Дискуссионный клуб ИЯФ

Измерение сечения процесса $e^+e^-\to\pi^+\pi^-\pi^0$ на Belle II и BESIII

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База отдыха «Разлив»

23.07.2025

Muon g-2 and HVP

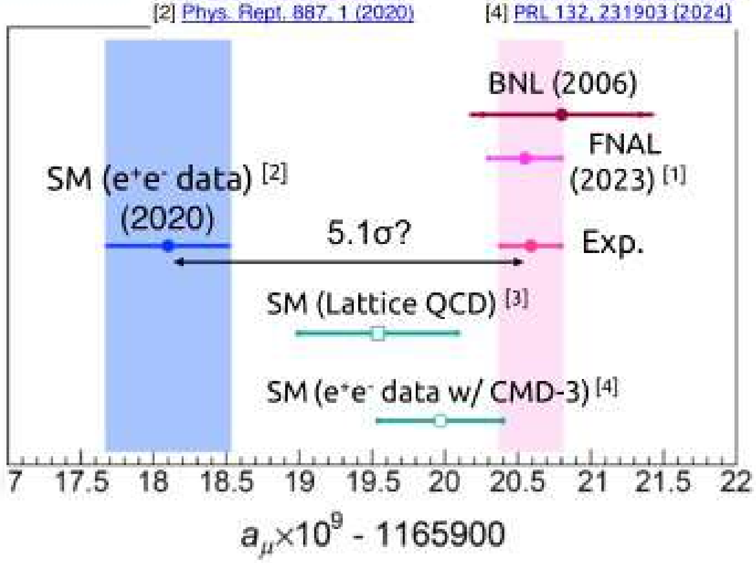
- Anomalous magnetic moment of muon in SM deviates from direct measurement by 5σ or $1\text{-}2\sigma$?

$$a_\mu^{SM} \equiv (g_\mu - 2)/2 = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{HVP} + a_\mu^{HLBL}$$

- Uncertainty is dominated (>80%) by the **leading order** (LO) **Hadronic Vacuum Polarization** (HVP)
  - Can be calculated by either **Lattice QCD** or
  - **Dispersion integral** over the bare cross section $\sigma^0(s)$ of $e^+e^- \to \text{hadrons}(\gamma)$

$$a_\mu^{HVP,LO} = \frac{\alpha^2}{3\pi^2}\int_{M_\pi^2}^{\infty}\frac{K(s)}{s}R(s)ds \qquad R(s) = \frac{\sigma^0(e^+e^- \to \text{hadrons}(\gamma))}{\sigma^0(e^+e^- \to \mu^+\mu^-(\gamma))}$$

[1] PRL 131, 161802 (2023)
[2] Phys. Rept. 887, 1 (2020)
[3] Nature 593, 7857 (2021)
[4] PRL 132, 231903 (2024)

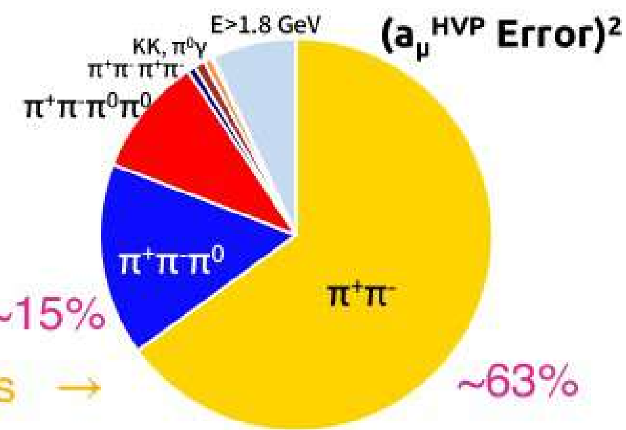


Belle II status

Measured →

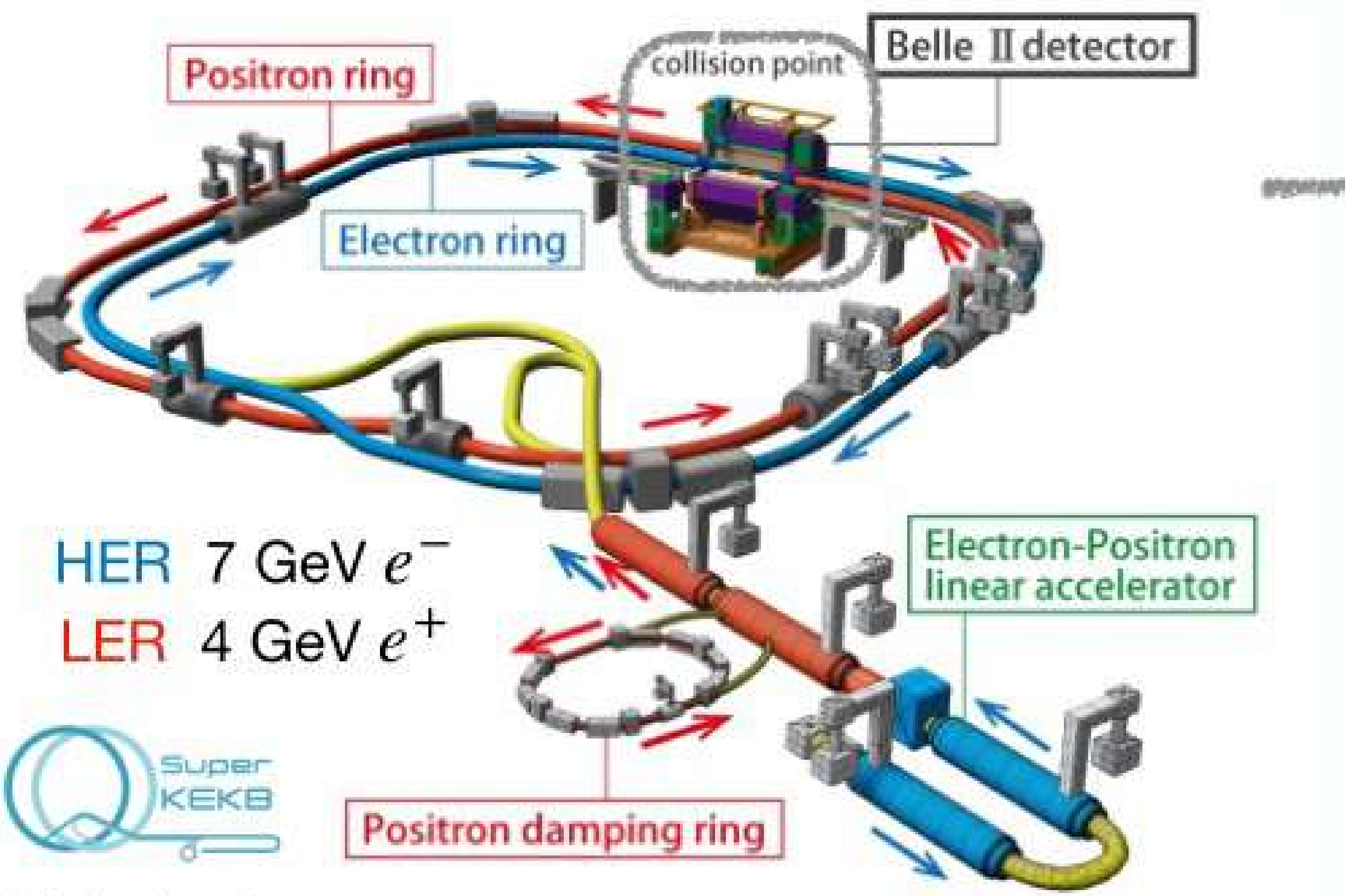
arXiv:2404.04915

Work in progress →



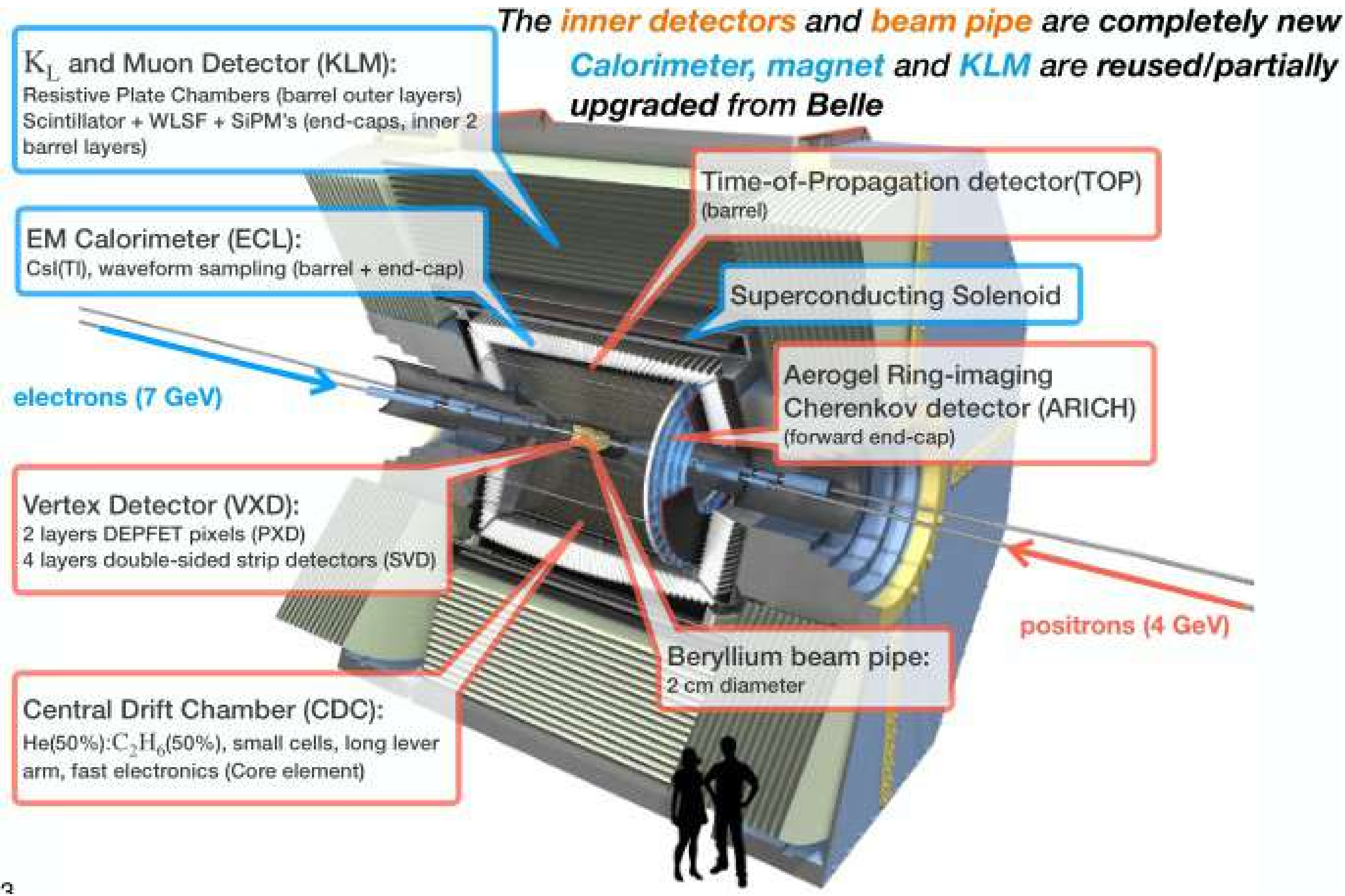
Установка: SuperKEKB

- **Asymmetric**-energy e^+e^- collider
- $E_{cm} = M_{\Upsilon(4S)} \approx 10.58$ GeV, B factory
- Goal: $L_{peak} = 6 \times 10^{35}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$
  - $4.7 \times 10^{34}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$ (**June 2022**, world record)



Tsukuba, Japan

Установка: Belle II

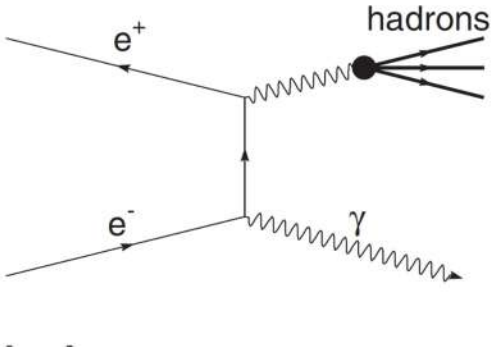


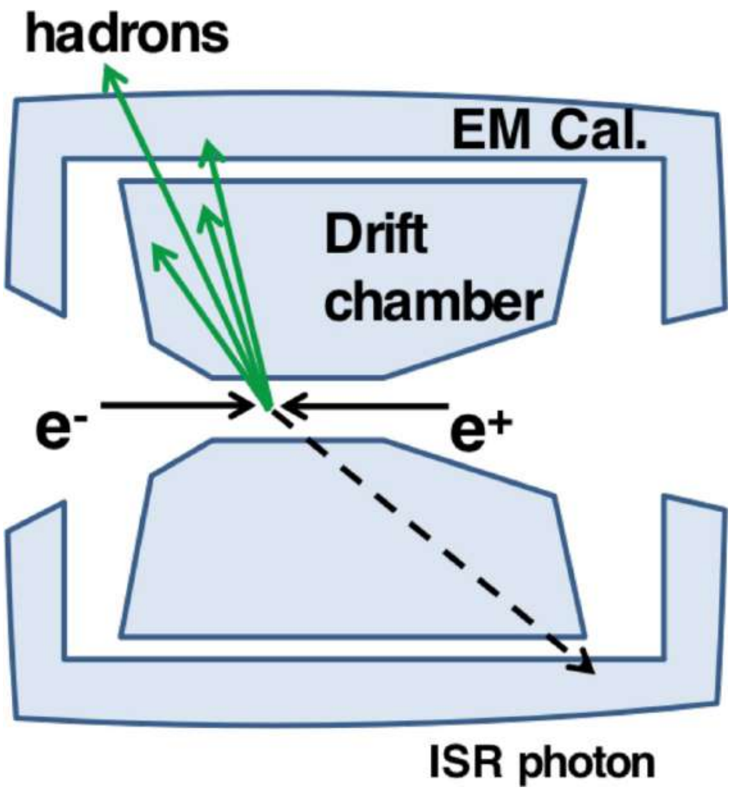
Метод ISR и триггер

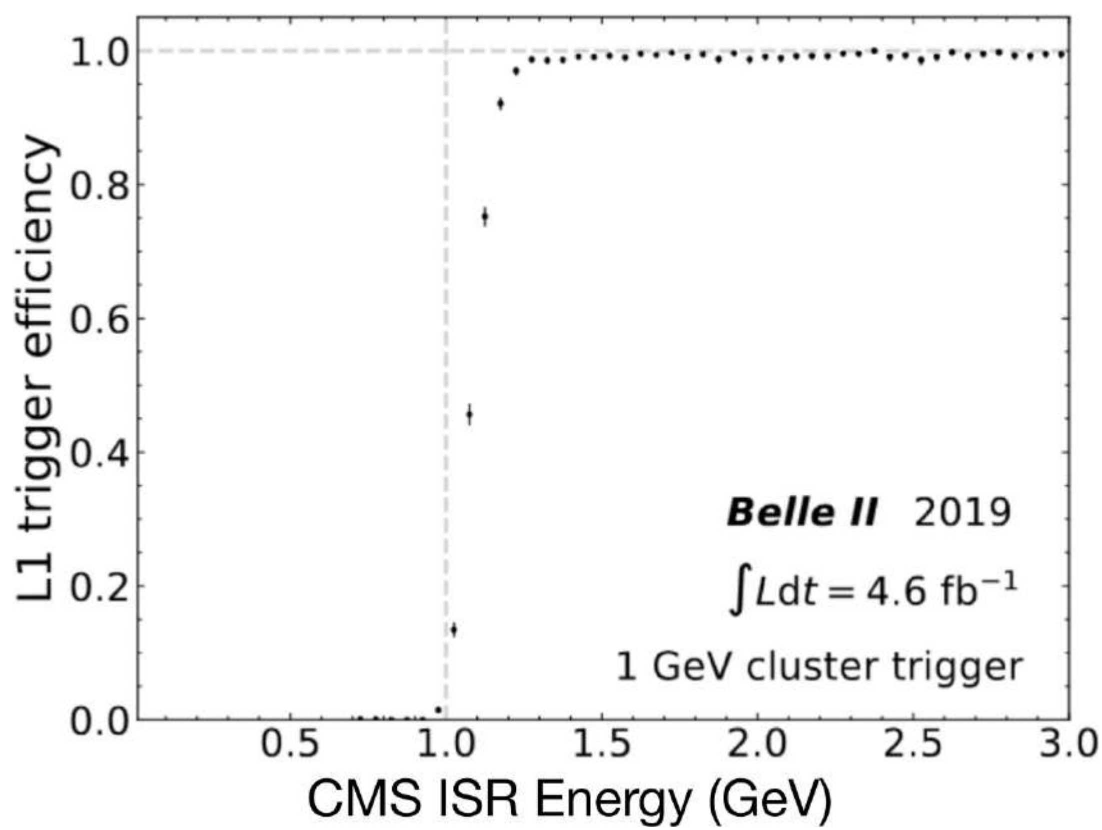
Scan over masses of the hadronic system via initial state radiation (ISR)

- Fixed center-of-mass energy $\sqrt{s} \approx 10.58$ GeV
- Scan $s' = (1 - 2E_\gamma^*/\sqrt{s})s$, E_γ^* is the ISR photon energy in c.m.s.
- Efficient **L1 trigger for ISR** events using ECL (cluster energy $\geq$ 2.0 GeV)
  - Studied with independent track trigger for **μμγ**: 99.9% in barrel region

    → 0.1% uncertainty   Not possible with Belle data !







План анализа

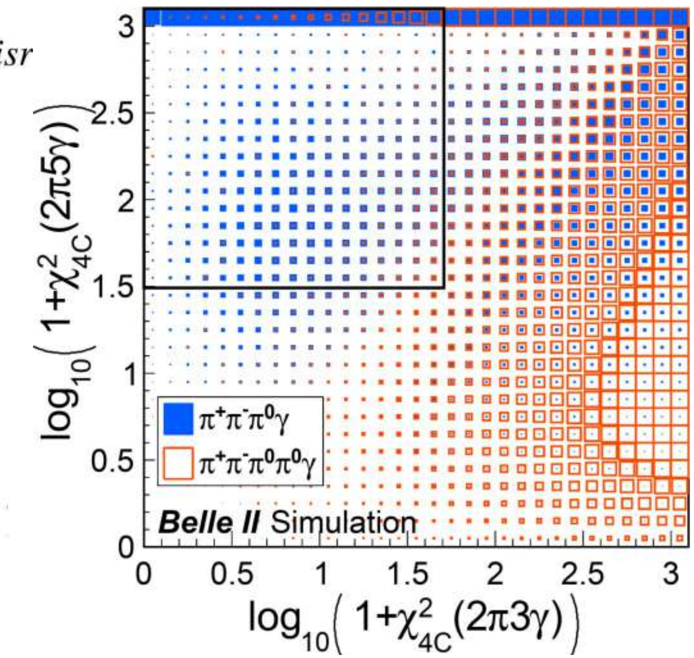
- Data set : 191 fb^{-1}

$$\sigma_{3\pi}(\mathrm{M}_{3\pi}) = \frac{\mathrm{N}_{\mathrm{signal}}}{\epsilon(\mathrm{M}_{3\pi}) \cdot \mathrm{L}_{\mathrm{eff}}(\mathrm{M}_{3\pi}) \cdot \mathrm{r}_{\mathrm{rad}}}$$

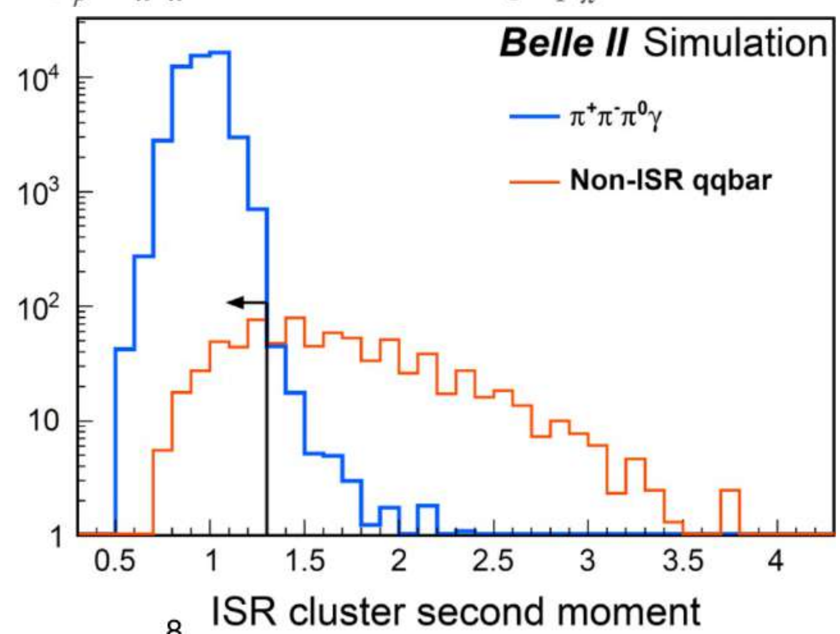
- $\sqrt{s'}$ range: 0.62 to 3.5 GeV
- Robust event selection to extract $e^+e^- \to \pi^+\pi^-\pi^0\gamma_{isr}$
  - Background determination and suppression (≤1% background at ω)
- Precise determination of the efficiency with ≤1% precision
- **Unfolding** the spectrum to mitigate detector resolution effects
- **Blind analysis**: all selections and corrections are determined with MC and control samples

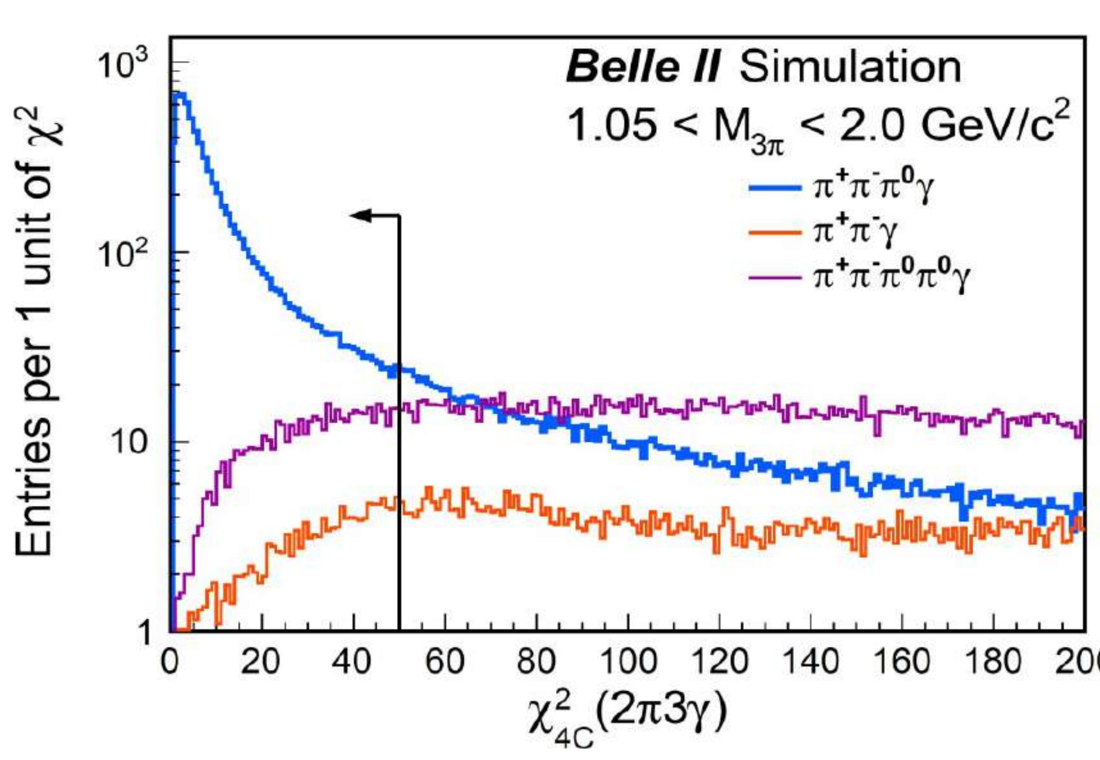
Условия отбора

- Reconstruct 2 tracks + 3 photons: $e^+e^- \to \pi^+\pi^-\pi^0\gamma_{isr} \to \pi^+\pi^-\gamma\gamma\gamma_{isr}$
  - ISR photon: $\mathrm{E}^*_\gamma > 4\,\mathrm{GeV}$ in ECL barrel region
  - $\pi^\pm$ from the IP with $\mathrm{p_T} > 0.2\,\mathrm{GeV/c}$, pion identification
  - π^0: $\mathrm{E}_\gamma > 0.1\,\mathrm{GeV}$, $\mathrm{M}_{\gamma\gamma} < 1\,\mathrm{GeV/c^2}$
- $\mathrm{M}^2_{\mathrm{recoil}}(\pi^+\pi^-) > 4\,\mathrm{GeV^2/c^4}$ against non-π^0 events: $e^+e^-\gamma$, $\pi^+\pi^-\gamma$, $\mu^+\mu^-\gamma$
- Four-momentum kinematic fit (**4C-Kfit**)
  - Constrain to initial e^+e^-
  - $\chi^2_{4C}(2\pi3\gamma) \le 50$ and $\chi^2_{4C}(2\pi5\gamma) > 30$
- Suppress non-ISR background
  - $\mathrm{M}_{\pi^\pm\gamma_{\mathrm{isr}}}$, $M_{\gamma_{isr}\gamma}$ and ECL cluster shape



High $p_{\rho^\pm\to\pi^\pm\pi^0}$ *High* p_{π^0}



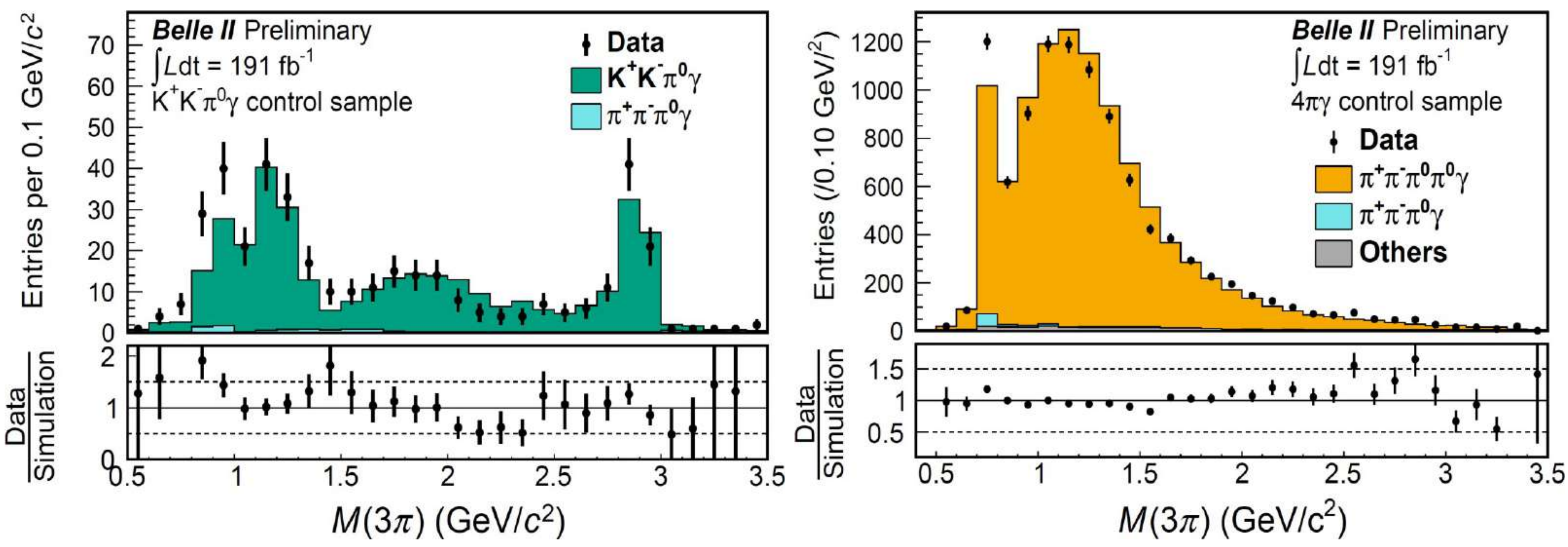


Оценка числа фоновых событий

Background enhanced data as a **control sample** to determine a **mass-dependent** data-MC **scale factor** :

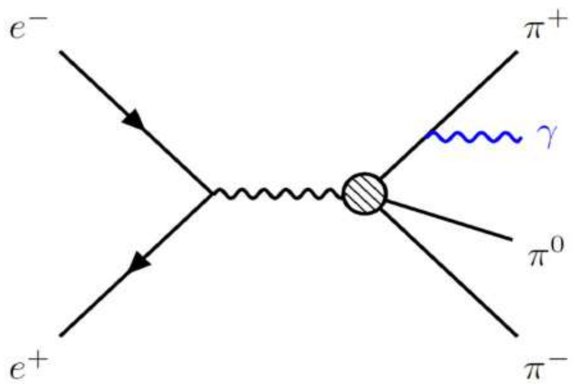
$$N^{\mathrm{data}}_{\mathrm{Signal}} = N^{\mathrm{MC}}_{\mathrm{Signal}} \cdot \frac{N^{\mathrm{data}}_{\mathrm{Control}}}{N^{\mathrm{MC}}_{\mathrm{Control}}}$$

- $e^+e^- \to K^+K^-\pi^0\gamma$: Inverted particle ID
- $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\gamma$: Reconstruct $\pi^+\pi^-\pi^0\pi^0\gamma$ and select $\chi^2_{4\pi\gamma} < 30$
- Non-ISR $q\bar{q}$: $0.10 < M_{\gamma_{isr}\gamma} < 0.17$ GeV/c² or large cluster second moment



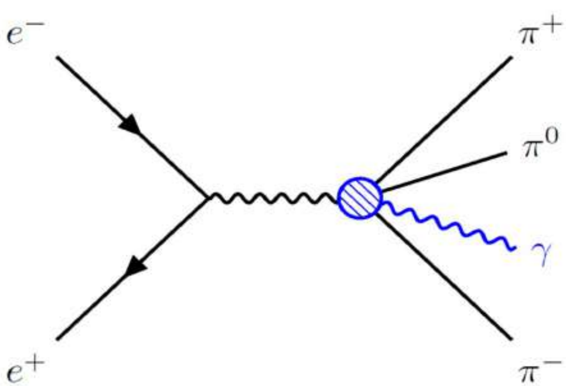
Оценка числа фоновых событий: FSR

- Difficult to reject FSR events or extract control samples
- Estimate FSR using pQCD prediction based on BaBar's [PhysRevD.104.112003]



FSR emission from final-state pions

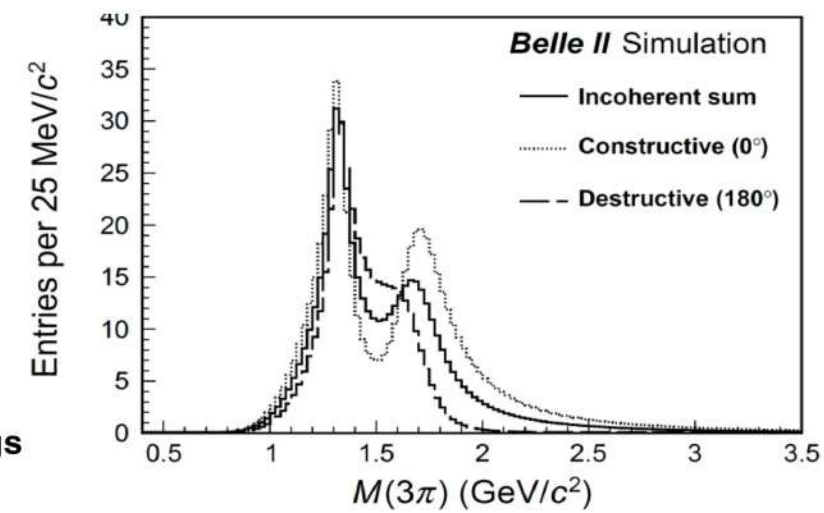
$\sim 0.001\text{fb} \rightarrow < 1$ event occur



FSR emission from the quark legs

$$e^+e^- \rightarrow M\gamma_{FSR} \rightarrow \pi^+\pi^-\pi^0\gamma_{FSR},$$

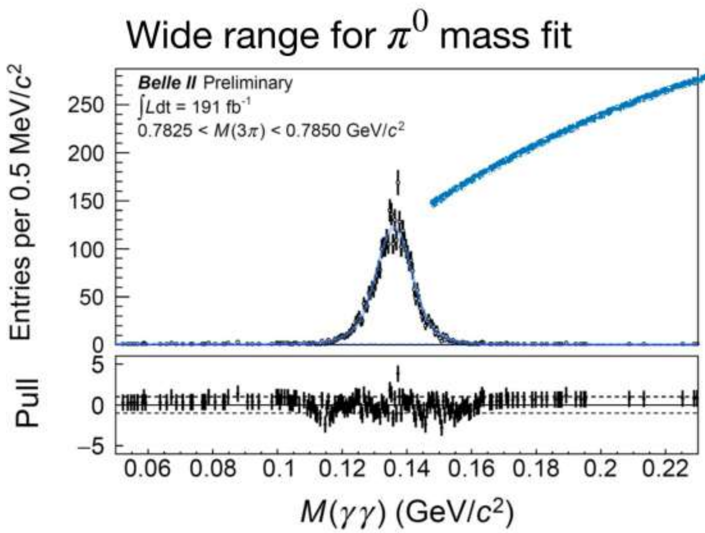
$$M = \eta, a_1(1260), a_2(1320), a_1(1640), a_2(1700), a_1(1930), a_2(2030)$$

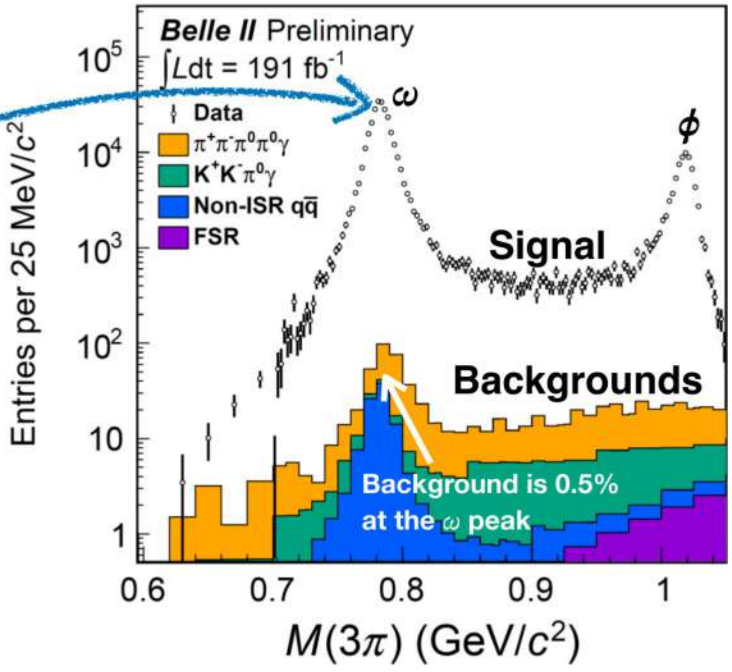


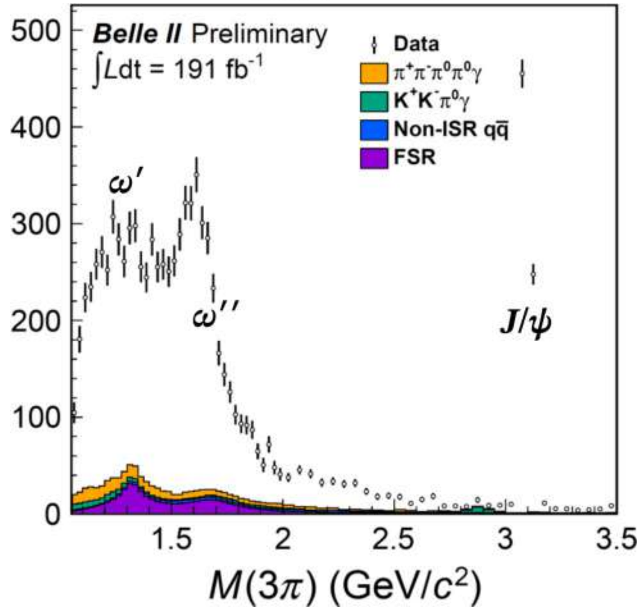
Considered in systematic uncertainty

Определение числа сигнальных событий

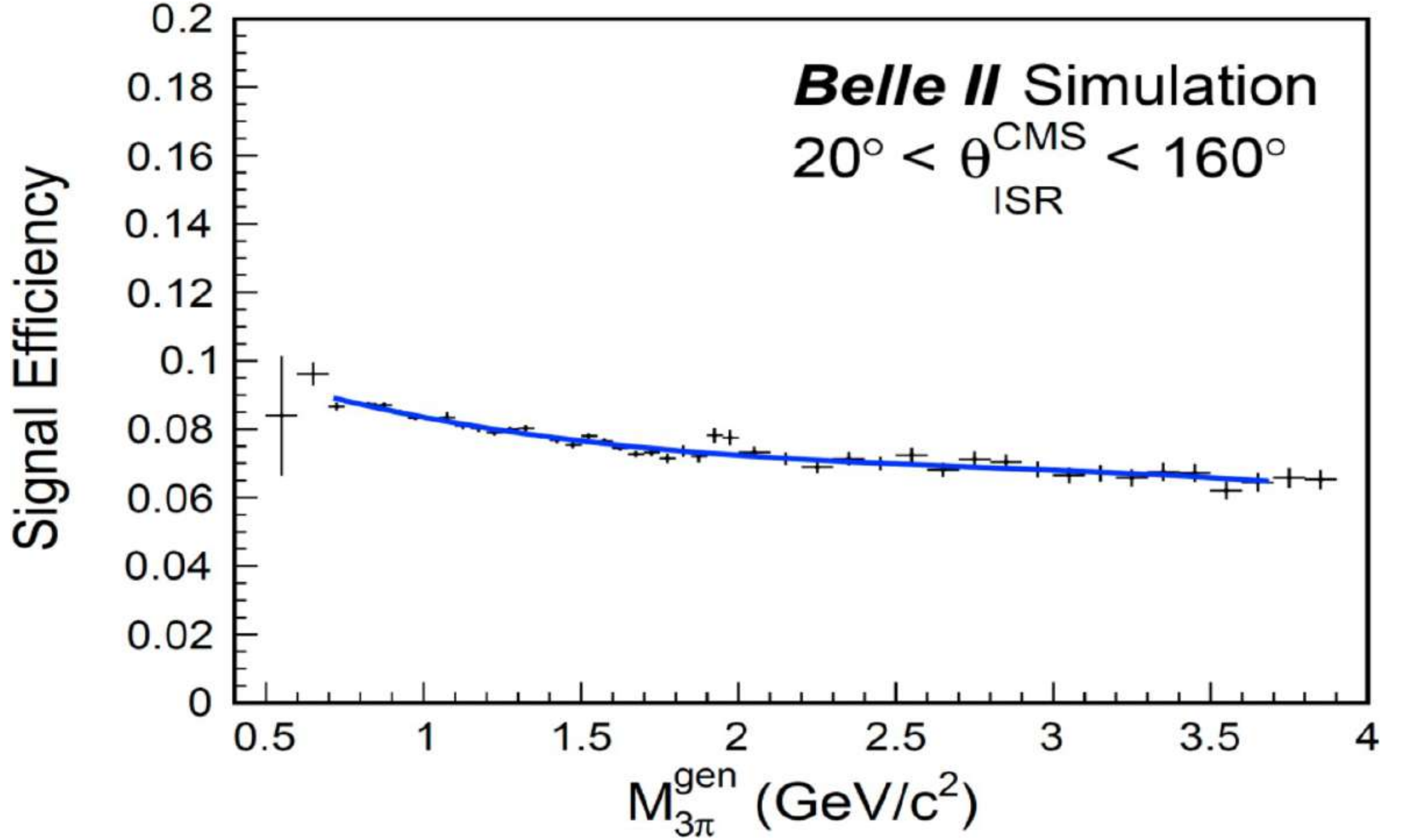
- Fitting $M_{\gamma\gamma}$ spectrum in each $M_{3\pi}$ bin to extract π^0 signal
- Residual background estimated with **data-MC correction** factors







Эффективность регистрации и поправки

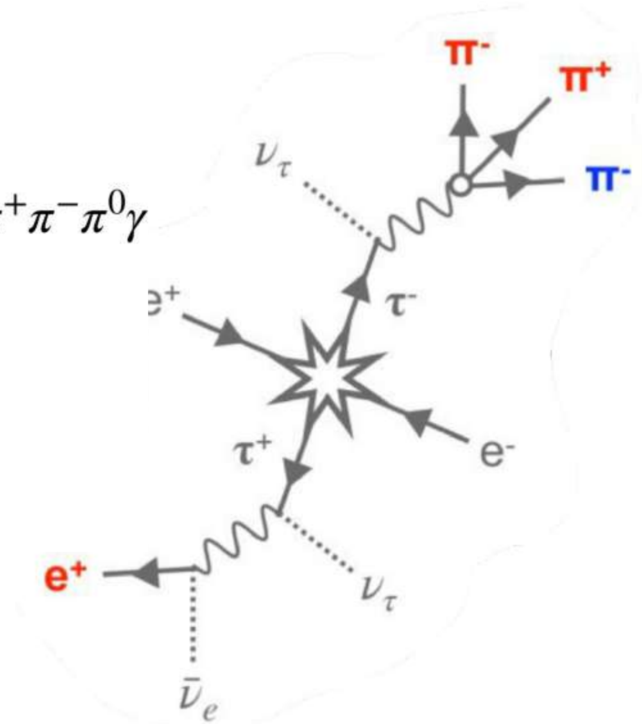


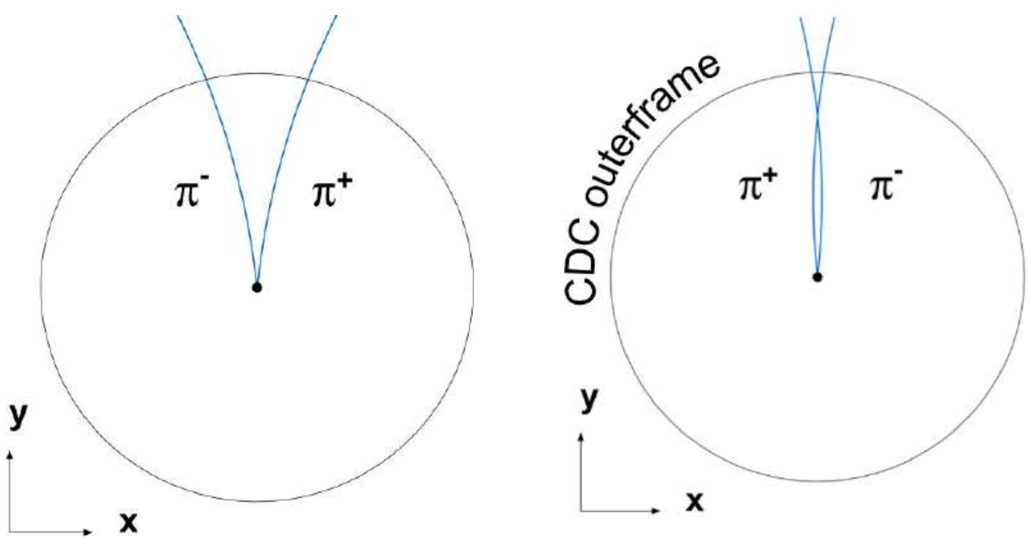
Efficiency $\epsilon = \epsilon_{MC} \prod_i (1 + \eta_i)$, Data-MC **correction** η_i ~ O(1)%

- Signal efficiency is estimated with **MC** of **10 x larger** statistics
- Data-MC correction factors are studied with **data-driven methods** and different **control samples**
  - **Background** suppression is studied with **signal yield before/after the suppression** criteria
  - **Tracking** and π^0 **detection** are **well understood**

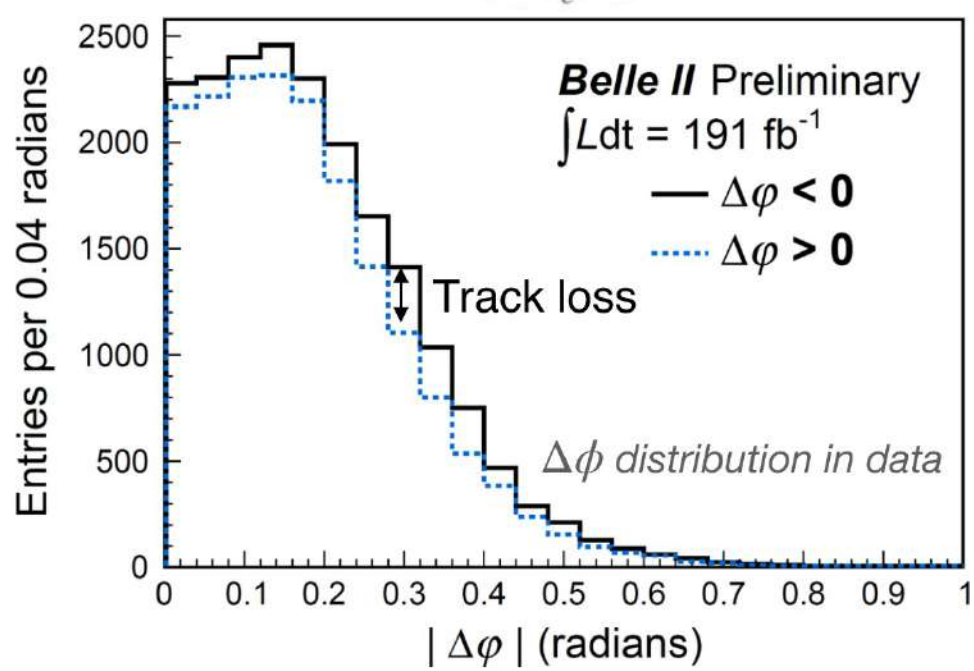
Эффективность регистрации: треки

- Studied with $e^+e^- \to \tau^+\tau^-$ process (1x3 prong)
  - Tag $\pi^+\pi^-e^\pm$ **or** $\pi^+\pi^-\mu^\pm$ and prob $\pi^\mp$
  - Small data-MC discrepancy → 0.3% uncertainty per track
- Correlated track loss due to shared hits in CDC is confirmed with $e^+e^- \to \pi^+\pi^-\pi^0\gamma$
  - Define $\Delta\phi \equiv \phi_{\pi^+} - \phi_{\pi^-}$, inefficiency: $f = \dfrac{N(\Delta\phi < 0) - N(\Delta\phi > 0)}{2N(\Delta\phi < 0)}$
    - 5% in data and 4% in MC
- Total correction factor for tracking: -1.4 ± 0.8 %
  - including dependency on no. of CDC hits and duplicated tracks



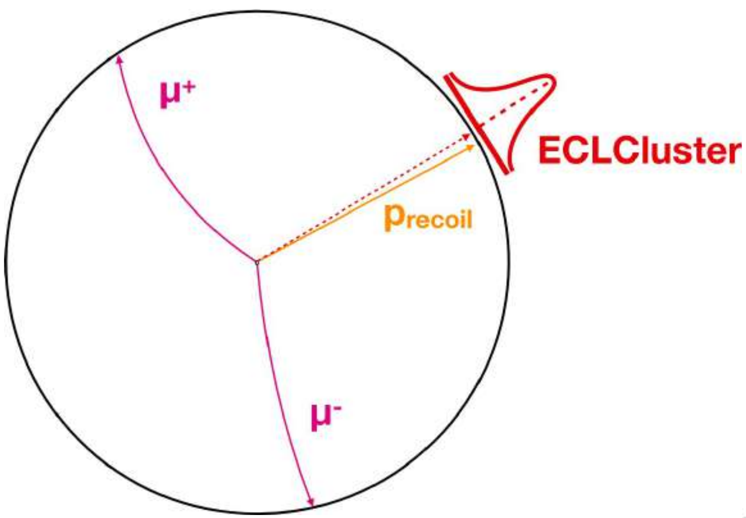


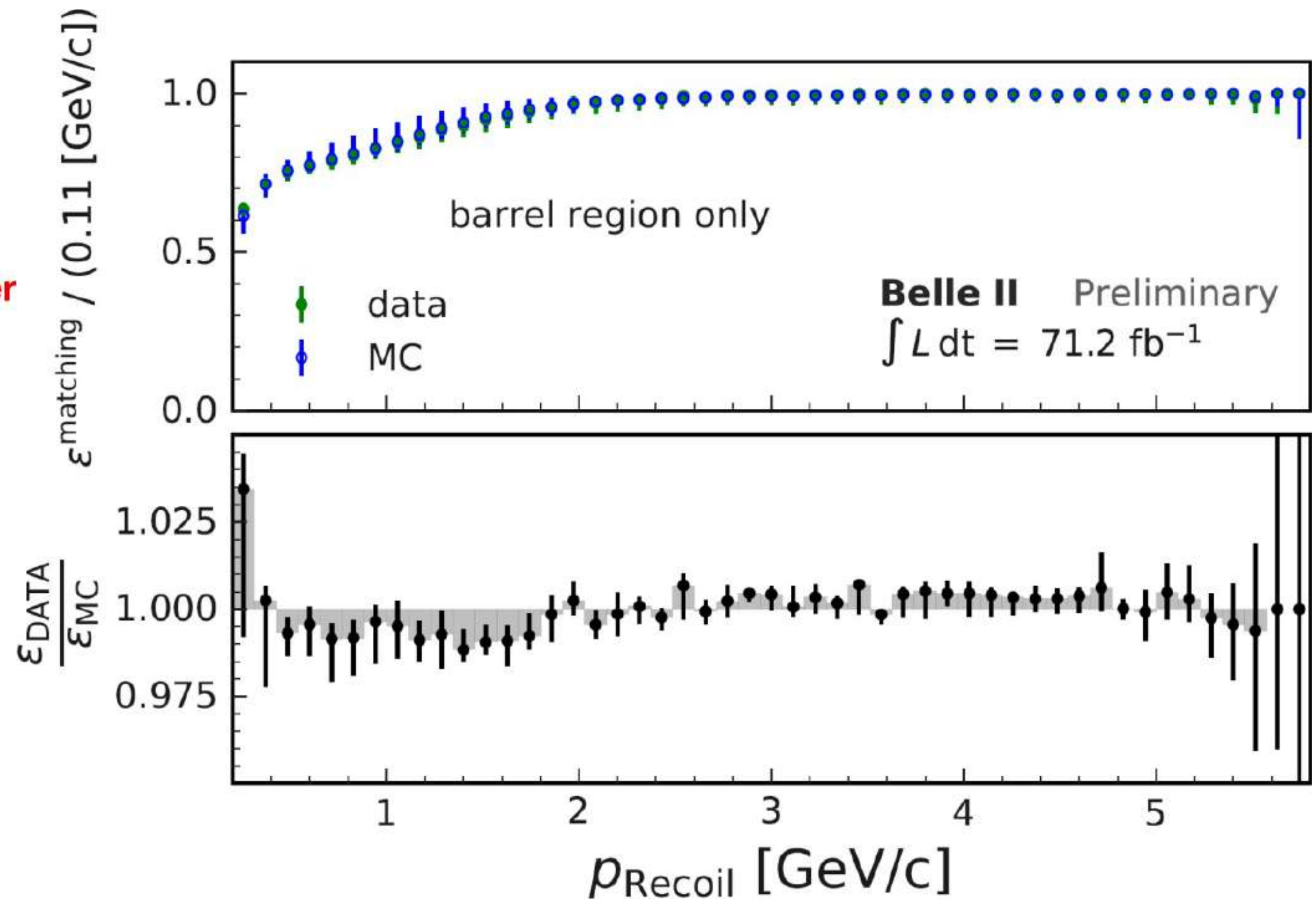
$\Delta\varphi > 0$



Эффективность регистрации: ISR

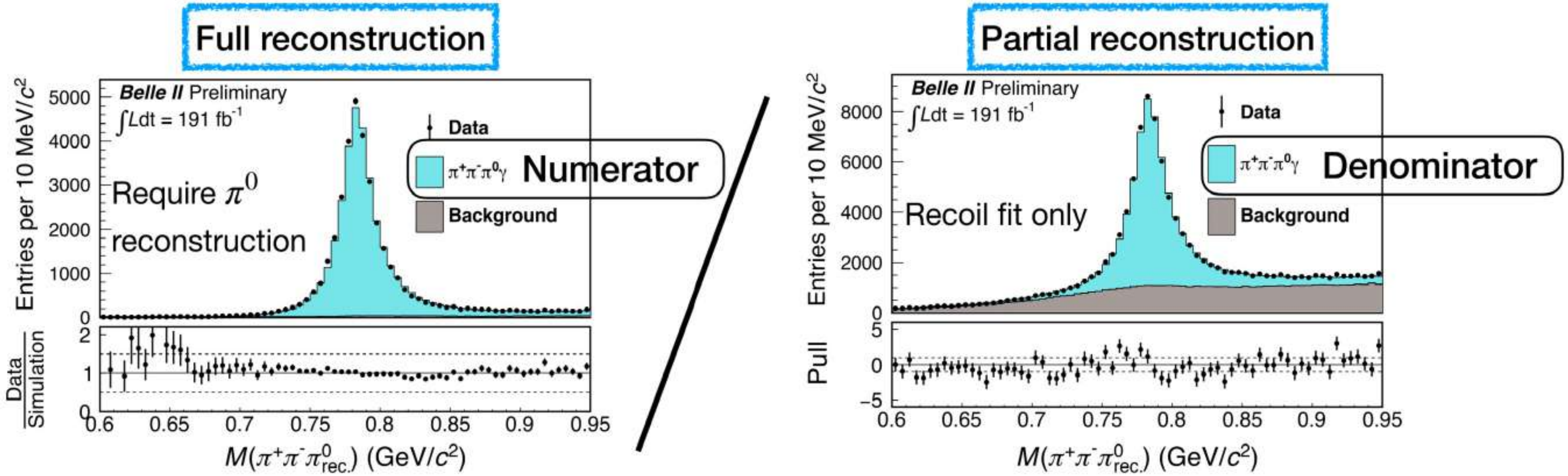
- Measured using $e^+e^- \to \mu^+\mu^-\gamma$ events
  - Matching a ECL cluster with missing momentum of the dimuon system
  - Good data-MC agreement → 0.7% systematic uncertainty





Эффективность регистрации: π^0

- Estimated using the exclusive process $e^+e^- \to \omega\gamma_{isr} \to \pi^+\pi^-\pi^0\gamma_{isr}$
  - Reconstruct only $\pi^+\pi^-\gamma_{isr}$, and constrain their recoil with π^0 mass (1C recoil fit) → counting $\omega \to \pi^+\pi^-\pi^0_{rec}$ as denominator
    - Events with successful π^0 reconstruction as numerator

$\epsilon_{\pi^0} =$ 

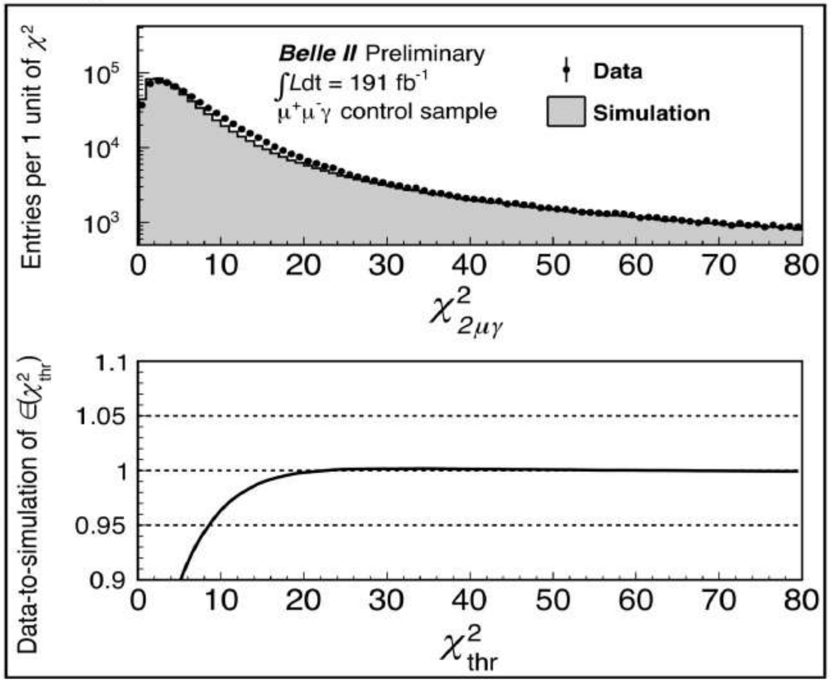
- ϵ_{π^0} is studied in data and MC respectively: Data/MC ratio = $0.986 \pm 0.006_{stat}$
- Related systematic uncertainty is 1.0% by varying M(γγ) signal pdf, background pdfs, and selections

Эффективность регистрации: χ^2

- Studied with $e^+e^- \to \mu^+\mu^-\gamma$
- Check effects from differences in vertex, momentum and energy of ISR and tracks
  - Agreement confirmed within ±0.6% uncertainty

$$\varepsilon(\chi^2_{\rm thr}) = \frac{N(\chi^2 < \chi^2_{\rm thr})}{N_{\rm all}}$$

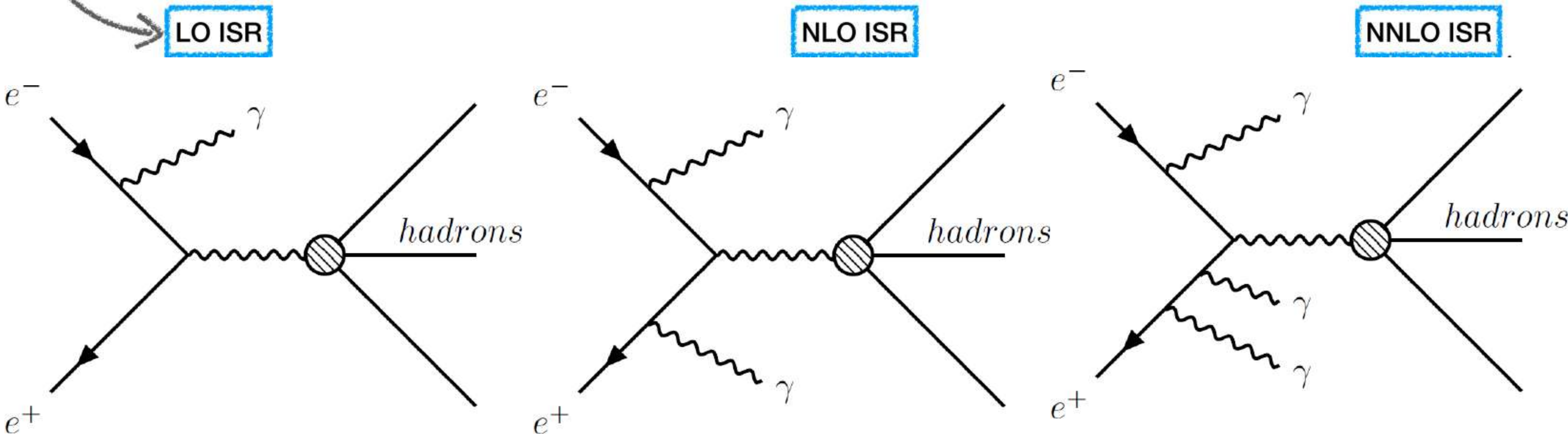
$$\frac{\varepsilon_{\rm data}(\chi^2_{\rm thr})}{\varepsilon_{\rm MC}(\chi^2_{\rm thr})}$$



- Multi-ISR photon is discussed on next slides

Эффективность регистрации: НО ISR

- **Signal** in this analysis: **single ISR** emission
  - In **reality**: There are processes **with multiple ISR** photon emissions
- Two effects of the existence of multiple ISR photons
  - Effective integrated luminosity L_{eff} (**radiative correction**): **0.5% unc.**
  - χ^2 **selection efficiency** due to ISR photon calculation in **generator**: **1.2% unc.**



Эффективность регистрации: НО χ^2

- **20% excess** of the fraction of **NLO** (two ISR) events on **PHOKHARA** is reported by BaBar [PhysRevD.108.L111103]
  - Also confirmed with Belle II data
  - Our χ^2 selection rejects most NLO events → efficiency change
    - Estimated with MC only: χ^2 **efficiency** is **underestimated** by **(2.4±0.7)%**
- **NNLO** (three ISR) is not included in the generator
  - **(3.4±0.4)%** observed by BaBar
  - Influence to this analysis: efficiency **overestimation** by **1.9%**
- No correction is applied to our result, but
  - **1.2% systematic uncertainty** is assigned as **MC generator derived error**
    - 0.7% (error from NLO excess) $\oplus$ 0.95% (half of NNLO effect) = 1.2%

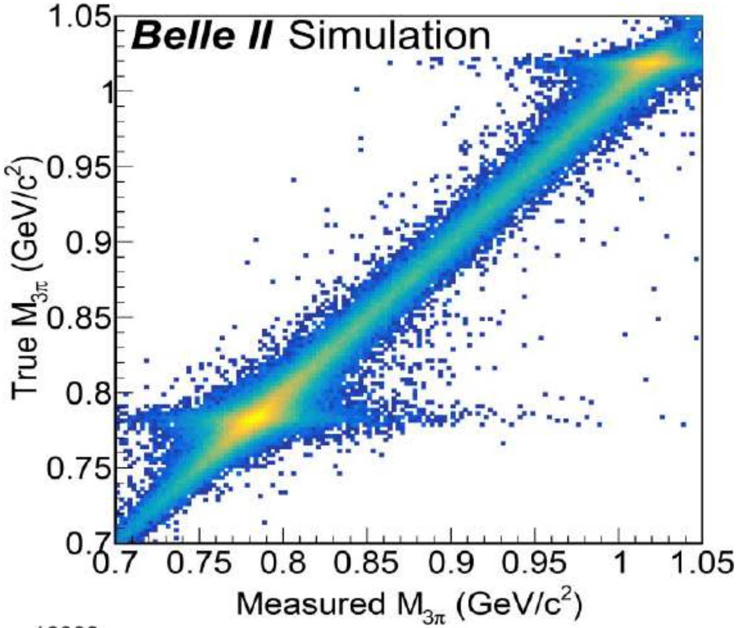
Эффективность регистрации: подавление фона

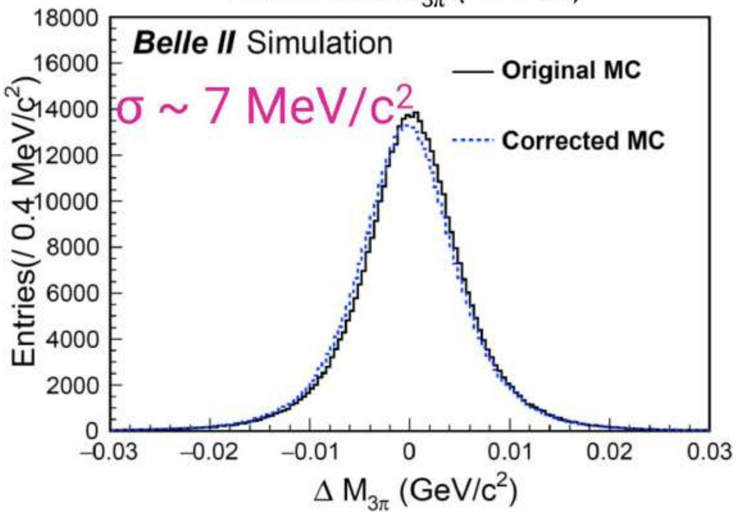
- Estimated by the ratio of signal yield before/after the suppression criteria
- Using ω and Φ, J/ψ resonances of good signal-to-noise ratio
- In $M_{3\pi} < 1.05\,\mathrm{GeV/c^2}$, efficiency is (89.5±0.2)% for data
  - $\epsilon_{\rm data}/\epsilon_{\rm MC} - 1$ = (-1.90±0.20)%
- In $M_{3\pi} > 1.05\,\mathrm{GeV/c^2}$, no. of J/ψ events is obtained by fitting $M_{3\pi}$
  - $\epsilon_{\rm data}/\epsilon_{\rm MC} - 1$ = (-1.78±1.85)%

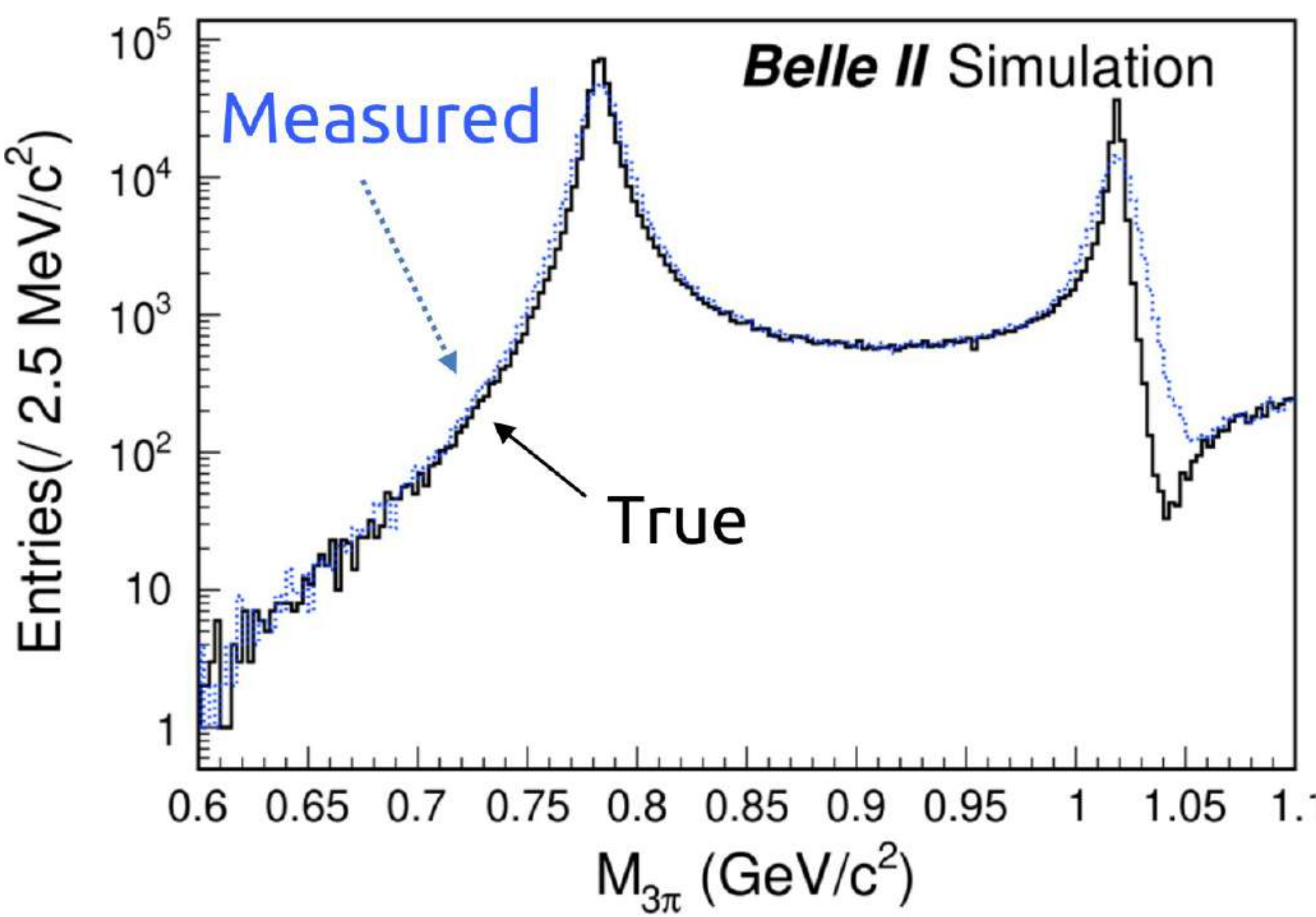
statistical errors in the sample

Восстановление истинного спектра $M_{3\pi}$

- Typical **mass resolution**: ~ 7-10 MeV/c^2
- **Data-MC difference** of mass bias and detector resolution is studied with narrow peaks at ω, Φ, and **J/ψ** in data
  - Correct MC by 1 MeV/c^2 for resolution and 0.5-1.5 MeV/c^2 for mass shift







Получение сечения

Cross section

$$\sigma_{ee\to 3\pi}(M_i(3\pi)) = \frac{N_{\mathrm{unfolded},i}}{\varepsilon(M_i(3\pi)) \cdot L_{\mathrm{eff}}(M_i(3\pi)) \cdot r_{\mathrm{rad}}}$$

3π mass at i-th bin

Radiative correction

Effective luminosity

Corrected Efficiency

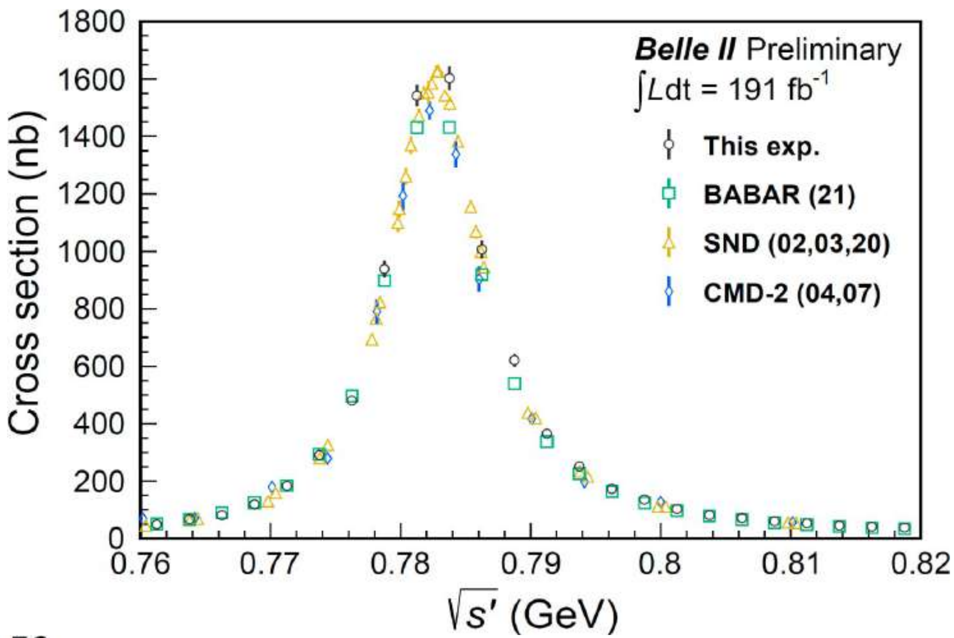
- **Leading order (LO) ISR luminosity** with L_{int} = 191/fb is given by:

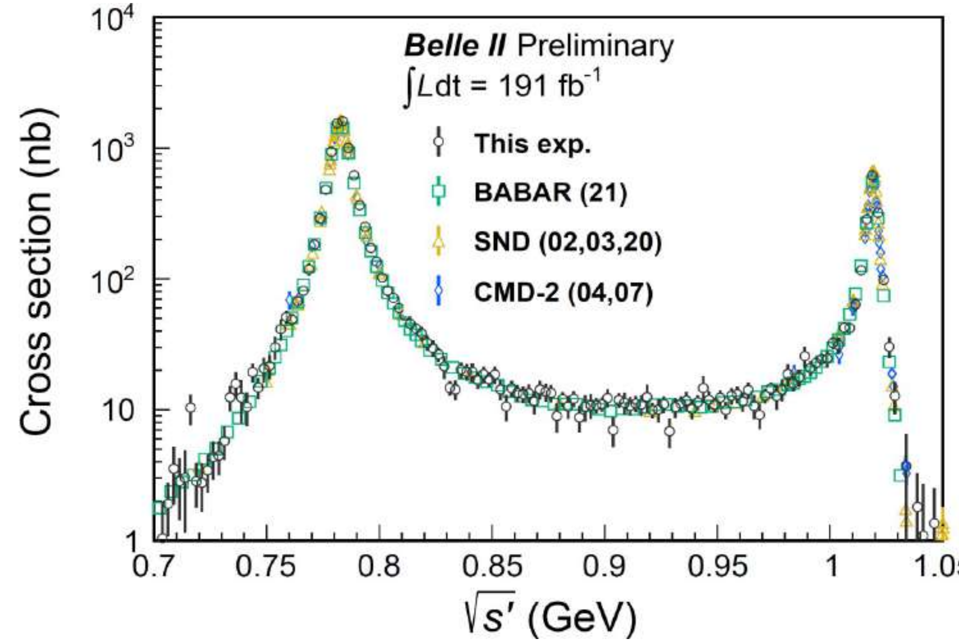
$$L_{eff} = \frac{2\sqrt{s'}}{s}\frac{\alpha}{\pi}\left(\frac{s^2+s'^2}{s(s-s')}\ln\frac{1+\cos\theta}{1-\cos\theta} - \frac{s-s'}{s}\cos\theta\right)L_{int}$$

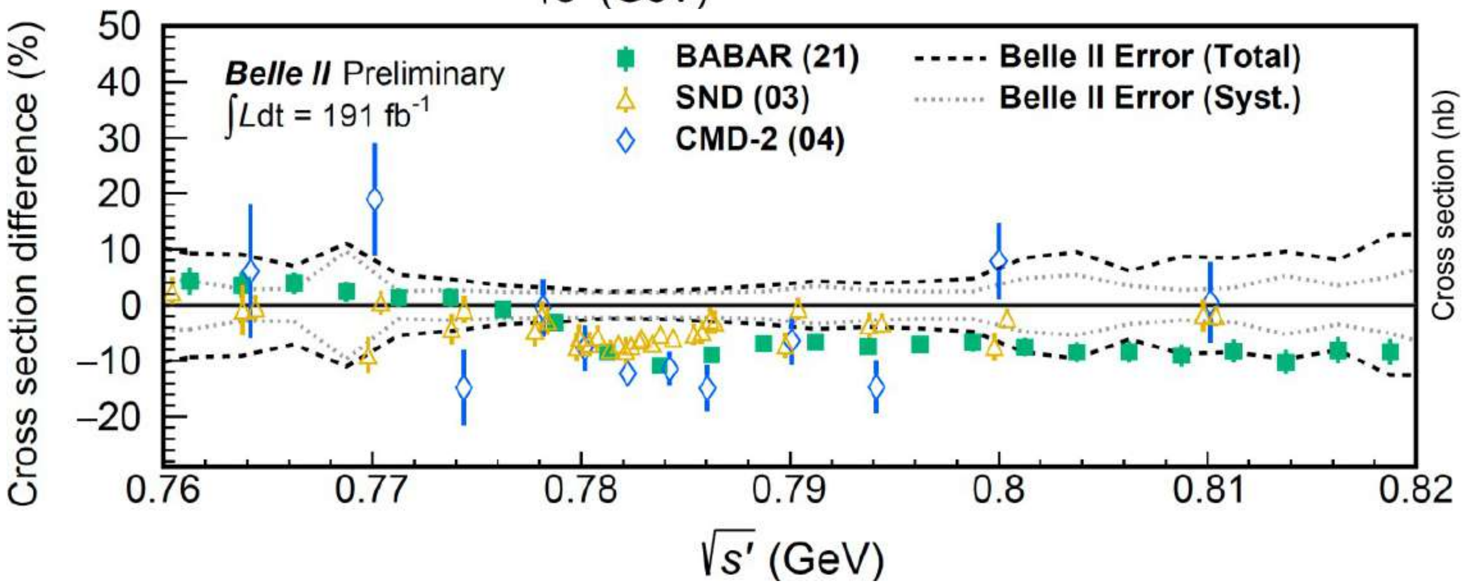
- **Radiative correction** is the ratio of the ISR emission probability including higher-order effects (LO+NLO+…) to LO
- Higher order (LO+NLO) effects **calculated by PHOKHARA**
  - Give us radiative correction of 1.008-1.013 depending on hadronic energy √s’
  - **0.5% uncertainty**

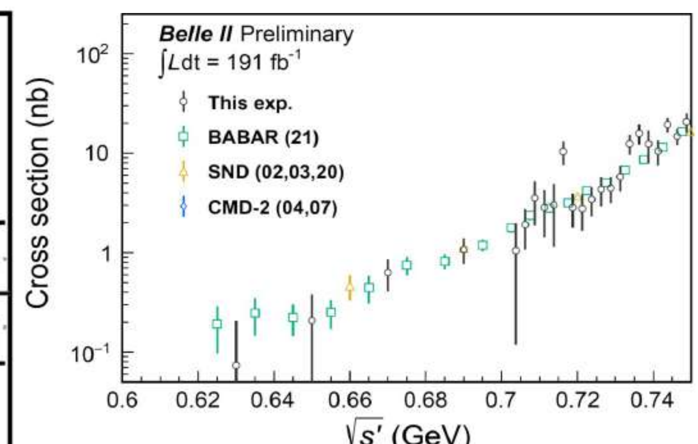
Сечение: область ниже 1.05 ГэВ

- **ω** resonance has a large cross section and contributes largely to $a_\mu(3\pi)$
- Our result is **5-10%** higher than BaBar, SND, and CMD-2

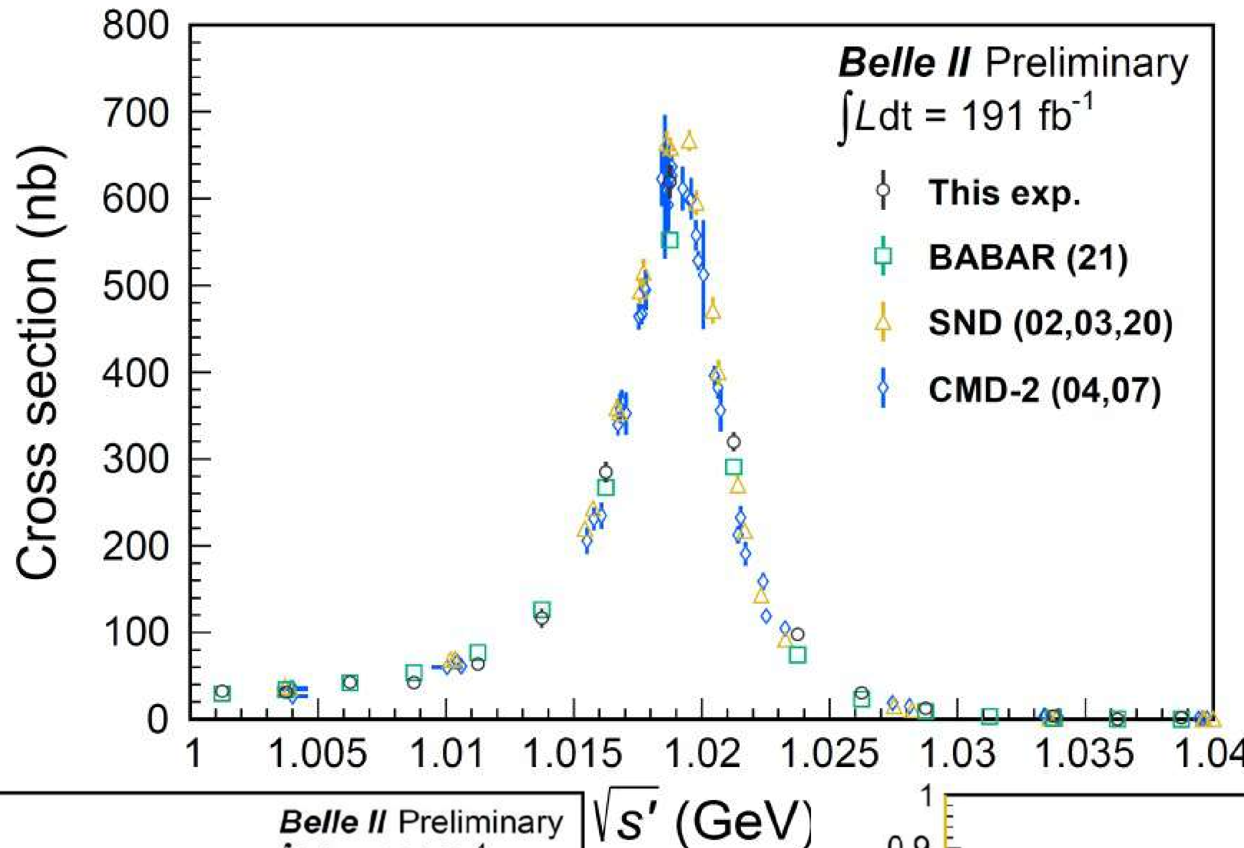


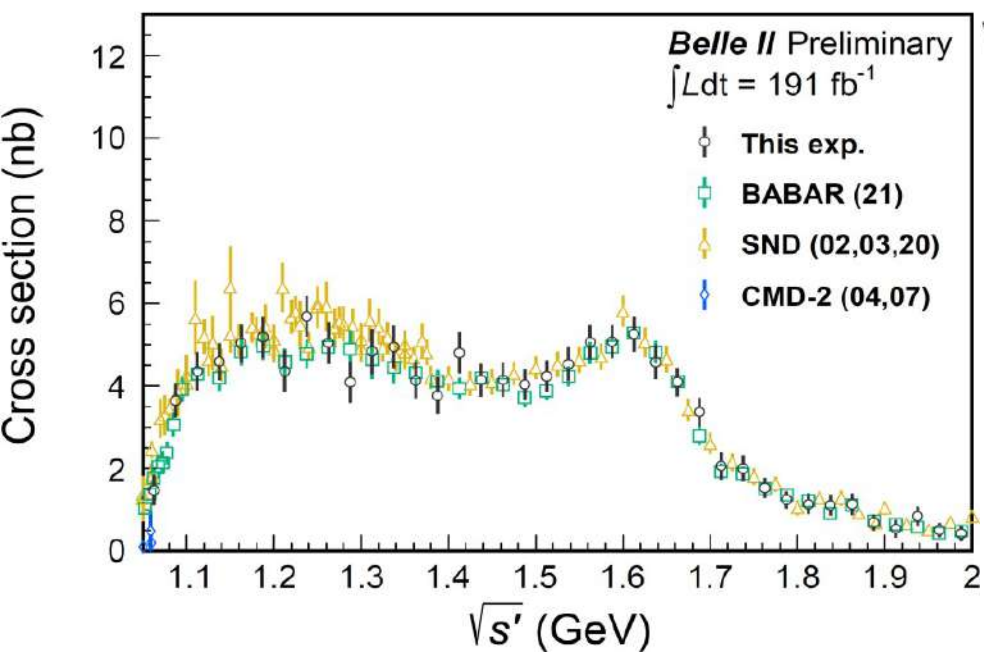


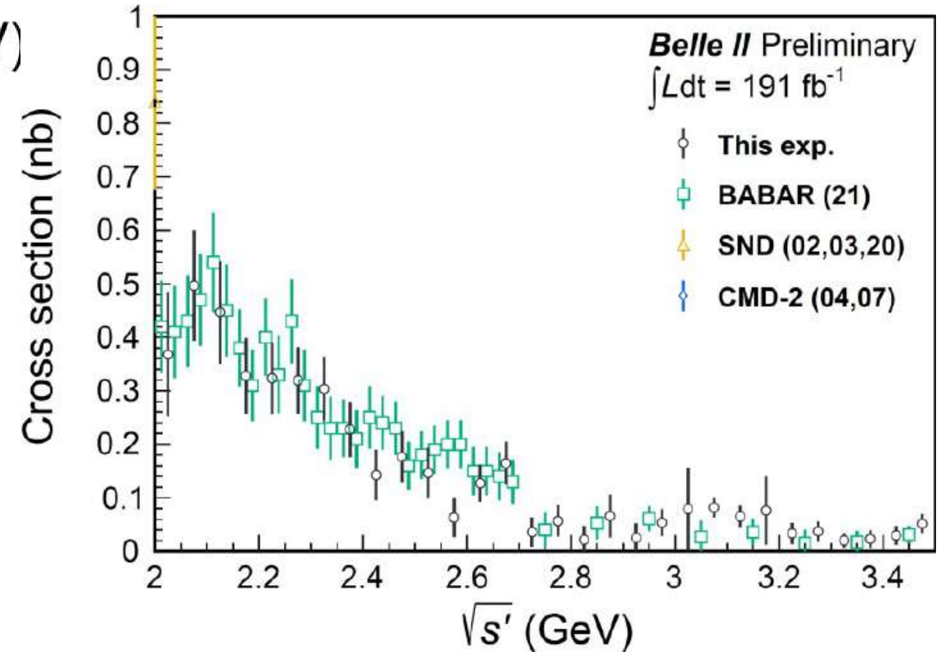




Сечение: область ϕ-мезона и выше





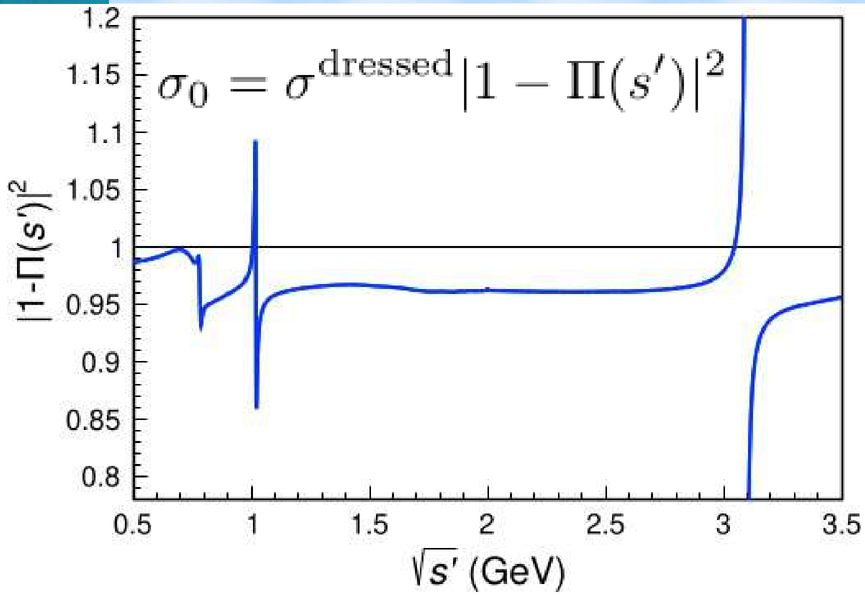


Систематические ошибки

Table I. Summary of fractional systematic uncertainty (%) in the $e^+e^- \to \pi^+\pi^-\pi^0$ cross section for three different energy regions. The values in parentheses are the efficiency correction factors η_i defined in Eq. (7). The systematic uncertainties due to the background subtraction and unfolding, which depend on the 3π mass bin, show the range from minimum to maximum in the energy regions of $M(3\pi)$ near the ω resonance, 1.1–1.8 GeV/c^2 and 2.0–3.0 GeV/c^2.

Source	Systematic uncertainty (efficiency correction factor η_i) %					
	0.62–1.05 GeV/c^2		1.05–2.00 GeV/c^2		2.0–3.5 GeV/c^2	
Tracking	0.8	(−1.35)	0.8	(−1.71)	0.8	(−1.71)
ISR photon detection	0.7	(+0.15)	0.7	(+0.15)	0.7	(+0.15)
π^0 detection	1.0	(−1.43)	1.0	(−1.43)	1.0	(−1.43)
Kinematic fit (χ^2)	0.6	(+0.0)	0.3	(+0.30)	0.3	(+0.30)
Trigger	0.1	(−0.09)	0.1	(−0.08)	0.1	(−0.06)
Background suppression	0.2	(−1.90)	1.9	(−1.78)	1.9	(−1.78)
Monte Carlo generator	1.2		1.2		1.2	
Integrated luminosity	0.6		0.6		0.6	
Radiative corrections	0.5		0.5		0.5	
Simulated sample size	0.2		0.2–0.5		0.5–1.6	
Background subtraction	0.2–2.3		0.4–7.2		4.4–44	
Unfolding	0.7–25		0.2–5.1		0.3–11	
Total uncertainty	2.3-25		2.9-8.8		6.4-44	
(Total correction $\varepsilon/\varepsilon_{\rm sim} - 1$)		(−4.61)		(−4.55)		(−4.53)

Вклад в a_μ



Source	Systematic uncertainty (%)
Efficiency corrections	1.63
Monte Carlo generator	1.20
Integrated luminosity	0.64
Simulated sample size	0.15
Background subtraction	0.02
Unfolding	0.12
Radiative corrections	0.50
Vacuum polarization corrections	0.04
Total	2.19

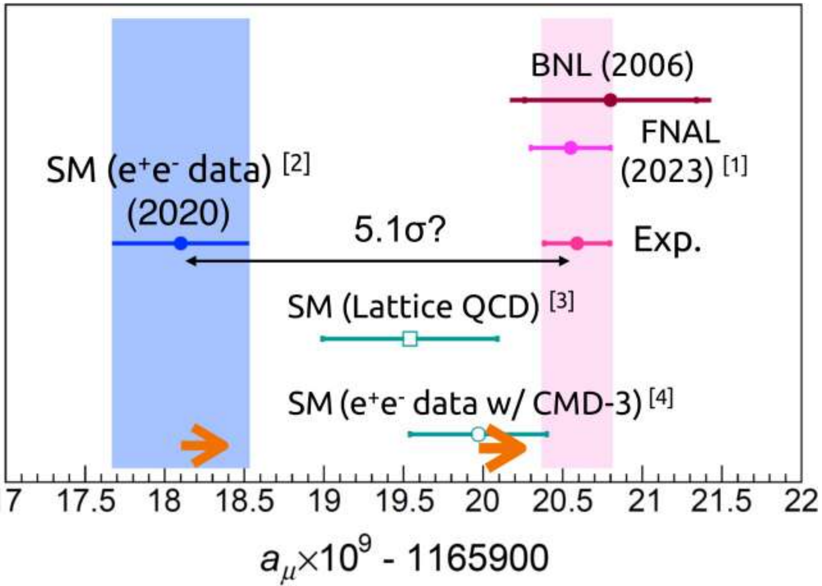
- Using our result: $a_\mu^{\text{LO,HVP},3\pi}(0.62-1.8\,\text{GeV}) = (48.91 \pm 0.23_{\text{stat}} \pm 1.07_{\text{syst}}) \times 10^{-10}$

	$a_\mu(3\pi)\times10^{10}$	Difference$\times10^{10}$
BABAR alone [PRD 104, 11 (2021)]	45.86 ± 0.14 ± 0.58	3.2±1.3 (6.9%)
Global fit* [JHEP 08, 208 (2023)]	45.91 ± 0.37 ± 0.38	3.0±1.2 (6.5%)

* Not includes BESIII preliminary result [arXiv:1912:11208]

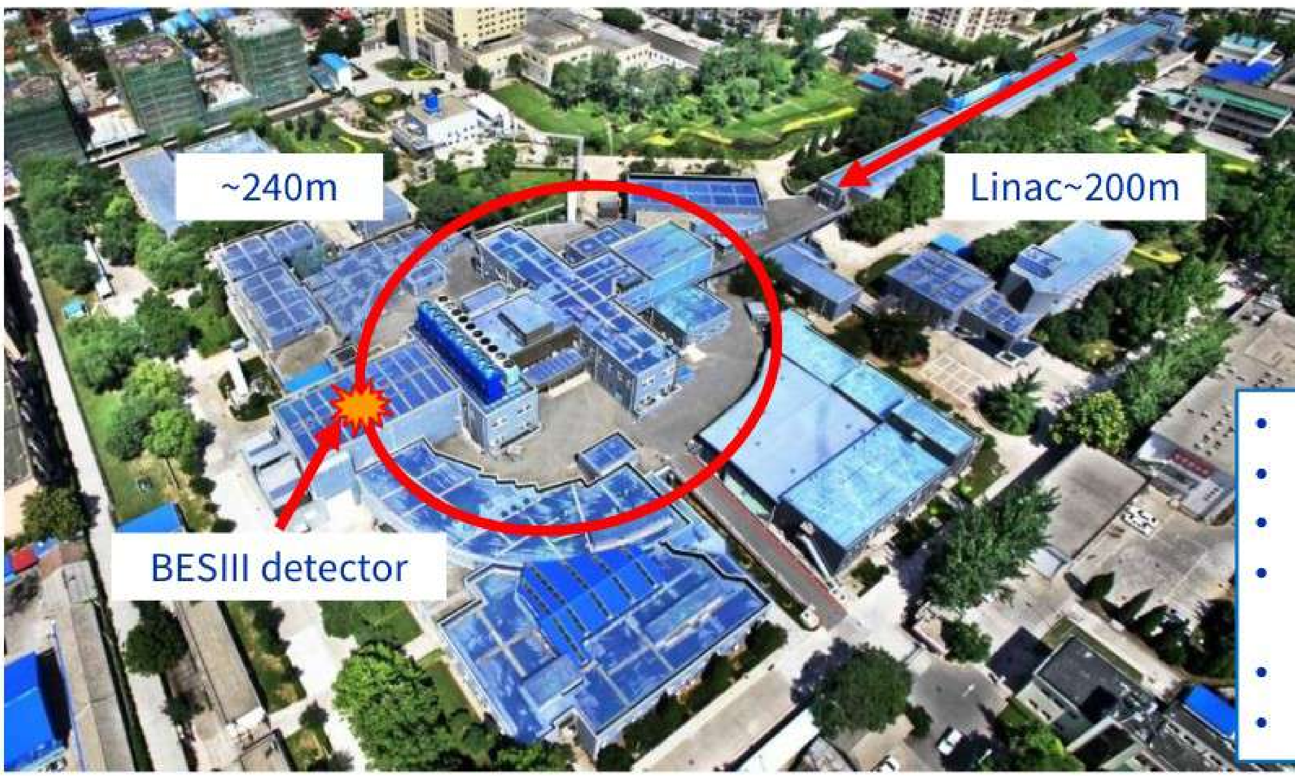
- 6.5% higher than the **global fit** result with $2.5\,\sigma$ significance
- The difference, 3×10^{-10}, corresponds to **10%** of $\Delta a_\mu = a_\mu(\text{Exp}) - a_\mu(\text{SM}) = 25 \times 10^{-10}$

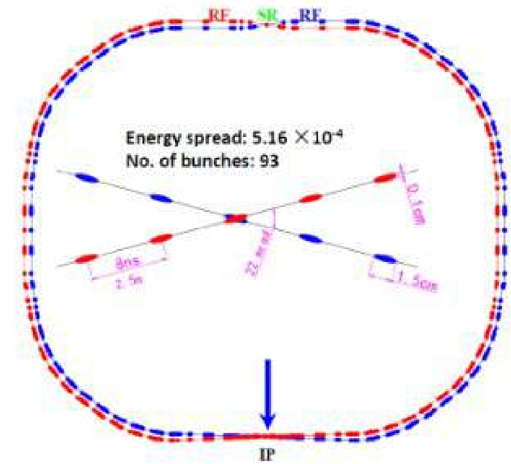
WP2020



Beijing Electron Positron Collider (BEPCII)

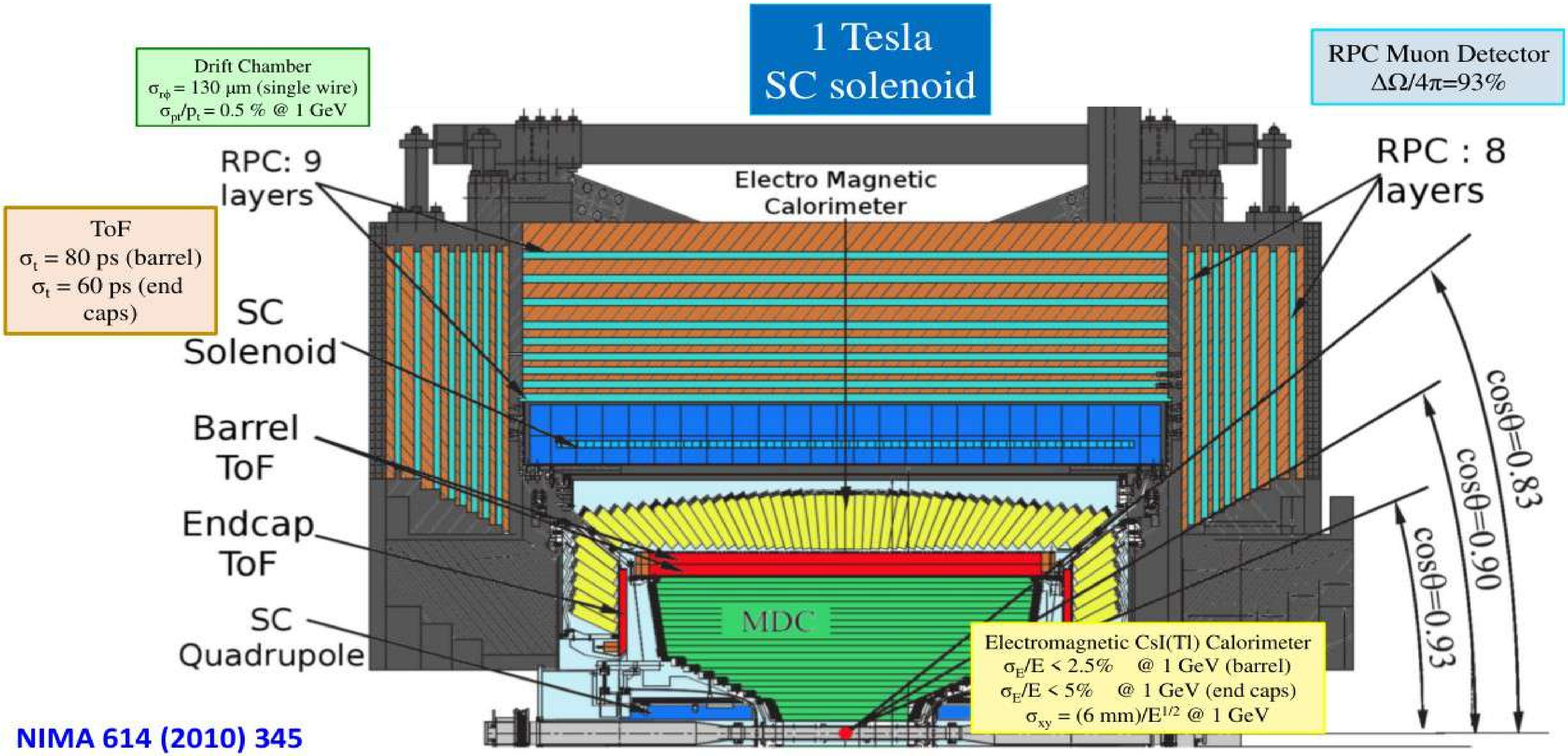
World unique e^+e^- accelerator in charm physics energy region





- 2004: started BEPCII/BESIII construction
- Double rings
- E_{cm} = 2.0~4.6 (4.9 since 2019) GeV
- Design luminosity: $1 \times 10^{33} cm^{-2}s^{-1}$ (reached 2016 @E_{cm} = 3.77 GeV)
- 2008: test run
- 2009~ today: BESIII physics runs

BESIII



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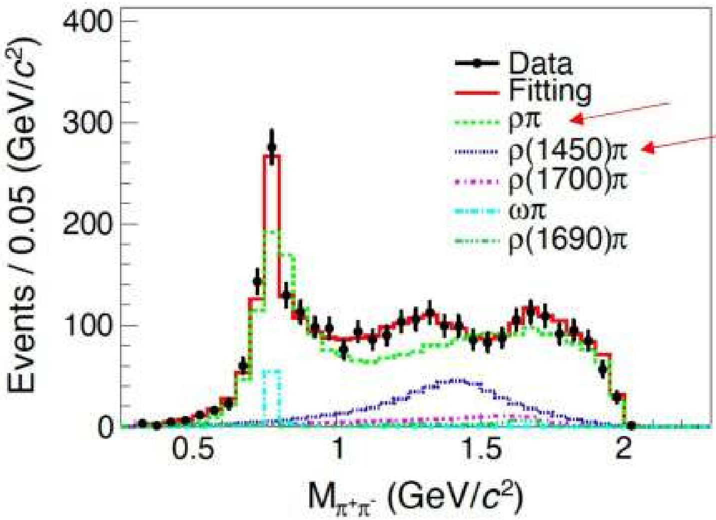
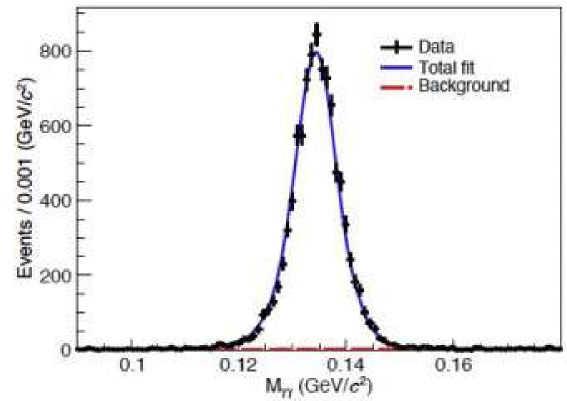
Сечение

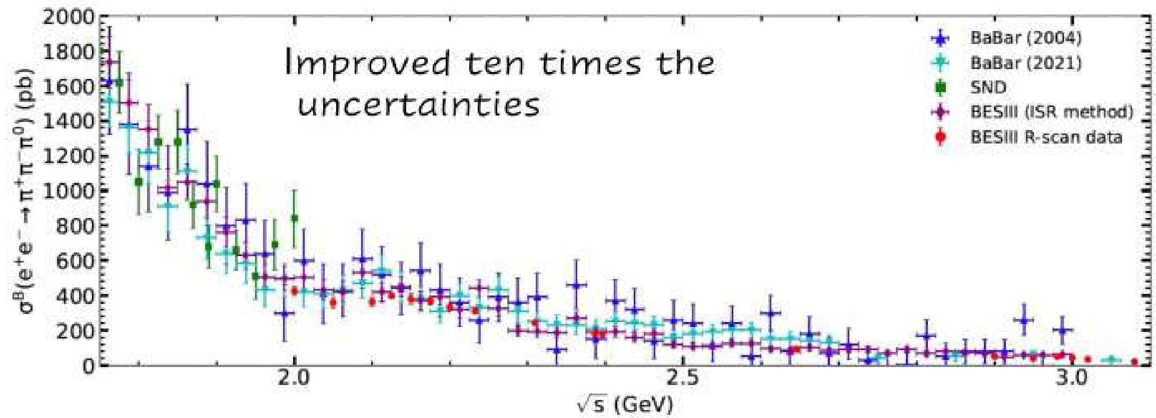


$\sqrt{s}$(GeV)	2.1250	2.3960	2.9000
$\rho\pi$	$> 50\sigma$	$> 50\sigma$	27σ
$\rho(1450)\pi$	18σ	10σ	13σ
$\rho(1700)\pi$	6.5σ	6.4σ	6.6σ
$\rho_3(1690)\pi$	14σ	9.2σ	6.8σ
$\rho(1570)\pi$	1.3σ	1.1σ	0.13σ
$\omega\pi$	31σ	11σ	9.7σ
$\omega(1420)\pi$	2.8σ	0.21σ	1.8σ
$\omega(1650)\pi$	3.5σ	0.29σ	0.53σ
$\omega_3(1670)\pi$	3.4σ	1.6σ	0.56σ

Francesca De Mori, ICHEP 2024, Prague



UML fit
For signal yield



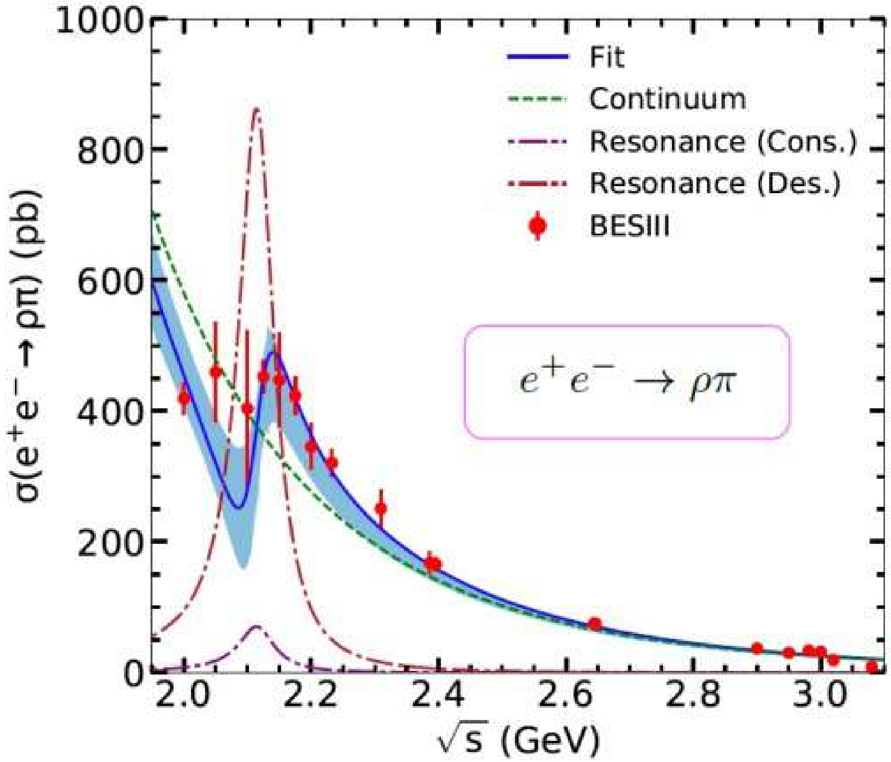
Improved ten times the uncertainties

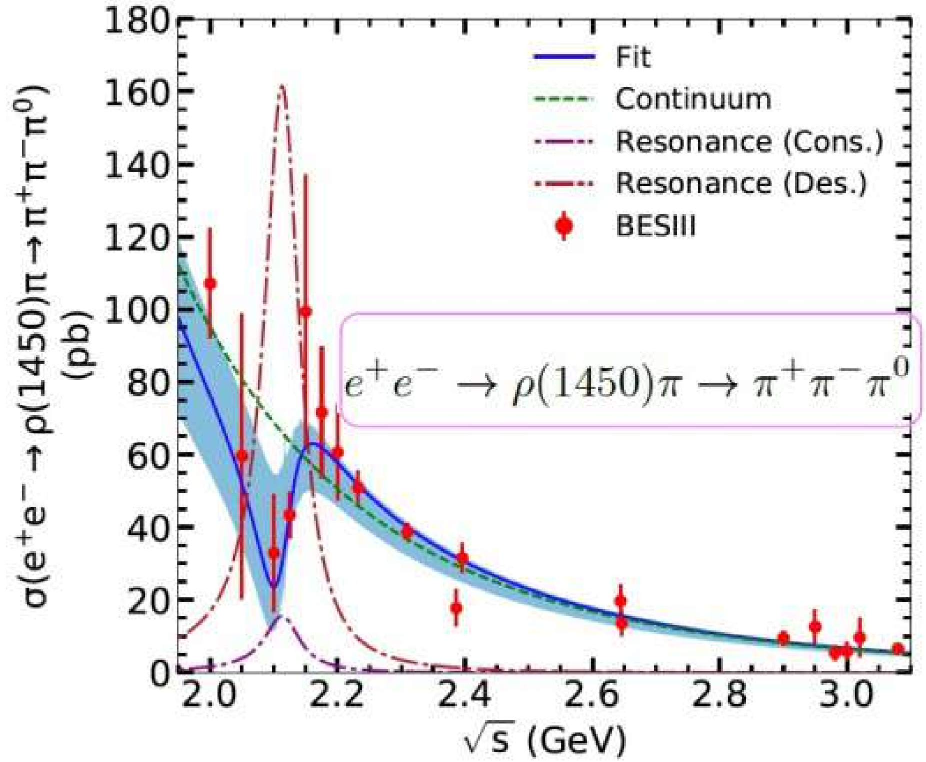
Born cross section also for major intermediate states(for $\omega\pi^0$ more precise BESIII result (Phys. Lett. B 813, 136059 (2021) already available)

10

Сечение: разделение вкладов

Simultaneous fit, assuming the same resonant structure (ω^*)

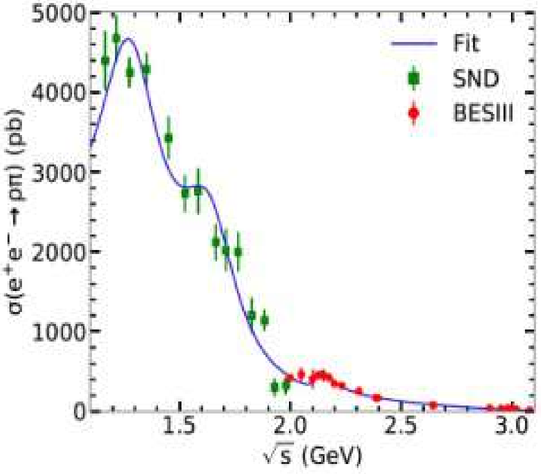




PDF: $|f_r(s)e^{i\phi_r} + f_c(s)|^2$ $\qquad f_r(s) = \frac{M_r}{\sqrt{s}} \frac{\sqrt{12\pi(\hbar c)^2 \Gamma^r_{ee} Br \Gamma_r}}{s - M_r^2 + iM_r\Gamma_r} \sqrt{\frac{\Phi(\sqrt{s})}{\Phi(M_r)}}, \qquad f_c(s) = a\frac{\sqrt{\Phi(\sqrt{s})}}{(\sqrt{s})^b}.$

Multiple solution: same M and width but different phase and $\Gamma^R_{ee} Br$

Сечение: разделение вкладов с учетом СНД



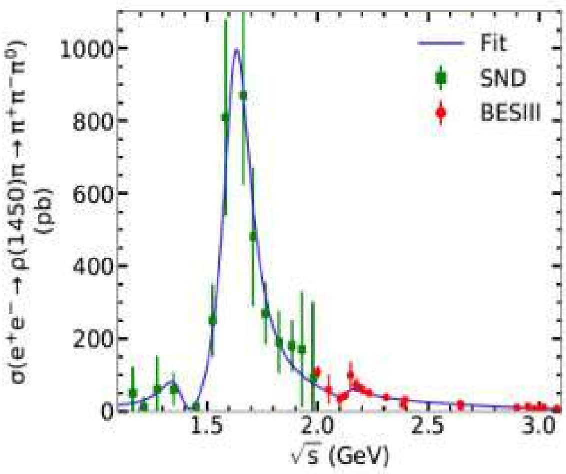


TABLE V. Fit results of the $e^+e^- \to \rho\pi$ and $e^+e^- \to \rho(1450)\pi \to \pi^+\pi^-\pi^0$ processes, where the uncertainties are statistical only.

	Nominal fit	Alternative fit
Mass	2119 ± 11 MeV$/c^2$	2134 ± 14 MeV$/c^2$
Width	69 ± 30 MeV	74 ± 31 MeV
χ^2	41.9	73.3
Number of free parameters	10	18
Degrees of freedom (ndf)	28	48
χ^2/ndf	1.50	1.53
Significance	5.9σ	5.2σ

Alternative fit with SND data @ low energies
SND-helps to constraint the continuum

To evaluate systematics

$M = 2119 \pm 11 \pm 15$ MeV$/c^2$ $\quad$ $\Gamma = 69 \pm 30 \pm 5$ MeV $\quad$ **> 5 σ significance**

For G-parity conservation it should be an exited ω or ϕ state.
ϕ State ruled out by OZI suppression in this final state channel.

ω* candidate

Further studies needed to clarify the situation.