T$ the trajectory comes to the point

$$\rho(t_1)=\rho_0, \quad p(t_1)=p_0-\sqrt{3}\omega\rho_0. \tag{A8}$$

At the scattering the transversal energy is changed

$$\varepsilon_\perp = \varepsilon_n = p_0^2/2 + \frac{1}{8}\omega^2\rho_0^2 \to \varepsilon_m = p^2/2 + \frac{1}{8}\omega^2\rho^2 =$$

$$= \varepsilon_n + \frac{3}{2}\omega^2\rho^2 \pm \sqrt{3}\,\omega\rho\sqrt{2(\varepsilon_n - \frac{1}{8}\omega^2\rho^2)} \tag{A9}$$

It is evident from this relation that the scattering strongly mixes the states. After simple calculation we find the limits in which ε_m lies

$$1/\lambda \le \varepsilon_m/\varepsilon_n \le \lambda, \qquad \lambda=7+4\sqrt{3} \approx 13.9. \tag{A9a}$$

To find the correction to the solution (A4) for z(t) we use the energy conservation low.

$$\frac{dz}{dt} = \pm\sqrt{2(1/z + \varepsilon - \varepsilon_\perp)} \approx \pm\sqrt{2/z}\,(1 + z\varepsilon/2 - z\varepsilon_\perp/2) \tag{A10a}$$

$$\varepsilon_\perp = \frac{\rho^2}{2} + \frac{1}{8}(\omega^2 + 4/z_0^3)\rho^2. \tag{A10b}$$

The transversal energy should be calculated using the Eqs. (A4), (A5). It should be noted that at $z \ll r_Q \sim a^2$ the correction to (A4) is small: $\delta z \ll z$. Nevertheless the production $p_z\delta z$ is not small and therefore the correction is essential. At $\omega t > 1$ ($z > a^{4/3}$) the $\varepsilon_\perp$ is conserved and the equation (A10a) can be easily integrated explicitly:

$$z(t)\approx z_0(t)\left(1+\frac{1}{5}(\varepsilon-\varepsilon_\perp)z_0(t)-0.030(\varepsilon-\varepsilon_\perp)^2 z_0^2(t)+\ldots\right). \qquad \text{(A11)}$$

To calculate the variation of action along the classical trajectory

$$\Delta\sigma = \int_{-T}^{+T}(p_z^2 + p_\rho^2)dt \qquad \text{(A12)}$$

we divide the trajectory into the three intervals: $[-T,-\tau]$, $[-\tau,\tau]$, $[\tau,T]$ with τ chosen in such a way that $na^{2/3} \ll z(\tau) \ll a^{4/3}$. At middle interval the magnetic field can be neglected and pure Coulomb action is

$$\Delta\sigma[-\tau,\ \tau] = 4\sqrt{2z_\tau + p} \approx 4\sqrt{2z_\tau} + p\sqrt{2/z_\tau}$$

$$z_\tau = z(\tau)\approx z(-\tau), \qquad p=(9\omega)^{-1/3}(d/\Gamma(5/6))^2. \qquad \text{(A13)}$$

At the first and second intervals let us represent the integrand in (A12) in the following form

$$(\dot z^2 + \dot\rho^2)dt = 2\left(1/z + \varepsilon - \frac{1}{8}(\omega^2 + 4/z^3)\rho^2\right)dt=$$

$$=\left(2/z + \varepsilon-\varepsilon_\perp\right)dt + \left(\varepsilon + \varepsilon_\perp - \frac{1}{4}(\omega^2 + 4/z^3)\rho^2\right)dt \approx$$

$$\approx \sqrt{2/z}\,dz + \left(\varepsilon + \dot\rho^2/2 - \frac{1}{8}(\omega^2 + 4/z_0^3)\rho^2\right)dt. \qquad \text{(A14)}$$

We have used here the relations (A10). After the partial integration of $\dot\rho^2$ with substitution of $\ddot\rho$ from Eq.(A3b) one find

$$\Delta\sigma[-T,\ -\tau] + \Delta\sigma[\tau,\ T] =$$

$$=2\sqrt{2}\left(\sqrt{z(-T)} - \sqrt{z(-\tau)} + \sqrt{z(T)} - \sqrt{z(\tau)}\ \right)+2\varepsilon T+$$

$$+\ \frac{1}{2}\left(\dot\rho\rho\big|_{-\tau} - \dot\rho\rho\big|_{-T} + \dot\rho\rho\big|_{T} - \dot\rho\rho\big|_{\tau}\right). \qquad \text{(A15)}$$

Using the explicit expressions (A6),(A7) for $\rho(t)$ we evaluate the sum of (A13) and (A15) which is the action on the trajectory

$$\Delta\sigma[-T,\ T] = 2\sqrt{2}\left(\sqrt{z(T)} + \sqrt{Z(-T)}\right) + 2\varepsilon T - \frac{\sqrt{3}}{2}\omega\rho^2. \qquad \text{(A16)}$$

Finally summing (A16) with the action before the scattering (A2) ($t=-T$) one find the action after the scattering ($t=+T$)

$$\sigma(\rho,\ z(T)) = 2\sqrt{2z(T)} + (\varepsilon+\varepsilon_n)T - \frac{\sqrt{3}}{2}\omega\rho^2 + \int_0^\rho p_\rho d\rho. \qquad \text{(A17)}$$

We take into account that due to (A8) at $t=T$ the trajectory comes to the starting ρ: $\rho(T)=\rho(-T)$. For the initial state (A2) all the trajecrories start from the different $\rho(-T)$ but from the same $z(-T)$. However they come to the different z, since due to (A9), (A11) $z(T)$ is function of the ρ. Therefore it is more correct to write $z_\rho(T)$. Thus the Eq. (A17) gives the phase of quasi-classical wave function (A1) on the line (ρ, $z(\rho)$).

Evaluation of $B(\rho,\ z)$ in (A1) is very simple. The momentum before the scattering is practically the same as that after the scattering: $p \approx p_z \gg p_\rho$. Classical trajectory comes to the same point ρ. Therefore from the current conservation equation

$$\mathrm{div}(|B|^2\vec{p}) = 0 \qquad \text{(A18)}$$

we can conclude that $|B(\rho,\ z_\rho(T)|\approx|B(\rho,\ z(-T)|$. Besides that it is very easy to verify that the trajectory (A4), (A5) touches the caustic $4l+2$ times. Therefore

$$B(\rho,\ z_\rho(T)\ \approx\ -B(\rho,\ z(-T). \tag{A19}$$

According to (A1), (A2), (A17), (A19) the wave function on the line ρ, $z_\rho(T)$ after the scattering is equal to

$$\psi(\rho,z_\rho(T))=-\frac{1}{\sqrt{p_z}}\exp\left(i\left[2\sqrt{2z_\rho(T)}+(\varepsilon+\varepsilon_n)T-\frac{\sqrt{3}}{2}\,\omega\rho^2\right]\right)\phi_n(\rho). \tag{A20}$$

$\phi_n(\rho)$ is the wave function of transversal motion. To find S-matrix we should decompose (A20) in the states (7) at $z=z_\rho(T)$. This gives

$$\langle m|S|n\rangle = -\exp[i(\varepsilon_m+\varepsilon_n)T]\langle\phi_m|\exp\left(-i\frac{\sqrt{3}}{2}\,\omega\rho^2\right)|\phi_n\rangle \tag{A21}$$

The standard calculation [2] using the representation of $\phi_n(\rho)$ in the form (4) gives

$$S_{mn} = \ (-i)^{m+n}\ \frac{1}{2}\left(\frac{\sqrt{3}}{2}\right)^{m+n}F(-m,-n,1,-1/3). \tag{A22}$$

F is the hypergeometric function.

We have calculated S_{mn} at $m,n\ \gg\ 1$. However (A22) probably is applicable with good accuracy up to $m,n\ =0$. In deriving (A22) we have supposed also that $\rho^2/p^2 \ll 1$ up to $z \ll a^{4/3}$. This condition is fulfilled at $m,n\ \ll\ a^{2/3}$.

In conclusion we would like to stress that due to Eq.(A21) the S-matrix is diagonal in the ρ-representation.

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Структура состояний непрерывного и дискретного спектра атома водорода в магнитном поле вблизи порога ионизации

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