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THE QUARK IMPACT FACTORS

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Abstract

We calculate in the next-to-leading approximation the non-forward quark impact factors for both singlet and octet color representation in the t-channel. The integral representation of the octet impact factor in the general case of arbitrary space-time dimension and massive quark flavors is used to check the so-called "second bootstrap condition" for the gluon Reggeization at the next-to-leading logarithmic approximation in perturbative QCD. We find that it is satisfied for both helicity conserving and non-conserving parts. The integrations are then performed for the explicit calculation of the impact factors in the massless quark case.

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1 Introduction

The BFKL equation [1] is very important for the theory of the Regge processes at high energy $\sqrt{s}$ in perturbative QCD. In particular, it can be used together with the DGLAP equation [2] for the description of the structure functions for the deep inelastic ep scattering at small values of the Bjorken variable x. It was derived more than twenty years ago in the leading logarithmic approximation (LLA) for the Regge region [1], that means summation of all the terms of the type $(\alpha_s \ln s)^n$. Recently also the radiative corrections to the kernel of the equation have been calculated [3]-[9] and the explicit form of the kernel in the next-to-leading approximation (NLA) for the case of forward scattering was found [10, 11]. A number of subsequent papers (see, for instance [12]) were devoted to the investigation of its consequences.

In the BFKL approach the high energy scattering amplitudes are expressed in terms of the Green function of two interacting Reggeized gluons and of the impact factors of the colliding particles [1, 10, 13, 14]. The BFKL equation allows to determine this Green function for forward scattering ($t = 0$ and singlet color state in the t-channel). The impact factors must be calculated separately and only in some cases (such as strongly-virtual photon or hard mesons) perturbation theory is applicable.

The key role in the derivation of the BFKL equation is played by the gluon Reggeization. In the LLA the Reggeization was noticed in the first orders of the perturbation theory and, assuming that it is correct to all orders, the equation for the t-channel partial waves of the elastic scattering amplitudes was derived [1]. It is evident that, for gluon quantum numbers in the t-channel, the solution of this equation must reproduce the gluon Reggeization. This was explicitly demonstrated in Ref. [1]. This "bootstrap" condition represents a stringent test of the gluon Reggeization, although it cannot be considered as a proof. In the LLA such proof was constructed in Ref. [15]. In the NLA the gluon Reggeization has only been checked in the first three orders of the perturbation theory [6]. Since it forms the base of the BFKL approach, a more stringent test is desirable. As well as in the LLA, such test is provided by the "bootstrap" condition. Using the gluon Reggeization as a base, it is possible to represent the high energy scattering amplitudes as a convolution of the impact factors of the colliding particles and of the Green function for two interacting Reggeized gluons (see Eq. (2.3) below) not only for the forward scattering, but also for non-zero momentum transfer $\sqrt{-t}$ and arbitrary color states in the t-channel [16]. For the case of octet color representation, the requirement of self-consistency leads to two "bootstrap" equations for the gluon Reggeization in the NLA (Eqs. (34) and (35) in Ref. [16]). Besides providing a stringent check of the gluon Reggeization, these equations are important since they contain almost all the values appearing in the NLA BFKL kernel and so provide a global test of calculations carried out over a long period of time [3]-[9] and only in a small part independently performed [8] or checked [17, 18].

The first bootstrap condition involves the kernel of the non-forward BFKL equation for octet color representation in the t-channel, expressed in terms of the effective vertices for the Reggeon-Reggeon interaction. The explicit demonstration that it is satisfied in the part concerning the quark–anti-quark contribution was given in Ref. [19] for arbitrary space-time dimension. The second bootstrap condition involves the impact factors of the scattered particles for octet color representation in the t-channel, expressed in terms of the effective vertices for the Reggeon-particle interaction. For the case of gluon impact factors, the explicit proof that this equation is satisfied has been given in Ref. [20] for arbitrary space-time dimension and massive quarks for both the helicity conserving and non-conserving parts. It must be stressed that for this proof it is sufficient to use the integral representation of the NLA non-forward gluon impact factors with color octet states in the t-channel and it is not necessary to perform explicit integration and to consider other color states. However, the impact factors have their own value, therefore in Ref. [20] they have been obtained in the gluon case for arbitrary color representation in the t-channel and the explicit integrations have been explicitly carried out in the massless quark case.

The main aim of this paper is to demonstrate the fulfillment of the second bootstrap condition also in the case of quark impact factors, along the same lines as in Ref. [20]. We determine an integral representation of the NLA non-forward quark impact factors for arbitrary space-time dimension and representation of the color group and check that the second bootstrap condition for the octet representation is satisfied for both the helicity conserving and non-conserving parts. The integrations are then carried out explicitly in the case of massless quarks.

The paper is organized as follows: in Section 2 we explain the method of calculation, in Section 3 and 4 we obtain the integral representation of the contributions to the quark impact factors from one-quark and from quark-gluon intermediate states, respectively; in Section 5 the check of the second bootstrap condition is explicitly demonstrated for both the helicity conserving and non-conserving parts; in Section 6 the integrations representing the NLA non-forward quark impact factors are carried out in the massless quark case. Section 7 contains the summary and a short discussion.

2 Method of calculation

The bootstrap conditions for the gluon Reggeization in the NLA were derived in Ref. [16]. The starting point is the elastic scattering process

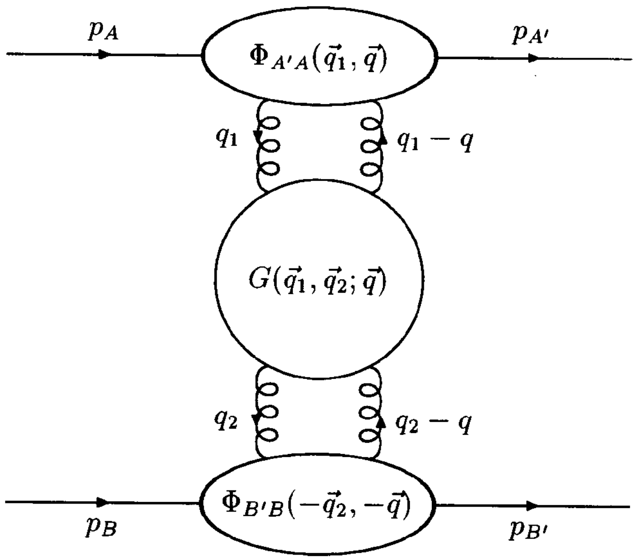

Figure 1: Diagrammatic representation of the elastic scattering amplitude $A + B \to A' + B'$.

$A + B \to A' + B'$ in the Regge kinematical region

$$s = (p_A + p_B)^2 = (p'_A + p'_B)^2 \to \infty \;, \quad t = (p_A - p'_A)^2 = (p'_B - p_B)^2 \quad \text{–fixed} \;, \tag{2.1}$$

where p_A, p_B and p'_A, p'_B are the momenta of the initial and final particles, respectively. We use the Sudakov decomposition for any vector p

$$p = \beta p_1 + \alpha p_2 + p_\perp, \qquad p_\perp^2 = -\vec{p}^{\,2} \;,$$

where the vectors (p_1, p_2) are the light-cone basis of the initial particle momenta plane (p_A, p_B)

$$p_A = p_1 + \frac{m_A^2}{s} p_2 \;, \qquad p_B = p_2 + \frac{m_B^2}{s} p_1 \;,$$
$$p_A^2 = m_A^2 \;, \qquad p_B^2 = m_B^2 \;, \qquad p_1^2 = p_2^2 = 0 \;, \qquad s \approx 2(p_1 p_2) \;.$$

In the Regge limit $s \gg -t$, the momentum transfer is dominated by its transverse part

$$q = p_A - p'_A \approx q_\perp \;, \qquad t = q^2 \approx q_\perp^2 = -\vec{q}^{\,2} \;.$$

In the case of gluon quantum numbers and negative signature in the t-channel, the amplitude for this elastic process has the Regge form

$$(\mathcal{A}_8^{(-)})_{AB}^{A'B'} = \Gamma^c_{A'A} \left[\left(\frac{-s}{-t} \right)^{j(t)} - \left(\frac{+s}{-t} \right)^{j(t)} \right] \Gamma^c_{B'B} \quad . \tag{2.2}$$

Here c is a color index, $\Gamma^c_{P'P}$ are the particle-particle-Reggeon (PPR) vertices which do not depend on s and $j(t) = 1 + \omega(t)$ is the Reggeized gluon trajectory. In the derivation of the BFKL equation, this form is assumed to be valid also in the NLA. On the other side, the s-channel unitarity of the scattering matrix leads to (see Fig. 1, where the wavy intermediate lines denote Reggeons)

$$\mathcal{I}m_s \left((\mathcal{A}_\mathcal{R})_{AB}^{A'B'} \right) = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}q_1}{\vec{q}_1^{\,2} \left(\vec{q}_1 - \vec{q}\right)^2} \int \frac{d^{D-2}q_2}{\vec{q}_2^{\,2} \left(\vec{q}_2 - \vec{q}\right)^2}$$
$$\times \sum_\nu \Phi^{(\mathcal{R},\nu)}_{A'A}(\vec{q}_1, \vec{q}; s_0) \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0} \right)^\omega G^{(\mathcal{R})}_\omega \left(\vec{q}_1, \vec{q}_2, \vec{q}\right) \right] \Phi^{(\mathcal{R},\nu)}_{B'B}(-\vec{q}_2, -\vec{q}; s_0) \,, \tag{2.3}$$

where $\mathcal{A}_\mathcal{R}$ stands for the scattering amplitude with the representation $\mathcal{R}$ of the color group in the t-channel. In the above equation, the index ν enumerates the states in the irreducible representation $\mathcal{R}$, $\Phi^{(\mathcal{R},\nu)}_{P'P}$ are the impact factors and $G^{(\mathcal{R})}_\omega$ is the Mellin transform of the Green function for the Reggeon-Reggeon scattering [16]. Here and below we do not indicate the signature since it is defined by the symmetry of the representation $\mathcal{R}$ in the product of the two octet representations. The parameter s_0 is an arbitrary energy scale introduced in order to define the partial wave expansion of the scattering amplitudes through the Mellin transform. The dependence on this parameter disappears in the full expressions for the amplitudes. The space-time dimension $D = 4 + 2\epsilon$ is kept different from four in order to regularize the infrared singularities. The Green function obeys the generalized BFKL equation

$$\omega G_\omega^{(\mathcal{R})}\left(\vec{q}_1, \vec{q}_2, \vec{q}\right) = \vec{q}_1^{\,2} \vec{q}_1^{\,\prime\,2} \delta^{(D-2)}\left(\vec{q}_1 - \vec{q}_2\right)$$
$$+ \int \frac{d^{D-2} q_r}{\vec{q}_r^{\,2} \vec{q}_r^{\,\prime\,2}} \mathcal{K}^{(\mathcal{R})}\left(\vec{q}_1, \vec{q}_r; \vec{q}\right) G_\omega^{(\mathcal{R})}\left(\vec{q}_r, \vec{q}_2; \vec{q}\right) \,, \qquad (2.4)$$

where $\mathcal{K}^{(\mathcal{R})}$ is the kernel in the NLA [16] and we have introduced the notation $q_i' \equiv q_i - q$ (which will be used also in the following).

The two bootstrap conditions derived in Ref. [16] follow from the comparison between the imaginary part of the amplitude (2.2) with the imaginary part given by Eq. (2.3) in the case of the gluon representation in the t-channel. In this paper we are interested in the second bootstrap condition (Eq. (35) in Ref. [16]) which includes the NLA correction $\Phi_{A'A}^{(8,a)(1)}$ to the octet impact factor and reads

$$-ig \int \frac{d^{D-2} q_1}{(2\pi)^{D-1}} \frac{\vec{q}^{\,2}}{\vec{q}_1^{\,2} \vec{q}_1^{\,\prime\,2}} \sqrt{N} \Phi_{A'A}^{(8,a)(1)}(\vec{q}_1, \vec{q}; s_0) = \Gamma_{A'A}^{(a)(1)} \, \omega^{(1)}(-\vec{q}^{\,2})$$
$$+\frac{1}{2} \Gamma_{A'A}^{(a)(B)} \left[\omega^{(2)}(-\vec{q}^{\,2}) + \left(\omega^{(1)}(-\vec{q}^{\,2}) \right)^2 \ln \left(\frac{s_0}{\vec{q}^{\,2}} \right) \right] , \qquad (2.5)$$

where g is the gauge coupling constant, N is the number of colors, $\omega^{(1)}$ and $\omega^{(2)}$ are the one- and two-loop contributions to the Reggeized gluon trajectory, $\Gamma_{A'A}^{(a)(B)}$ and $\Gamma_{A'A}^{(a)(1)}$ are the Born and one-loop parts of the PPR effective vertex.

The definition of the non-forward impact factors with color state ν of the irreducible representation $\mathcal{R}$ was given in Ref. [16]. This definition can be written as

$$\Phi_{A'A}^{(\mathcal{R},\nu)}(\vec{q}_1, \vec{q}; s_0) = \langle cc' | \hat{\mathcal{P}}_\mathcal{R} | \nu \rangle \Phi_{AA'}^{cc'}(\vec{q}_1, \vec{q}; s_0) \,, \qquad (2.6)$$

where $\hat{\mathcal{P}}_\mathcal{R}$ is the projector of the two-gluon color states in the t-channel on the irreducible representation $\mathcal{R}$ of the color group and the remaining part $\Phi_{AA'}^{cc'}$ is the unprojected impact factor of the particle A,

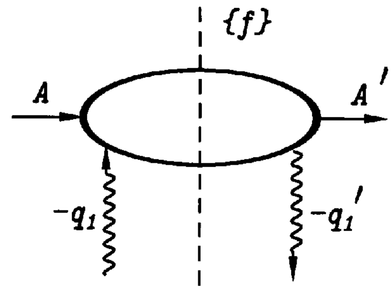

Figure 2: Schematic description of the intermediate states contributions to the impact factors.

$$\Phi^{cc'}_{AA'}(\vec{q}_1, \vec{q}; s_0)\}$$

$$= \left(\frac{s_0}{\vec{q}_1^{\,2}}\right)^{\frac{1}{2}\omega(-\vec{q}_1^{\,2})} \left(\frac{s_0}{\vec{q}_1^{\,\prime\,2}}\right)^{\frac{1}{2}\omega(-\vec{q}_1^{\,\prime\,2})} \sum_{\{f\}} \int \frac{ds_{AR}}{2\pi} d\rho_f \theta(s_\Lambda - s_{AR}) \Gamma^c_{\{f\}A} \left(\Gamma^{c'}_{\{f\}A'}\right)^*$$
$$-\frac{1}{2}\int \frac{d^{D-2}q_r}{\vec{q}_r^{\,2}\vec{q}_r^{\,\prime\,2}} \Phi^{c_1 c_1'(B)}_{AA'}(\vec{q}_r, \vec{q}) (\mathcal{K}^B_r)^{c_1' c'}_{c_1 c}(\vec{q}_r, \vec{q}_1; \vec{q}) \ln\left(\frac{s_\Lambda^2}{s_0(\vec{q}_r - \vec{q}_1)^{\,2}}\right). \quad (2.7)$$

For brevity we do not perform here and below an explicit expansion in α_s; evidently this expansion is assumed and only the leading and the next-to-leading terms should be kept. Here $\omega(t)$ is the Reggeized gluon trajectory which can be taken at leading order, $\Gamma^c_{\{f\}A}$ is the effective amplitude for the production of the system $\{f\}$ (see Fig. 2) in the collision of the particle A with the Reggeized gluon carrying color index c and momentum

$$-q_1 = \alpha p_B - q_{1\perp}, \quad \alpha \approx \left(s_{AR} - m_A^2 + \vec{q}_1^{\,2}\right)/s \ll 1\,, \quad q_1^2 = -\vec{q}_1^{\,2}\,, \quad (2.8)$$

s_{AR} is the particle-Reggeon squared invariant mass and the momentum of the other Reggeon is $q_1' = q_1 - q$, with $q_1'^{\,2} = -(\vec{q}_1 - \vec{q})^{\,2}$. In the fragmentation region of the particles A and A', all the transverse momenta as well as the squared invariant squared mass s_{AR} are of the order of the typical virtuality, i.e. $\vec{q}_1^{\,2}$, $s_{AR} \sim \vec{q}^{\,2}$.

In Eq. (2.7) the summation is over all the possible intermediate states which can be produced in the NLA and the integration is over the particle-Reggeon squared invariant mass and over the phase space of the produced system

$$d\rho_f = (2\pi)^D \delta^{(D)}(p_A - q_1 - \sum_{m=1}^{n} k_m) \prod_{m=1}^{n} \frac{d^{D-1} k_m}{2\epsilon_m (2\pi)^{D-1}} , \qquad (2.9)$$

in the case of an n-particle system. The parameter s_Λ, which limits the integration over s_{AR} in Eq. (2.7), is introduced in order to separate the contributions from multi-Regge and quasi-multi-Regge kinematics (MRK and QMRK) and is to be considered as tending to infinity. The dependence on this parameter disappears due to the cancellation between the first and the second terms in the R.H.S. of Eq. (2.7). The second term of Eq. (2.7) contains the s_0-independent Born contribution to the impact factor, $\Phi^{cc'(B)}_{AA'}$, and the unprojected part of the non-forward BFKL kernel, connected with real gluon production, at the Born level

$$(\mathcal{K}_r^B)^{c_1'c'}_{c_1c}(\vec{q}_1, \vec{q}_2, \vec{q}) = \frac{1}{2(2\pi)^{D-1}} \sum_{\lambda_G} \gamma^{G(B)}_{c_1c}(q_1, q_2) \left(\gamma^{G(B)}_{c_1'c'}(q_1', q_2')\right)^* , \quad (2.10)$$

being $\gamma^{G(B)}_{c_1c}(k_1, q_1)$ the Born effective amplitude for the production of one gluon G with helicity λ_G in the collision of two Reggeized gluons carrying color indices c_1, c and momenta q_1, $-q_2$, respectively. The definitions (2.6) and (2.7) apply also in the case of colorless particle as well as in the case of charged QCD particles. Of course, the impact factor in the octet color representation, entering the bootstrap condition (2.5), makes sense only for colored particles.

In this paper we consider the non-forward quark impact factors. At the NLA the only intermediate states $\{f\}$ which can contribute to the quark impact factor (2.7) are one-quark and a quark-gluon system. The second term in the R.H.S of Eq. (2.7), which is a counterterm for the LLA part of the first term, will be attributed to the quark-gluon intermediate state. In the following, we will determine the integral representation of the non-forward quark impact factors in arbitrary space-time dimension $D = 4 + 2\epsilon$ and keeping the quark massive. Then, after checking that the bootstrap condition (2.5) is satisfied, we perform the integrations for the case of massless quarks, using the expansion in ϵ when necessary.

3 One-quark contribution

In the case of the one-quark contribution, the squared invariant mass s_{AR} is equal to the m_A^2, the squared mass of the colliding quark flavor. From the definition (2.7) one easily gets

$$\Phi_{AA'}^{cc'\{Q\}}(\vec{q}_1, \vec{q}; s_0) = \left(\frac{s_0}{\vec{q}_1^{\,2}}\right)^{\omega(-\vec{q}_1^{\,2})/2} \left(\frac{s_0}{\vec{q}_1^{\,\prime\,2}}\right)^{\omega(-\vec{q}_1^{\,\prime\,2})/2} \sum_{\lambda_Q} \Gamma_{QA}^{c} \left(\Gamma_{QA'}^{c'}\right)^* , \tag{3.1}$$

where Γ_{QA}^c is the quark-quark-Reggeon (QQR) effective vertex, which was obtained in Refs. [21, 22] and has the form

$$\Gamma_{QA}^{c} = g(t^c)_{QA} \left[\delta_{\lambda_Q,\lambda_A} \left(1 + \Gamma_{QQ}^{(+)(1)}(q_1^2)\right) + \delta_{\lambda_Q,-\lambda_A} \Gamma_{QQ}^{(-)(1)}(q_1^2)\right] . \tag{3.2}$$

Here $\Gamma_{QQ}^{(\pm)(1)}$ represent the radiative corrections to the helicity conserving and to the helicity non-conserving parts of the QQR effective interaction vertex, t_{QA}^c is the (Q, A) matrix element of the color group generator t^c in the fundamental representation and the δ-symbols on the helicities λ_Q and λ_A of the quarks A and Q are defined as

$$\delta_{\lambda_Q,\lambda_A} \equiv \overline{u}(p_Q) \frac{\not{p}_B}{s} u(p_A) , \tag{3.3}$$

$$\begin{aligned} \delta_{\lambda_Q,-\lambda_A} &\equiv \frac{i}{\sqrt{-(p_A - p_Q)^2}} \overline{u}(p_Q)(\not{p}_A - \not{p}_Q) \frac{\not{p}_B}{s} u(p_A) \\ &= \frac{i}{\sqrt{-(p_A - p_Q)^2}} \overline{u}(p_Q) \left(1 - 2m_A \frac{\not{p}_B}{s}\right) u(p_A) . \end{aligned} \tag{3.4}$$

The convolution in Eq. (3.1) can be easily calculated and gives

$$\Phi^{cc'\{Q\}}_{AA'}(\vec{q}_1,\vec{q};s_0) = g^2(t^{c'}t^c)_{A'A}\left[\delta_{\lambda_{A'},\lambda_A}\left(1+\frac{1}{2}\omega^{(1)}(-\vec{q}_1^{\,2})\ln\left(\frac{s_0}{\vec{q}_1^{\,2}}\right)\right.\right.$$

$$+\frac{1}{2}\omega^{(1)}(-\vec{q}_1^{\,\prime\,2})\ln\left(\frac{s_0}{\vec{q}_1^{\,\prime\,2}}\right)+\Gamma^{(+)(1)}_{QQ}(-\vec{q}_1^{\,2})+\Gamma^{(+)(1)}_{QQ}(-\vec{q}_1^{\,\prime\,2})\Bigg)$$

$$+\Gamma^{(-)(1)}_{QQ}(-\vec{q}_1^{\,2})\frac{i}{\sqrt{\vec{q}_1^{\,2}}}\overline{u}(p_{A'})\frac{\not{q}_{1_\perp}\not{p}_B}{s}u(p_A)$$

$$\left.+\Gamma^{(-)(1)}_{QQ}(-\vec{q}_1^{\,\prime\,2})\frac{i}{\sqrt{\vec{q}_1^{\,\prime\,2}}}\overline{u}(p_{A'})\frac{\not{p}_B\not{q}'_{1_\perp}}{s}u(p_A)\right]\,. \qquad (3.5)$$

Here the one-loop contribution to the Reggeized gluon trajectory $\omega^{(1)}(-\vec{v}^{\,2})$ is given by [23]

$$\omega^{(1)}(-\vec{v}^{\,2}) = -\frac{g^2N}{2}\int\frac{d^{D-2}k}{(2\pi)^{D-1}}\frac{\vec{v}^{\,2}}{\vec{k}^{\,2}(\vec{k}-\vec{v})^2} = -\frac{g^2N}{(4\pi)^{2+\epsilon}}\,\Gamma(1-\epsilon)\,\frac{[\Gamma(\epsilon)]^2}{\Gamma(2\epsilon)}(\vec{v}^{\,2})^\epsilon\,. \qquad (3.6)$$

The radiative corrections $\Gamma^{(+)(1)}_{QQ}$ have the form [22]:

$$\Gamma^{(+)(1)}_{QQ}(-\vec{v}^{\,2}) = a^{(+)}_f(-\vec{v}^{\,2})+a^{(+)}_Q(-\vec{v}^{\,2},m_A^2)+a^{(+)}_g(-\vec{v}^{\,2})+\delta^{(+)}_g(-\vec{v}^{\,2},m_A^2)\,, \qquad (3.7)$$

with

$$a^{(+)}_f(-\vec{v}^{\,2}) = -g^2\frac{2}{(4\pi)^{2+\epsilon}}\Gamma(-\epsilon)\sum_f\int_0^1 dx\,x(1-x)\,[m_f^2+x(1-x)\vec{v}^{\,2}]^\epsilon\,, \qquad (3.8)$$

$$a^{(+)}_Q(-\vec{v}^{\,2},m_A^2) = \frac{g^2}{2N}\frac{\Gamma(-\epsilon)}{(4\pi)^{2+\epsilon}}\left\{\int_0^1\frac{dx}{[m_A^2+x(1-x)\vec{v}^{\,2}]^{1-\epsilon}}\right.$$

$$\left.\times\left[-\vec{v}^{\,2}\left(\frac{1}{1+2\epsilon}+\frac{\epsilon}{2}\right)-\frac{2m_A^2}{1+2\epsilon}\right]+\frac{2}{1+2\epsilon}(m_A^2)^\epsilon\right\}\,, \qquad (3.9)$$

$$a^{(+)}_g(-\vec{v}^{\,2}) = g^2N\,\frac{\Gamma(-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{[\Gamma(1+\epsilon)]^2}{\Gamma(1+2\epsilon)}(\vec{v}^{\,2})^\epsilon\left\{\psi(1-\epsilon)-2\psi(\epsilon)+\psi(1)\right.$$

$$\left.+\frac{1}{1+2\epsilon}\left[\frac{1}{4(3+2\epsilon)}-\frac{1}{\epsilon}-\frac{7}{4}\right]\right\}\,, \qquad (3.10)$$

$$\delta_g^{(+)}(-\vec{v}^{\,2}, m_A^2) = \frac{g^2 N}{(4\pi)^{2+\epsilon}}\Gamma(1-\epsilon)\left\{\int_0^1 dx_1 \int_0^1 dx_2\ \theta(1-x_1-x_2)\right.$$

$$\times\left[\frac{-\vec{v}^{\,2}(1-x_1)}{x_1}\left(1-x_1+\frac{1+\epsilon}{2}x_1^2\right)\left(\frac{1}{[m_A^2 x_1^2 + x_2(1-x_1-x_2)\vec{v}^{\,2}]^{1-\epsilon}}\right.\right.$$

$$\left.\left.-\frac{1}{[x_2(1-x_1-x_2)\vec{v}^{\,2}]^{1-\epsilon}}\right) - \frac{2m_A^2 x_1}{[m_A^2 x_1^2 + x_2(1-x_1-x_2)\vec{v}^{\,2}]^{1-\epsilon}}\right] + \left.\frac{(m_A^2)^\epsilon}{\epsilon(1+2\epsilon)}\right\}. \tag{3.11}$$

The expression for $\Gamma_{QQ}^{(-)(1)}$ can be found in Ref. [21]:

$$\Gamma_{QQ}^{(-)(1)}(-\vec{v}^{\,2}) = a_g^{(-)}(-\vec{v}^{\,2}) + a_Q^{(-)}(-\vec{v}^{\,2}, m_A^2)\ , \tag{3.12}$$

with

$$a_g^{(-)}(-\vec{v}^{\,2}) = -i\sqrt{\vec{v}^{\,2}}\, m_A\, \frac{g^2}{2N}\, \frac{\Gamma(1-\epsilon)}{(4\pi)^{1+\epsilon}}\, \frac{1-2\epsilon}{1+2\epsilon} \int_0^1 \frac{dx}{[m_A^2 + x(1-x)\vec{v}^{\,2}]^{1-\epsilon}} \tag{3.13}$$

and

$$a_Q^{(-)}(-\vec{v}^{\,2}, m_A^2)$$

$$= -i\sqrt{\vec{v}^{\,2}}\, m_A\, g^2 N\, \frac{\Gamma(1-\epsilon)}{(4\pi)^{1+\epsilon}} \int_0^1 dx_1 \int_0^1 dx_2\, \frac{\theta(1-x_1-x_2)[x_1-(1+\epsilon)x_1^2]}{[m_A^2 x_1^2 + x_2(1-x_1-x_2)\vec{v}^{\,2}]^{1-\epsilon}}\ . \tag{3.14}$$

In the above equations, $\Gamma(x)$ is the Euler Γ-function and $\psi(x)$ its logarithmic derivative. We observe, moreover, that $\delta_g^{(+)}$, $a_g^{(-)}$ and $a_Q^{(-)}$ vanish in the zero quark mass limit.

4 Quark-gluon contribution

In this Section we calculate the NLA contribution to the quark impact factor from the quark-gluon production in the fragmentation region

$$\Phi_{AA'}^{cc'(1)\{QG\}}(\vec{q}_1, \vec{q})$$

$$= \sum_{\substack{\lambda_Q,\lambda_G \\ Q,g}} \int \frac{ds_{AR}}{(2\pi)} d\rho_{\{QG\}}\ \theta(s_\Lambda - s_{AR})\Gamma_{\{QG\}A}^{c}(q_1)\left(\Gamma_{\{QG\}A'}^{c'}(q_1')\right)^*\ , \tag{4.1}$$

where the summation is performed over the helicities λ_G and λ_Q and over the color indices g and Q of the produced gluon and quark, having momenta k_1 and k_2, respectively (although we use the notation g also for the coupling constant, this can not be misleading here). We remind that the above expression must be completed including the counterterm, i.e. the second term in the R.H.S. of Eq. (2.7). We will consider this term at the end of the Section. Introducing the Sudakov representation for the momenta of the produced particles,

$$k_1 = \beta_1 p_1 + \frac{\vec{k}_1^{\,2}}{s\beta_1} p_2 + k_{1_\perp} \ , \quad k_1^2 = 0 \ , \tag{4.2}$$

$$k_2 = \beta_2 p_1 + \frac{m_A^2 + \vec{k}_2^{\,2}}{s\beta_2} p_2 + k_{2_\perp} \ , \quad k_2^2 = m_A^2 \ . \tag{4.3}$$

we have

$$s_{AR} = (k_1 + k_2)^2 = \frac{\beta_1(\beta_1 + \beta_2)m_A^2 + (\vec{k}_1\beta_2 - \vec{k}_2\beta_1)^2}{\beta_1\beta_2} \ , \tag{4.4}$$

$$\frac{ds_{AR}}{2\pi} d\rho_{\{QG\}} = \delta(1-\beta_1-\beta_2)\delta^{(D-2)}\left((k_1 + k_2 + q_1)_\perp\right) \frac{d\beta_1 d\beta_2}{\beta_1\beta_2} \frac{d^{D-2}k_1 \, d^{D-2}k_2}{2(2\pi)^{D-1}} . \tag{4.5}$$

In the last of these equations we have used also Eq. (2.8).

The amplitude for quark-gluon production in the quark-Reggeon collision $\Gamma^c_{\{QG\}A}$ was obtained in Ref. [22] using for convenience the the gauge $(ep_2) = 0$, although all the calculations could be performed in a gauge invariant way. It can be written in the following form:

$$\Gamma^c_{\{QG\}A} = g^2 \left[(t^c t^g)_{QA} \mathcal{A}_1 - (t^g t^c)_{QA} \mathcal{A}_2 \right] \ , \tag{4.6}$$

with

$$\mathcal{A}_1 = \overline{u}(k_2) \left[L(k_{1_\perp}) - L(-k_{2_\perp}) \right] \frac{\not{p}_B}{s} u(p_A) \tag{4.7}$$

and

$$\mathcal{A}_2 = \overline{u}(k_2) \left[L(\beta_2 k_{1_\perp} - \beta_1 k_{2_\perp}) - L(-k_{2_\perp}) \right] \frac{\not{p}_B}{s} u(p_A) \ , \tag{4.8}$$

where

$$L(k_\perp) = L_\mu(k_\perp) e_\perp^{*\,\mu} \;, \qquad L^\mu(k_\perp) = \frac{\gamma_\perp^\mu (m_A \beta_1^2 - \beta_1 \not{k}_\perp) + 2k_\perp^\mu}{m_A^2 \beta_1^2 - k_\perp^2} \;, \tag{4.9}$$

being e^μ the gluon polarization vector and $\gamma_\perp^\mu$ the transverse component of the Dirac γ^μ, defined as

$$\gamma_\perp^\mu = g_\perp^{\mu\nu} \gamma_\nu \;, \qquad g^{\mu\nu} = \frac{p_A^\mu p_B^\nu + p_A^\nu p_B^\mu}{(p_A p_B)} + g_\perp^{\mu\nu} \;. \tag{4.10}$$

The amplitude $\Gamma^{c'}_{\{QG\}A'}$ has the same form of the amplitude $\Gamma^{c}_{\{QG\}A}$, except that the Sudakov basis (p_1, p_2) must be replaced by the *primed* one $(p_{1'}, p_{2'})$, i.e. by the light-cone basis of the final particle momenta plane ($p_{A'}$, $p_{B'}$). This leads to

$$\Gamma^{c'}_{\{QG\}A'} = g^2 \left[(t^{c'} t^g)_{QA'} \mathcal{A}_1' - (t^g t^{c'})_{QA'} \mathcal{A}_2' \right] , \tag{4.11}$$

with

$$\mathcal{A}_1' = \overline{u}(k_2) \left[L(k_{1_\perp} + \beta_1 q_\perp) - L(-k_{2_\perp} - \beta_2 q_\perp)] \right] \frac{\not{p}_B}{s} u(p_{A'}) \tag{4.12}$$

and

$$\mathcal{A}_2' = \overline{u}(k_2) \left[L(\beta_2 k_{1_\perp} - \beta_1 k_{2_\perp}) - L(-k_{2_\perp} - \beta_2 q_\perp) \right] \frac{\not{p}_B}{s} u(p_{A'}) \;. \tag{4.13}$$

We can now calculate the convolution appearing in the integrand of Eq. (4.1):

$$\sum_{\substack{\lambda_Q, \lambda_G \\ Q, g}} \Gamma^c_{\{QG\}A} \left(\Gamma^{c'}_{\{QG\}A'} \right)^* =$$

$$g^4 \Bigg[(t^g t^{c'} t^c t^g)_{A'A} \sum_{\lambda_Q, \lambda_G} \mathcal{A}_1 (\mathcal{A}_1')^* - (t^{c'} t^g t^c t^g)_{A'A} \sum_{\lambda_Q, \lambda_G} \mathcal{A}_1 (\mathcal{A}_2')^*$$

$$- (t^g t^{c'} t^g t^c)_{A'A} \sum_{\lambda_Q, \lambda_G} \mathcal{A}_2 (\mathcal{A}_1')^* + (t^{c'} t^g t^g t^c)_{A'A} \sum_{\lambda_Q, \lambda_G} \mathcal{A}_2 (\mathcal{A}_2')^* \Bigg]$$

$$= \frac{g^4}{4}\delta_{A'A}\delta^{cc'} \sum_{\lambda_Q,\lambda_G} \mathcal{A}_1(\mathcal{A}_1')^* + \frac{g^4}{2N}(t^{c'}t^c)_{A'A} \sum_{\lambda_Q,\lambda_G} \Big[N^2\, \mathcal{A}_2(\mathcal{A}_2')^*$$

$$+\mathcal{A}_1(\mathcal{A}_2')^* + \mathcal{A}_2(\mathcal{A}_1')^* - \mathcal{A}_1(\mathcal{A}_1')^* - \mathcal{A}_2(\mathcal{A}_2')^* \Big] \,. \tag{4.14}$$

In the last equality the following relations have been used:

$$(t^g t^g)_{A'A} = \frac{N^2-1}{2N}\delta_{A'A} \;, \quad (t^g t^c t^g)_{A'A} = -\frac{1}{2N}(t^c)_{A'A} \;,$$

$$(t^g t^{c'} t^c t^g)_{A'A} = \frac{1}{4}\delta_{A'A}\delta^{cc'} - \frac{1}{2N}(t^{c'}t^c)_{A'A} \;. \tag{4.15}$$

Using the definitions of the quantities $\mathcal{A}_{1,2}$ and $\mathcal{A}'_{1,2}$ given in Eqs. (4.7), (4.8), (4.12) and (4.13) and recalling that

$$\sum_\lambda e_\perp^\mu(k,\lambda) e_\perp^{*\,\nu}(k,\lambda) = -g_\perp^{\mu\nu} \;, \tag{4.16}$$

it is easy to find

$$\sum_{\lambda_Q,\lambda_G} \mathcal{A}_2(\mathcal{A}_2')^* \equiv -A_Q = -\overline{u}(p_{A'})\frac{\not p_B}{s}\Big[\overline{L}_\mu(\beta_2 k_{1_\perp} - \beta_1 k_{2_\perp}) - \overline{L}_\mu(-k_{2_\perp} - \beta_2 q_\perp)\Big]$$

$$\times(\not k_2 + m_A)\Big[L^\mu(\beta_2 k_{1_\perp} - \beta_1 k_{2_\perp}) - L^\mu(-k_{2_\perp})\Big]\frac{\not p_B}{s}u(p_A) \;, \tag{4.17}$$

$$\sum_{\lambda_Q,\lambda_G}\Big[\mathcal{A}_1(\mathcal{A}_2')^* + \mathcal{A}_2(\mathcal{A}_1')^* - \mathcal{A}_1(\mathcal{A}_1')^* - \mathcal{A}_2(\mathcal{A}_2')^*\Big] \equiv B_Q =$$

$$\overline{u}(p_{A'})\frac{\not p_B}{s}\Big[\overline{L}_\mu(k_{1_\perp} + \beta_1 q_\perp) - \overline{L}_\mu(\beta_2 k_{1_\perp} - \beta_1 k_{2_\perp})\Big]$$

$$\times(\not k_2 + m_A)\Big[L^\mu(k_{1_\perp}) - L^\mu(\beta_2 k_{1_\perp} - \beta_1 k_{2_\perp})\Big]\frac{\not p_B}{s}u(p_A) \;, \tag{4.18}$$

$$\sum_{\lambda_Q,\lambda_G} \mathcal{A}_1(\mathcal{A}_1')^* \equiv -C_Q = -\overline{u}(p_{A'})\frac{\not p_B}{s}\Big[\overline{L}_\mu(k_{1_\perp} + \beta_1 q_\perp) - \overline{L}_\mu(-k_{2_\perp} - \beta_2 q_\perp)\Big]$$

$$\times(\not k_2 + m_A)\Big[L^\mu(k_{1_\perp}) - L^\mu(-k_{2_\perp})\Big]\frac{\not p_B}{s}u(p_A) \;, \tag{4.19}$$

where

$$\overline{L}^{\mu}(k_\perp) = \frac{(m_A\beta_1^2 - \beta_1 \not{k}_\perp)\gamma_\perp^\mu + 2k_\perp^\mu}{m_A^2\beta_1^2 - k_\perp^2} \; . \tag{4.20}$$

The above expressions for A_Q, B_Q and C_Q can be put in the following form

$$A_Q = A_Q^{(+)} + A_Q^{(-)} \; , \tag{4.21}$$

$$A_Q^{(+)} = \beta_2^2 \Big\{ [2(1-\beta_1)^2 + (1+\epsilon)\beta_1^2(1-\beta_1)] d(k_{2_\perp}) d(k_{2_\perp} + \beta_2 q_\perp) d(\beta_2 k_{1_\perp} - \beta_1 k_{2_\perp})$$

$$\times \left[\frac{\vec{q}^{\,2}}{d(\beta_2 k_{1_\perp} - \beta_1 k_{2_\perp})} - \frac{\vec{q}_1^{\,2}}{d(k_{2_\perp} + \beta_2 q_\perp)} - \frac{\vec{q}_1^{\,\prime\,2}}{d(k_{2_\perp})} \right] + 4m_A^2\beta_1^2$$

$$\times \Big[d(\beta_2 k_{1_\perp} - \beta_1 k_{2_\perp}) - d(k_{2_\perp} + \beta_2 q_\perp) \Big] \Big[d(\beta_2 k_{1_\perp} - \beta_1 k_{2_\perp}) - d(k_{2_\perp}) \Big] \Big\} \delta_{\lambda_{A'},\lambda_A} \; ,$$

$$A_Q^{(-)} = 2m_A\beta_1^2\beta_2(1-\beta_1)[(1+\epsilon)\beta_1 - 1]\overline{u}(p_{A'}) \Big[\not{q}_\perp d(k_{2_\perp}) d(k_{2_\perp} + \beta_2 q_\perp) \tag{4.22}$$

$$- \not{q}_{1_\perp} d(k_{2_\perp}) d(\beta_2 k_{1_\perp} - \beta_1 k_{2_\perp}) + \not{q}_{1_\perp}^{\prime} d(k_{2_\perp} + \beta_2 q_\perp) d(\beta_2 k_{1_\perp} - \beta_1 k_{2_\perp}) \Big] \frac{\not{p}_B}{s} u(p_A) \; ;$$

$$B_Q = B_Q^{(+)} + B_Q^{(-)} \; , \tag{4.23}$$

$$B_Q^{(+)} = \beta_1^2\beta_2 \Big\{ [2\beta_2 + (1+\epsilon)\beta_1^2] \, d(k_{1_\perp}) d(k_{1_\perp} + \beta_1 q_\perp) d(\beta_2 k_{1_\perp} - \beta_1 k_{2_\perp})$$

$$\times \left[\frac{\vec{q}^{\,2}}{d(\beta_2 k_{1_\perp} - \beta_1 k_{2_\perp})} - \frac{\vec{q}_1^{\,2}}{d(k_{1_\perp} + \beta_1 q_\perp)} - \frac{\vec{q}_1^{\,\prime\,2}}{d(k_{1_\perp})} \right] + 4m_A^2\beta_2$$

$$\times \Big[d(\beta_2 k_{1_\perp} - \beta_1 k_{2_\perp}) - d(k_{1_\perp} + \beta_1 q_\perp) \Big] \Big[d(\beta_2 k_{1_\perp} - \beta_1 k_{2_\perp}) - d(k_{1_\perp}) \Big] \Big\} \delta_{\lambda_{A'},\lambda_A} ,$$

$$B_Q^{(-)} = 2m_A\beta_1^3\beta_2[(1+\epsilon)\beta_1 - 1]\overline{u}(p_{A'}) \Big[- \not{q}_\perp \, d(k_{1_\perp}) d(k_{1_\perp} + \beta_1 q_\perp) \tag{4.24}$$

$$+ \not{q}_{1_\perp} d(k_{1_\perp}) d(\beta_2 k_{1_\perp} - \beta_1 k_{2_\perp}) - \not{q}_{1_\perp}^{\prime} d(k_{1_\perp} + \beta_1 q_\perp) d(\beta_2 k_{1_\perp} - \beta_1 k_{2_\perp}) \Big] \frac{\not{p}_B}{s} u(p_A) \; ;$$

$$C_Q = C_Q^{(+)} + C_Q^{(-)} \ , \tag{4.25}$$

$$C_Q^{(+)} = \beta_2 \Big\{ [2\beta_2 + (1+\epsilon)\beta_1^2] \Big[\beta_1^2 \vec{q}^{\,2} \ d(k_{1_\perp}) d(k_{1_\perp} + \beta_1 q_\perp)$$

$$+ \beta_2^2 \vec{q}^{\,2} \ d(k_{2_\perp}) d(k_{2_\perp} + \beta_2 q_\perp)$$

$$- (\vec{q}_1 - \beta_2 \vec{q})^2 \ d(k_{1_\perp}) d(k_{2_\perp} + \beta_2 q_\perp) - (\vec{q}_1 - \beta_1 \vec{q})^2 \ d(k_{2_\perp}) d(k_{1_\perp} + \beta_1 q_\perp) \Big]$$

$$+ 4 m_A^2 \beta_1^2 \beta_2 \Big[d(k_{1_\perp} + \beta_1 q_\perp) - d(k_{2_\perp} + \beta_2 q_\perp) \Big] \Big[d(k_{1_\perp}) - d(k_{2_\perp}) \Big] \Big\} \delta_{\lambda_{A'}, \lambda_A} \ ,$$

$$C_Q^{(-)} = 2 m_A \beta_1^2 \beta_2 [(1+\epsilon)\beta_1 - 1] \overline{u}(p_{A'}) \Big[-\beta_1 \not{q}_\perp \ d(k_{1_\perp}) d(k_{1_\perp} + \beta_1 q_\perp) \tag{4.26}$$

$$+ \beta_2 \not{q}_\perp \ d(k_{2_\perp}) d(k_{2_\perp} + \beta_2 q_\perp) - (\not{q}_{1_\perp} - \beta_1 \not{q}_\perp) \ d(k_{2_\perp}) d(k_{1_\perp} + \beta_1 q_\perp)$$

$$+ (\not{q}_{1_\perp} - \beta_2 \not{q}_\perp) \ d(k_{1_\perp}) d(k_{2_\perp} + \beta_2 q_\perp) \Big] \frac{\not{p}_B}{s} u(p_A) \ ;$$

where we have used $d(l) = 1/(m_A^2 \beta_1^2 - l^2)$ and have explicitly separated the helicity conserving terms (labeled by (+)) from the terms which, after integration, give the helicity non-conserving contribution to the bootstrap condition (2.5) (labeled by (−)).

Putting together the above results and using Eq. (4.5), the quark-gluon production contribution to the impact factor can be written as

$$\Phi_{AA'}^{cc'(1)\{QG\}}(\vec{q}_1, \vec{q}) =$$

$$g^4 \int \frac{d^{D-2} k_1}{(2\pi)^{D-1}} \int_{\beta_0}^1 \frac{d\beta}{2\beta(1-\beta)} \left\{ -\delta_{A'A} \delta^{cc'} \frac{C_Q}{4} + \frac{1}{2N} (t^{c'} t^c)_{A'A} \Big[-N^2 A_Q + B_Q \Big] \right\}, \tag{4.27}$$

where now $\beta = \beta_1$ and in the expressions for A_Q, B_Q and C_Q the variables β_2 and $k_{2_\perp}$ are replaced with $(1-\beta)$ and $-q_{1_\perp} - k_{1_\perp}$, respectively. The lower limit in the integration over β is $\beta_0 = \vec{k}_1^{\,2}/s_\Lambda$ and follows from the cut in the integration over s_{AR} given by the θ-function in the Eq. (4.1).

Let us consider first the integral

$$I_A^{(+)}(\vec{q}_1, \vec{q}) = \int \frac{d^{D-2}k_1}{(2\pi)^{D-1}} \int_{\beta_0}^{1} \frac{d\beta}{2\beta(1-\beta)} A_Q^{(+)} \; . \qquad (4.28)$$

It is convenient to split this integral into $I_A^{(+)}(\vec{q}_1, \vec{q}, m_A = 0)$ and the remaining part, which we call $\delta I_A^{(+)}(\vec{q}_1, \vec{q})$. Then, we can use Eq. (93) of Ref. [22] to write

$$I_A^{(+)}(\vec{q}_1, \vec{q}, m_A = 0) = \int \frac{d^{D-2}k_1}{(2\pi)^{D-1}} \left[\frac{\vec{q}^{\,2}}{(\vec{k}_1 - \vec{q}_1)^2 (\vec{k}_1 - \vec{q}_1^{\,\prime})^2} - \frac{\vec{q}_1^{\,2}}{\vec{k}_1^{\,2} (\vec{k}_1 - \vec{q}_1)^2} \right.$$
$$\left. - \frac{\vec{q}_1^{\,\prime\,2}}{\vec{k}_1^{\,2} (\vec{k}_1 - \vec{q}_1^{\,\prime})^2} \right] \left[\ln\left(\frac{s_\Lambda}{\vec{k}_1^{\,2}}\right) + \psi(1) - \psi(1+2\epsilon) - \frac{3}{4(1+2\epsilon)} \right] \delta_{\lambda_{A'}, \lambda_A} \; , \qquad (4.29)$$

where we recall that $\vec{q}_1^{\,\prime} = \vec{q}_1 - \vec{q}$.

In the calculation of $\delta I_A^{(+)}(\vec{q}_1, \vec{q})$ we can put the lower limit in the integration over β equal to zero, since there is no divergence for $s_\Lambda \to \infty$. The result is

$$\delta I_A^{(+)}(\vec{q}_1, \vec{q}) = \delta_{\lambda_{A'}, \lambda_A} \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \int_0^1 d\beta \, \frac{(1-\beta)^2}{\beta} \int_0^1 dx \left\{ [2(1-\beta) + (1+\epsilon)\beta^2] \right.$$
$$\times \left[\frac{\vec{q}^{\,2}}{c(m_A\beta, (1-\beta)q_\perp)} - \frac{\vec{q}^{\,2}}{c(0, (1-\beta)q_\perp)} - \frac{\vec{q}_1^{\,2}}{c(m_A\beta, (1-\beta)q_{1_\perp})} \right.$$
$$\left. + \frac{\vec{q}_1^{\,2}}{c(0, (1-\beta)q_{1_\perp})} - \frac{\vec{q}_1^{\,\prime\,2}}{c(m_A\beta, (1-\beta)q'_{1_\perp})} + \frac{\vec{q}_1^{\,\prime\,2}}{c(0, (1-\beta)q'_{1_\perp})} \right]$$
$$+ 4m_A^2 \frac{\beta^2}{1-\beta} \left[\frac{1}{c(m_A\beta, 0)} \right.$$
$$\left. \left. - \frac{1}{c(m_A\beta, (1-\beta)q'_{1_\perp})} - \frac{1}{c(m_A\beta, (1-\beta)q_{1_\perp})} + \frac{1}{c(m_A\beta, (1-\beta)q_\perp)} \right] \right\} \; , \qquad (4.30)$$

where we have used $c(m, l) = [m^2 - l^2 x(1-x)]^{1-\epsilon}$.

We consider now the integral

$$I_A^{(-)}(\vec{q}_1,\vec{q}) = \int_0^1 \frac{d\beta}{2\beta(1-\beta)} \int \frac{d^{D-2}k_1}{(2\pi)^{D-1}} A_Q^{(-)} \ , \tag{4.31}$$

where we have put again $\beta_0 = 0$ since there is no divergence for infinite s_Λ. The result of the integration over $\vec{k}_1$ is

$$I_A^{(-)}(\vec{q}_1,\vec{q}) = \frac{2m_A}{(4\pi)^{2+\epsilon}}\Gamma(1-\epsilon) \int_0^1 d\beta\, \beta(1-\beta)[(1+\epsilon)\beta - 1] \int_0^1 dx\, \overline{u}(p_{A'})$$
$$\times \left[\frac{\not{q}_\perp}{c(m_A\beta,(1-\beta)q_\perp)} - \frac{\not{q}_{1_\perp}}{c(m_A\beta,(1-\beta)q_{1_\perp})} \right.$$
$$\left. + \frac{\not{q}'_{1_\perp}}{c(m_A\beta,(1-\beta)q'_{1_\perp})} \right] \frac{\not{p}_B}{s} u(p_A). \tag{4.32}$$

The next integrals we consider are

$$I_B^{(\pm)}(\vec{q}_1,\vec{q}) = \int_0^1 \frac{d\beta}{2\beta(1-\beta)} \int \frac{d^{D-2}k_1}{(2\pi)^{D-1}} B_Q^{(\pm)} \ , \tag{4.33}$$

for which again β_0 can be put equal to zero. We have

$$I_B^{(+)}(\vec{q}_1,\vec{q}) = \frac{\Gamma(-\epsilon)}{(4\pi)^{2+\epsilon}} \int_0^1 dx \left\{ \left(\frac{\epsilon}{2} + \frac{1}{1+2\epsilon} \right) \right.$$
$$\times \left[-\frac{\vec{q}^{\,2}}{c(m_A,q_\perp)} + \frac{\vec{q}_1^{\,2}}{c(m_A,q_{1_\perp})} + \frac{\vec{q}_1^{\,\prime\,2}}{c(m_A,q'_{1_\perp})} \right]$$
$$\left. - \frac{2m_A^2}{1+2\epsilon} \left[\frac{1}{c(m_A,0)} - \frac{1}{c(m_A,q'_{1_\perp})} - \frac{1}{c(m_A,q_{1_\perp})} + \frac{1}{c(m_A,q_\perp)} \right] \right\} \delta_{\lambda_{A'},\lambda_A} \tag{4.34}$$

and

$$I_B^{(-)}(\vec{q}_1,\vec{q}) = m_A \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{2\epsilon-1}{2\epsilon+1} \int_0^1 dx\, \overline{u}(p_{A'}) \left[\frac{-\not{q}_\perp}{c(m_A,q_\perp)} \right.$$
$$\left. + \frac{\not{q}_{1_\perp}}{c(m_A,q_{1_\perp})} - \frac{\not{q}'_{1_\perp}}{c(m_A,q'_{1_\perp})} \right] \frac{\not{p}_B}{s} u(p_A) \ . \tag{4.35}$$

Finally, we consider the integrals involving C_Q, i.e.

$$I_C^{(+)}(\vec{q}_1,\vec{q}) = \int \frac{d^{D-2}k_1}{(2\pi)^{D-1}} \int_{\beta_0}^1 \frac{d\beta}{2\beta(1-\beta)} C_Q^{(+)} \tag{4.36}$$

and

$$I_C^{(-)}(\vec{q}_1,\vec{q}) = \int_0^1 \frac{d\beta}{2\beta(1-\beta)} \int \frac{d^{D-2}k_1}{(2\pi)^{D-1}} C_Q^{(-)} \; , \tag{4.37}$$

where only in the latter integration β_0 can be put equal to zero. The integral $I_C^{(+)}(\vec{q}_1,\vec{q})$ can be expressed in the following form:

$$\begin{aligned} I_C^{(+)}(\vec{q}_1,\vec{q}) \; &= \; I_A^{(+)}(\vec{q}_1,\vec{q}) \; + \; I_B^{(+)}(\vec{q}_1,\vec{q}) + \\ &+\delta_{\lambda_{A'},\lambda_A} \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \int_0^1 d\beta \int_0^1 dx \Bigg\{ \frac{2(1-\beta)+(1+\epsilon)\beta^2}{\beta} \\ &\times \Bigg[-\frac{(\vec{q}_1-(1-\beta)\vec{q})^2}{c(m_A\beta, q_{1_\perp}-(1-\beta)q_\perp)} - \frac{(\vec{q}_1-\beta\vec{q})^2}{c(m_A\beta, q_{1_\perp}-\beta q_\perp)} + \frac{\beta^2\vec{q}_1^{\,2}}{c(m_A\beta, \beta q_{1_\perp})} \\ &+ \frac{(1-\beta)^2\vec{q}_1^{\,2}}{c(m_A\beta,(1-\beta)q_{1_\perp})} + \frac{(1-\beta)^2\vec{q}_1^{\,\prime\,2}}{c(m_A\beta,(1-\beta)q'_{1_\perp})} + \frac{\beta^2\vec{q}_1^{\,\prime\,2}}{c(m_A\beta,\beta q'_{1_\perp})} \Bigg] \\ &+4m_A^2\beta \Bigg[\frac{1}{c(m_A\beta,\beta q_{1_\perp})} + \frac{1}{c(m_A\beta,(1-\beta)q_{1_\perp})} - \frac{1}{c(m_A\beta, q_{1_\perp}-\beta q_\perp)} \\ &- \frac{1}{c(m_A\beta, q_{1_\perp}-(1-\beta)q_\perp)} + \frac{1}{c(m_A\beta,(1-\beta)q'_{1_\perp})} \\ &+ \frac{1}{c(m_A\beta,\beta q'_{1_\perp})} - 2\,\frac{1}{c(m_A\beta,0)} \Bigg] \Bigg\} \; , \end{aligned} \tag{4.38}$$

while for the integral $I_C^{(-)}(\vec{q}_1,\vec{q})$ we have

$$\begin{aligned} I_C^{(-)}(\vec{q}_1,\vec{q}) &= 2m_A \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \int_0^1 d\beta\beta[(1+\epsilon)\beta-1] \int_0^1 dx \overline{u}(p_{A'}) \Bigg[-\frac{\beta \not{q}_\perp}{c(m_A\beta,\beta q_\perp)} \\ &+ \frac{(1-\beta)\not{q}_\perp}{c(m_A\beta,(1-\beta)q_\perp)} - \frac{\not{q}_{1_\perp} - \beta\not{q}_\perp}{c(m_A\beta, q_{1_\perp}-\beta q_\perp)} \\ &+ \frac{\not{q}_{1_\perp} - (1-\beta)\not{q}_\perp}{c(m_A\beta, q_{1_\perp}-(1-\beta)q_\perp)} \Bigg] \frac{\not{p}_B}{s} u(p_A) \; . \end{aligned} \tag{4.39}$$

As anticipated at the beginning of this Section, we take now into account the contribution to the impact factors from the second term in the R.H.S. of Eq. (2.7), i.e.

$$
-\frac{1}{2}\int \frac{d^{D-2}q_r}{\vec{q}_r^{\,2}\vec{q}_r^{\,\prime\,2}}\,\Phi^{c_1c_1'(B)}_{AA'}(\vec{q}_r,\vec{q})(\mathcal{K}^B_r)^{c_1'c'}_{c_1c}(\vec{q}_r,\vec{q}_1;\vec{q})\ln\left(\frac{s_\Lambda^2}{s_0(\vec{q}_r-\vec{q}_1)^{\,2}}\right)
$$

$$
\equiv \quad \text{counterterm}\ .
$$

Using the expression for the unprojected quark impact factor at the Born level (see Eq. (3.5))

$$
\Phi^{c_1c_1'(B)}_{AA'}(\vec{k}_1,\vec{q}) = g^2(t^{c_1'}t^{c_1})_{A'A}\delta_{\lambda_{A'},\lambda_A}\ , \tag{4.40}
$$

and recalling that [1]

$$
(\mathcal{K}^B_r)^{c_1'c'}_{c_1c}(\vec{q}_r,\vec{q}_1;\vec{q}) = \frac{g^2}{(2\pi)^{D-1}}\sum_d T^d_{c_1c}(T^d_{c_1'c'})^*\left(\frac{\vec{q}_r^{\,2}\vec{q}_1^{\,\prime\,2}+\vec{q}_1^{\,2}\vec{q}_r^{\,\prime\,2}}{(\vec{q}_r-\vec{q}_1)^2}-\vec{q}^{\,2}\right)\ , \tag{4.41}
$$

where T^d are color group generators in the adjoint representation, it is easy to obtain the following expression for the counterterm:

$$
\text{counterterm} = -g^4\left(\frac{1}{4}\delta_{A'A}\delta^{cc'}+\frac{N}{2}(t^{c'}t^c)_{A'A}\right)\int\frac{d^{D-2}k_1}{2(2\pi)^{D-1}}\ln\left(\frac{s_\Lambda^2}{\vec{k}_1^{\,2}\,s_0}\right)
$$

$$
\times\left[\frac{\vec{q}_1^{\,\prime\,2}}{\vec{k}_1^{\,2}(\vec{k}_1-\vec{q}_1^{\,\prime})^2}+\frac{\vec{q}_1^{\,2}}{\vec{k}_1^{\,2}(\vec{k}_1-\vec{q}_1)^2}-\frac{\vec{q}^{\,2}}{(\vec{k}_1-\vec{q}_1)^2(\vec{k}_1-\vec{q}_1^{\,\prime})^2}\right]\delta_{\lambda_{A'},\lambda_A} \tag{4.42}
$$

This counterterm leads to the cancellation of the s_Λ-dependence in Eq. (4.27), which comes from the integrals $I_A^{(+)}(\vec{q}_1,\vec{q},m_A=0)$ and $I_C^{(+)}(\vec{q}_1,\vec{q})$, as it can be easily checked comparing Eqs. (4.27), (4.29), (4.38) and (4.42).

5 The check of the bootstrap condition

We have now all the contributions needed to check the bootstrap condition given in Eq. (2.5). First of all, we must consider the quark impact factors in the octet color representation in the t-channel. According to Eq. (2.6), this means that the contributions to the unprojected quark impact factors from one-quark intermediate state, Eq. (3.1), from quark-gluon intermediate state, Eq. (4.27), and from the counterterm, Eq. (4.42), must be contracted with

$$\langle cc'|\hat{\mathcal{P}}_8|a\rangle = \frac{f_{acc'}}{\sqrt{N}} \;, \tag{5.1}$$

where f_{abc} are the $SU(N)$ structure constants. Since

$$\langle cc'|\hat{\mathcal{P}}_8|a\rangle \, \delta^{cc'} = 0 \;, \qquad \langle cc'|\hat{\mathcal{P}}_8|a\rangle \, (t^{c'}t^{c})_{A'A} = -i\frac{\sqrt{N}}{2}(t^a)_{A'A} \;, \tag{5.2}$$

we have for the octet quark impact factor at 1-loop order the following expression:

$$\Phi^{(8,a)(1)}_{A'A}(\vec{q}_1,\vec{q};s_0) = -i\frac{\sqrt{N}}{2}(t^a)_{A'A}\left\{g^2\,\left[\delta_{\lambda_{A'},\lambda_A}\left(\frac{1}{2}\omega^{(1)}(-\vec{q}_1^{\,2})\ln\left(\frac{s_0}{\vec{q}_1^{\,2}}\right)\right.\right.\right.$$
$$+\frac{1}{2}\omega^{(1)}(-\vec{q}_1^{\,\prime\,2})\ln\left(\frac{s_0}{\vec{q}_1^{\,\prime\,2}}\right) + a_f^{(+)}(-\vec{q}_1^{\,2}) + a_Q^{(+)}(-\vec{q}_1^{\,2},m_A^2)$$
$$+a_g^{(+)}(-\vec{q}_1^{\,2}) + \delta_g^{(+)}(-\vec{q}_1^{\,2},m_A^2)$$
$$\left.+a_f^{(+)}(-\vec{q}_1^{\,\prime\,2}) + a_Q^{(+)}(-\vec{q}_1^{\,\prime\,2},m_A^2) + a_g^{(+)}(-\vec{q}_1^{\,\prime\,2}) + \delta_g^{(+)}(-\vec{q}_1^{\,\prime\,2},m_A^2)\right)$$
$$+\left(a_g^{(-)}(-\vec{q}_1^{\,2}) + a_Q^{(-)}(-\vec{q}_1^{\,2},m_A^2)\right)\frac{i}{\sqrt{\vec{q}_1^{\,2}}}\overline{u}(p_{A'})\frac{\not{q}_{1\perp}\;\not{p}_B}{s}u(p_A)$$
$$\left.+\left(a_g^{(-)}(-\vec{q}_1^{\,\prime\,2}) + a_Q^{(-)}(-\vec{q}_1^{\,\prime\,2},m_A^2)\right)\frac{i}{\sqrt{\vec{q}_1^{\,\prime\,2}}}\overline{u}(p_{A'})\frac{\not{p}_B\;\not{q}'_{1\perp}}{s}u(p_A)\right]$$
$$-g^4\,\frac{N}{2}\left(I_A^{(+)}(\vec{q}_1,\vec{q},m_A=0) + \delta I_A^{(+)}(\vec{q}_1,\vec{q}) + I_A^{(-)}(\vec{q}_1,\vec{q})\right)$$
$$+\frac{g^4}{2N}\left(I_B^{(+)}(\vec{q}_1,\vec{q}) + I_B^{(-)}(\vec{q}_1,\vec{q})\right) - g^4\,\frac{N}{2}\int\frac{d^{D-2}k_1}{2(2\pi)^{D-1}}\ln\left(\frac{s_\Lambda^2}{\vec{k}_1^{\,2}\,s_0}\right)$$
$$\left.\times\left[\frac{\vec{q}_1^{\,\prime\,2}}{\vec{k}_1^{\,2}(\vec{k}_1-\vec{q}_1^{\,\prime})^2} + \frac{\vec{q}_1^{\,2}}{\vec{k}_1^{\,2}(\vec{k}_1-\vec{q}_1)^2} - \frac{\vec{q}^{\,2}}{(\vec{k}_1-\vec{q}_1^{\,\prime})^2(\vec{k}_1-\vec{q})^2}\right]\delta_{\lambda_{A'},\lambda_A}\right\} \;, \tag{5.3}$$

where $a_{f,Q,g}^{(+)}$ and $\delta_g^{(+)}$ are given in Eqs. (3.8), (3.9), (3.10) and (3.11), respectively, $a_{f,Q}^{(-)}$ in Eqs. (3.13) and (3.14), $I_A^{(+)}(\vec{q}_1,\vec{q},m_A=0)$, $\delta I_A^{(+)}(\vec{q}_1,\vec{q})$, $I_A^{(-)}(\vec{q}_1,\vec{q})$ and $I_B^{(\pm)}(\vec{q}_1,\vec{q})$ are given in Eqs. (4.29), (4.30), (4.32), (4.34) and (4.35), respectively. We can now proceed to check the fulfillment of the bootstrap condition (2.5), whose L.H.S. and the R.H.S. read

$$\text{L.H.S.} = -g\frac{N}{2}(t^a)_{A'A}\int\frac{d^{D-2}q_1}{(2\pi)^{D-1}}\frac{\vec{q}^{\,2}}{\vec{q}_1^{\,2}\vec{q}_1^{\,\prime\,2}}\left\{g^2\delta_{\lambda_{A'},\lambda_A}\left(\omega^{(1)}(-\vec{q}_1^{\,2})\ln\left(\frac{s_0}{\vec{q}_1^{\,2}}\right)\right.\right.$$
$$\left.+2\left(a_f^{(+)}(-\vec{q}_1^{\,2})+a_Q^{(+)}(-\vec{q}_1^{\,2},m_A^2)+a_g^{(+)}(-\vec{q}_1^{\,2})+\delta_g^{(+)}(-\vec{q}_1^{\,2},m_A^2)\right)\right)$$
$$+2g^2\left(a_g^{(-)}(-\vec{q}_1^{\,2})+a_Q^{(-)}(-\vec{q}_1^{\,2},m_A^2)\right)\frac{i}{\sqrt{\vec{q}_1^{\,2}}}\overline{u}(p_{A'})\frac{\not{q}_{1\perp}\not{p}_B}{s}u(p_A)$$
$$-g^4\frac{N}{2}\left(I_A^{(+)}(\vec{q}_1,\vec{q},m_A=0)+\delta I_A^{(+)}(\vec{q}_1,\vec{q})+I_A^{(-)}(\vec{q}_1,\vec{q})\right)$$
$$+\frac{g^4}{2N}\left(I_B^{(+)}(\vec{q}_1,\vec{q})+I_B^{(-)}(\vec{q}_1,\vec{q})\right)-g^4\frac{N}{2}\int\frac{d^{D-2}k_1}{2(2\pi)^{D-1}}\ln\left(\frac{s_\Lambda^2}{\vec{k}_1^{\,2}\,s_0}\right)$$
$$\left.\times\left[\frac{\vec{q}_1^{\,\prime\,2}}{\vec{k}_1^{\,2}(\vec{k}_1-\vec{q}_1^{\,\prime})^2}+\frac{\vec{q}_1^{\,2}}{\vec{k}_1^{\,2}(\vec{k}_1-\vec{q}_1)^2}-\frac{\vec{q}^{\,2}}{(\vec{k}_1-\vec{q}_1)^2(\vec{k}_1-\vec{q}_1^{\,\prime})^2}\right]\delta_{\lambda_{A'},\lambda_A}\right\}\,, \quad (5.4)$$

and

$$\text{R.H.S} = g(t^a)_{A'A}\left\{\delta_{\lambda_{A'},\lambda_A}\left[\frac{1}{2}\omega^{(2)(Q)}(-\vec{q}^{\,2})+\frac{1}{2}\omega^{(2)(G)}(-\vec{q}^{\,2})\right.\right.$$
$$+\frac{1}{2}(\omega^{(1)}(-\vec{q}^{\,2}))^2\ln\left(\frac{s_0}{\vec{q}^{\,2}}\right)$$
$$\left.+\omega^{(1)}(-\vec{q}^{\,2})\left(a_f^{(+)}(-\vec{q}^{\,2})+a_Q^{(+)}(-\vec{q}^{\,2},m_A^2)+a_g^{(+)}(-\vec{q}^{\,2})+\delta_g^{(+)}(-\vec{q}^{\,2},m_A^2)\right)\right]$$
$$\left.+\delta_{\lambda_{A'},-\lambda_A}\,\omega^{(1)}(-\vec{q}^{\,2})\left(a_g^{(-)}(-\vec{q}^{\,2})+a_Q^{(-)}(-\vec{q}^{\,2},m_A^2)\right)\right\}. \quad (5.5)$$

In the last equation, $\omega^{(2)(Q)}(-\vec{q}^{\,2})$ and $\omega^{(2)(G)}(-\vec{q}^{\,2})$ are the quark and gluon contributions to the two-loop gluon trajectory [6]:

$$\omega^{(2)(Q)}(-\vec{q}^{\,2}) = 2g^4N\frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{1}{\epsilon}\int\frac{d^{D-2}q_1}{(2\pi)^{D-1}}\frac{\vec{q}^{\,2}}{\vec{q}_1^{\,2}\vec{q}_1^{\,\prime\,2}}\sum_f\int_0^1 dx\,x(1-x)$$

$$\times \left[\left(m_f^2 + x(1-x)\vec{q}^{\,2} \right)^\epsilon - 2 \left(m_f^2 + x(1-x)\vec{q}_1^{\,2} \right)^\epsilon \right] \tag{5.6}$$

and

$$\omega^{(2)(G)}(-\vec{q}^{\,2}) = \frac{g^4 N^2}{2} \int \frac{d^{D-2}q_1}{(2\pi)^{D-1}} \frac{d^{D-2}q_2}{(2\pi)^{D-1}} \frac{\vec{q}^{\,2}}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}} \left\{ \frac{\vec{q}^{\,2}}{2\vec{q}_1^{\,\prime\,2}\vec{q}_2^{\,\prime\,2}} \ln\left(\frac{\vec{q}^{\,2}}{(\vec{q}_1 - \vec{q}_2)^2} \right) \right.$$
$$- \frac{1}{(\vec{q}_1 + \vec{q}_2 - \vec{q})^2} \ln\left(\frac{(\vec{q}_1 + \vec{q}_2)^2}{\vec{q}_1^{\,2}} \right) + \left(\frac{-\vec{q}^{\,2}}{2\vec{q}_1^{\,\prime\,2}\vec{q}_2^{\,\prime\,2}} + \frac{1}{(\vec{q}_1 + \vec{q}_2 - \vec{q})^2} \right)$$
$$\left. \times \left[\frac{1}{1+2\epsilon} \left(\frac{1}{4(3+2\epsilon)} - \frac{1}{\epsilon} - \frac{1}{4} \right) + 2\psi(1+2\epsilon) + \psi(1-\epsilon) - 2\psi(\epsilon) - \psi(1) \right] \right\} . \tag{5.7}$$

The check of the bootstrap condition is now a matter of recognizing similar terms in the L.H.S. and the R.H.S. written above and performing cancellations. We can separate this task into two parts, namely we can concentrate separately on helicity conserving and on helicity non-conserving terms. The first part is not quite independent of the calculation of the two-loop correction to the gluon trajectory [6], since it was performed assuming that the gluon Reggeization holds, by comparison of the s-channel discontinuity dictated by the Regge form (2.2) with that calculated from the unitarity. Therefore, it represents more a test of correctness of all the calculations involved in the determination of the trajectory and of the impact factors. The part related to the helicity non-conserving terms is a quite new check of the gluon Reggeization, for the spin structure which is absent in the LLA. It is worthwhile to mention that the Reggeization in this structure is not necessary for the derivation of the BFKL equation. Nevertheless, it is straightforward to verify that it holds, by considering that the following cancellations occur between a set of terms in the L.H.S. and a set of terms in the R.H.S.:

- L.H.S.: terms with $I_B^{(-)}$ and $a_g^{(-)}$;
  R.H.S.: term with $a_g^{(-)}$;
- L.H.S.: terms with $I_A^{(-)}$ and $a_Q^{(-)}$;
  R.H.S.: term with $a_Q^{(-)}$.

The check of the bootstrap for the helicity conserving part is not less straightforward, although the list of cancellations to perform is longer:

- L.H.S.: term with $a_f^{(+)}$;
  R.H.S.: terms with $a_f^{(+)}$ and $\omega^{(2)(Q)}$;
- L.H.S.: term with $a_Q^{(+)}$ and $I_B^{(+)}$;
  R.H.S.: term with $a_Q^{(+)}$;
- L.H.S.: terms with $\delta_g^{(+)}$ and $\delta I_A^{(+)}$;
  R.H.S.: term with $\delta_g^{(+)}$;
- L.H.S.: terms with $\ln(s_0/\vec{q}_1^{\,2})$, $a_g^{(+)}$, $I_A^{(+)}$ and $\ln(s_\Lambda^2/\vec{k}_1^{\,2}s_0)$ (counterterm);
  R.H.S.: terms with $a_g^{(+)}$, $\omega^{(2)(G)}$ and $\ln(s_0/\vec{q}^{\,2})$.

In the second cancellation for the helicity non-conserving part and in the third cancellation for the helicity conserving part, it has been used that

$$\int_0^1 dx_1 \int_0^1 dx_2 \theta(1-x_1-x_2) \longrightarrow \int_0^1 d\beta \int_0^1 dx\,(1-\beta)$$

under the change of variables $x_1 = \beta$, $x_2 = (1-\beta)x$.

This completes the check of the bootstrap condition for quark impact factors.

6 Quark impact factors in massless QCD

In this Section we will calculate explicitly the integrals which contribute to the quark impact factors, restricting ourselves to the case of massless quark, which is acceptable for all practical applications. In this case, only the helicity conserving part of the quark impact factor survives and most of the remaining integrals can be calculated for arbitrary ϵ. We will always consider the unprojected impact factor $\Phi_{AA'}^{cc'}(\vec{q}_1, \vec{q}; s_0)$ defined in Eq. (2.7). In order to obtain the quark impact factor in a definite (octet or singlet) color representation in the t-channel, it is sufficient to use Eq. (2.6), with the help of Eqs. (5.1) and (5.2), for the octet case, and of

$$\langle cc'|\hat{\mathcal{P}}_0|0\rangle = \frac{\delta_{cc'}}{\sqrt{N^2-1}} \;, \tag{6.1}$$

and

$$\langle cc'|\hat{\mathcal{P}}_0|0\rangle\, \delta^{cc'} = \sqrt{N^2-1}\;, \qquad \langle cc'|\hat{\mathcal{P}}_0|0\rangle\, (t^{c'}t^c)_{A'A} = \frac{\sqrt{N^2-1}}{2N}\delta_{A'A}\;, \tag{6.2}$$

for the singlet case.

We start for completeness from the quark impact factors at the Born level, which was already given in Eq. (4.40) and reads

$$\Phi^{cc'(B)}_{AA'}(\vec{k}_1,\vec{q}) = g^2 (t^{c'}t^c)_{A'A}\delta_{\lambda_{A'},\lambda_A}\;. \tag{6.3}$$

At one-loop level, we must consider the contribution from one-quark and quark-gluon intermediate states and from the counterterm – Eqs. (3.5), (4.27) and (4.42), respectively. Let's start from the one-quark contribution to the color unprojected quark impact factor which was given in Eq. (3.5). In the massless quark case it becomes

$$\Phi^{cc'(1)\{Q\}}_{AA'}(\vec{q}_1,\vec{q};s_0) = g^2 (t^{c'}t^c)_{A'A}\left[\delta_{\lambda_{A'},\lambda_A}\left(\frac{1}{2}\omega^{(1)}(-\vec{q}_1^{\,2})\ln\left(\frac{s_0}{\vec{q}_1^{\,2}}\right)\right.\right.$$
$$\left.\left.+\frac{1}{2}\omega^{(1)}(-\vec{q}_1^{\,\prime\,2})\ln\left(\frac{s_0}{\vec{q}_1^{\,\prime\,2}}\right) + \Gamma^{(+)(1)}_{QQ}(-\vec{q}_1^{\,2})\Big|_{m_f=0} + \Gamma^{(+)(1)}_{QQ}(-\vec{q}_1^{\,\prime\,2})\Big|_{m_f=0}\right)\right]\;, \tag{6.4}$$

where

$$\Gamma^{(+)(1)}_{QQ}(-\vec{v}^{\,2})\Big|_{m_f=0} = a^{(+)}_f(-\vec{v}^{\,2})\Big|_{m_f=0} + a^{(+)}_Q(-\vec{v}^{\,2},0) + a^{(+)}_g(-\vec{v}^{\,2})\;, \tag{6.5}$$

and m_f stands for the mass of any quark flavor. The integral expressions for $a^{(+)}_f$ and $a^{(+)}_Q$ were given in Eqs. (3.8) and (3.9), respectively, while $a^{(+)}_g$ was already given in explicit form in Eq. (3.10). The explicit forms for $a^{(+)}_f$ and

$a_Q^{(+)}$ can be easily calculated in the massless quark case giving

$$a_f^{(+)}(-\vec{v}^{\,2})\Big|_{m_f=0} = -\frac{g^2}{(4\pi)^{2+\epsilon}} n_f \Gamma(-\epsilon) \frac{[\Gamma(1+\epsilon)]^2}{\Gamma(1+2\epsilon)} \frac{(1+\epsilon)}{(1+2\epsilon)(3+2\epsilon)} (\vec{v}^{\,2})^\epsilon \tag{6.6}$$

and

$$a_Q^{(+)}(-\vec{v}^{\,2},0) = -\frac{g^2}{(4\pi)^{2+\epsilon}} \frac{1}{N} \Gamma(-\epsilon) \frac{[\Gamma(1+\epsilon)]^2}{\Gamma(1+2\epsilon)} \left[\frac{1}{\epsilon(1+2\epsilon)} + \frac{1}{2} \right] (\vec{v}^{\,2})^\epsilon \,. \tag{6.7}$$

Using Eq. (3.6) for the one-loop gluon trajectory and summing up all the terms in the R.H.S. of Eq. (6.4), we obtain

$$\Phi_{AA'}^{cc'(1)\{Q\}}(\vec{q}_1, \vec{q}; s_0) = \Bigg((t^{c'} t^c)_{A'A} \delta_{\lambda_{A'},\lambda_A} \frac{g^4}{(4\pi)^{2+\epsilon}} \Gamma(-\epsilon) \frac{[\Gamma(1+\epsilon)]^2}{\Gamma(1+2\epsilon)} (\vec{q}_1^{\,2})^\epsilon$$

$$\times \Bigg\{ N \ln\left(\frac{s_0}{\vec{q}_1^{\,2}}\right) - n_f \frac{(1+\epsilon)}{(1+2\epsilon)(3+2\epsilon)} - \frac{1}{N} \left[\frac{1}{\epsilon(1+2\epsilon)} + \frac{1}{2} \right]$$

$$+ N \Big[\psi(1-\epsilon) - 2\psi(\epsilon) + \psi(1)$$

$$+ \frac{1}{4(1+2\epsilon)(3+2\epsilon)} - \frac{1}{\epsilon(1+2\epsilon)} - \frac{7}{4(1+2\epsilon)} \Big] \Bigg\} \Bigg) + \left(\vec{q}_1 \longrightarrow \vec{q}_1^{\,\prime} \right). \tag{6.8}$$

Next, we consider the contributions to the quark impact factors from quark-gluon intermediate state, Eq. (4.27), and from the counterterm, Eq. (4.42). In the massless quark case, we have

$$\Phi_{AA'}^{cc'(1)\{QG\}}(\vec{q}_1, \vec{q}; s_0) + \text{counterterm} = g^4 \Bigg[-\delta_{A'A} \delta^{cc'} \frac{I_C^{(+)}(\vec{q}_1, \vec{q})}{4}$$

$$+ \frac{1}{2N} (t^{c'} t^c)_{A'A} \Big(-N^2 I_A^{(+)}(\vec{q}_1, \vec{q}, m_A = 0) + I_B^{(+)}(\vec{q}_1, \vec{q}) \Big) \Bigg]$$

$$- g^4 \left(\frac{1}{4} \delta_{A'A} \delta^{cc'} + \frac{N}{2} (t^{c'} t^c)_{A'A} \right) \int \frac{d^{D-2} k_1}{2(2\pi)^{D-1}} \ln\left(\frac{s_\Lambda^2}{\vec{k}_1^{\,2} s_0} \right)$$

$$\times \left[\frac{\vec{q}_1^{\,\prime 2}}{\vec{k}_1^{\,2} (\vec{k}_1 - \vec{q}_1^{\,\prime})^2} + \frac{\vec{q}_1^{\,2}}{\vec{k}_1^{\,2} (\vec{k}_1 - \vec{q}_1)^2} - \frac{\vec{q}^{\,2}}{(\vec{k}_1 - \vec{q}_1)^2 (\vec{k}_1 - \vec{q}_1^{\,\prime})^2} \right] \delta_{\lambda_{A'},\lambda_A} \,, \tag{6.9}$$

where $I_A^{(+)}(\vec{q}_1,\vec{q},m_A=0)$, $I_B^{(+)}(\vec{q}_1,\vec{q})$ and $I_C^{(+)}(\vec{q}_1,\vec{q})$ were given in Eqs. (4.29), (4.34) and (4.38), respectively. Now, let us consider the sum of the term with $I_A^{(+)}(\vec{q}_1,\vec{q},m_A=0)$ with the part of the counterterm having the same color structure. Leaving out the overall factor $-g^4N/2\,(t^{c'}t^c)_{A'A}$, we have

$$\tilde{I}_A^{(+)}(\vec{q}_1,\vec{q}) = I_A^{(+)}(\vec{q}_1,\vec{q},m_A=0) + \int \frac{d^{D-2}k_1}{2(2\pi)^{D-1}} \ln\left(\frac{s_\Lambda^2}{\vec{k}_1^{\,2}\, s_0}\right)$$

$$\times \left[\frac{\vec{q}_1^{\,\prime\,2}}{\vec{k}_1^{\,2}(\vec{k}_1-\vec{q}_1^{\,\prime})^2} + \frac{\vec{q}_1^{\,2}}{\vec{k}_1^{\,2}(\vec{k}_1-\vec{q}_1)^2} - \frac{\vec{q}^{\,2}}{(\vec{k}_1-\vec{q}_1)^2(\vec{k}_1-\vec{q}_1^{\,\prime})^2}\right]\delta_{\lambda_{A'},\lambda_A}\,. \tag{6.10}$$

It is easy to show that

$$\tilde{I}_A^{(+)}(\vec{q}_1,\vec{q}) = \Bigg(\delta_{\lambda_{A'},\lambda_A}\left\{\frac{1}{(4\pi)^{2+\epsilon}}\,\Gamma(-\epsilon)\frac{[\Gamma(1+\epsilon)]^2}{\Gamma(1+2\epsilon)}\right.$$

$$\times\left[-2(\vec{q}^{\,2})^\epsilon\left(\frac{1}{2}\ln\left(\frac{s_0}{\vec{q}^{\,2}}\right)+\psi(1)-\psi(1+2\epsilon)-\frac{3}{4(1+2\epsilon)}\right)\right.$$

$$+(\vec{q}_1^{\,2})^\epsilon\left(-\frac{1}{\epsilon}-\frac{3}{1+2\epsilon}+2\psi(1-\epsilon)-2\psi(1+2\epsilon)+2\psi(1)-2\psi(\epsilon)\right)$$

$$\left.\left.+2(\vec{q}_1^{\,2})^\epsilon\ln\left(\frac{s_0}{\vec{q}_1^{\,2}}\right)-\epsilon K_1\right]\right\}\Bigg)+\left(\vec{q}_1\longrightarrow\vec{q}_1^{\,\prime}\right)\,, \tag{6.11}$$

where the cancellation of the s_Λ-dependence occurred as anticipated in Section 4. The term K_1 in the R.H.S. of the above expression stands for

$$K_1 = \frac{(4\pi)^{2+\epsilon}}{4}\frac{1}{\Gamma(1-\epsilon)}\frac{\Gamma(1+2\epsilon)}{[\Gamma(1+\epsilon)]^2}\int\frac{d^{D-2}k}{(2\pi)^{D-1}}\ln\left(\frac{\vec{q}^{\,2}}{\vec{k}^{\,2}}\right)\frac{\vec{q}^{\,2}}{(\vec{k}-\vec{q}_1)^2(\vec{k}-\vec{q}_1^{\,\prime})^2}\,. \tag{6.12}$$

The integral K_1 can be calculated only in the ϵ-expansion. Its explicit form up to the order ϵ has been determined in the Appendix of Ref. [20]. Here we simply quote the result

$$K_1 = \frac{1}{2}(\vec{q}^{\,2})^\epsilon\left[\frac{1}{\epsilon^2}\left(2-\left(\frac{\vec{q}_1^{\,\prime\,2}}{\vec{q}^{\,2}}\right)^\epsilon-\left(\frac{\vec{q}_1^{\,2}}{\vec{q}^{\,2}}\right)^\epsilon\right)+4\psi''(1)\epsilon+\ln\left(\frac{\vec{q}_1^{\,\prime\,2}}{\vec{q}^{\,2}}\right)\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}^{\,2}}\right)\right]\,. \tag{6.13}$$

Similarly, we consider now the sum of the term with $I_C^{(+)}(\vec{q}_1, \vec{q})$ with the part of the counterterm having the same color structure. Leaving out the overall factor $-g^4/4\,\delta_{A'A}\delta^{cc'}$, we have

$$\tilde{I}_C^{(+)}(\vec{q}_1, \vec{q}) = I_C^{(+)}(\vec{q}_1, \vec{q}) + \int \frac{d^{D-2}k_1}{2(2\pi)^{D-1}} \ln\left(\frac{s_\Lambda^2}{\vec{k}_1^{\,2} s_0}\right)$$

$$\times \left[\frac{\vec{q}_1^{\,\prime\,2}}{\vec{k}_1^{\,2}(\vec{k}_1 - \vec{q}_1^{\,\prime})^2} + \frac{\vec{q}_1^{\,2}}{\vec{k}_1^{\,2}(\vec{k}_1 - \vec{q}_1)^2} - \frac{\vec{q}^{\,2}}{(\vec{k}_1 - \vec{q}_1)^2(\vec{k}_1 - \vec{q}_1^{\,\prime})^2}\right] \delta_{\lambda_{A'},\lambda_A}. \qquad (6.14)$$

This expression can be put in the following form:

$$\tilde{I}_C^{(+)}(\vec{q}_1, \vec{q}) = \tilde{I}_A^{(+)}(\vec{q}_1, \vec{q}) + \frac{2\Gamma(-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{[\Gamma(1+\epsilon)]^2}{\Gamma(1+2\epsilon)}$$

$$\times \left[-\left(\frac{1}{\epsilon(1+2\epsilon)} + \frac{1}{2}\right)(\vec{q}^{\,2})^\epsilon + K_2'\right] \delta_{\lambda_{A'},\lambda_A}, \qquad (6.15)$$

where K_2' is the integral analogous to K_2 from Ref. [20]:

$$K_2' = \int_0^1 \frac{d\beta}{\beta}[2(1-\beta)+(1+\epsilon)\beta^2]\left\{\left[\left((\beta\vec{q}_1 + (1-\beta)\vec{q}_1^{\,\prime})^2\right)^\epsilon - \left((1-\beta)^2\vec{q}_1^{\,\prime\,2}\right)^\epsilon\right]\right.$$

$$\left. + \left[\vec{q}_1 \longrightarrow \vec{q}_1^{\,\prime}\right]\right\}. \qquad (6.16)$$

This integral can be easily calculated in the ϵ-expansion up to the order ϵ and the result is the following:

$$K_2' = \epsilon\left[1 + \frac{1}{2}\ln^2\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_1^{\,\prime\,2}}\right) - \frac{3}{2}\frac{(\vec{q}_1^{\,2} - \vec{q}_1^{\,\prime\,2})}{\vec{q}^{\,2}}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_1^{\,\prime\,2}}\right)\right.$$

$$\left. -6\frac{|\vec{q}_1||\vec{q}_1^{\,\prime}|}{\vec{q}^{\,2}}\theta\sin\theta + 8\psi'(1) - 2\theta^2\right], \qquad (6.17)$$

being θ the angle between $\vec{q}_1^{\,\prime}$ and $\vec{q}_1$ defined so that $|\theta| \leq \pi$.

The last integral we need to calculate is $I_B^{(+)}(\vec{q}_1, \vec{q})$. In the massless quark case, it can be easily found that

$$I_B^{(+)}(\vec{q}_1, \vec{q}) = -\frac{2}{(4\pi)^{2+\epsilon}}\Gamma(-\epsilon)\frac{[\Gamma(1+\epsilon)]^2}{\Gamma(1+2\epsilon)}\left[\frac{1}{\epsilon(1+2\epsilon)} + \frac{1}{2}\right]$$

$$\times\left[(\vec{q}^{\,2})^\epsilon - (\vec{q}_1^{\,2})^\epsilon - (\vec{q}_1^{\,\prime\,2})^\epsilon\right]\delta_{\lambda_{A'},\lambda_A}. \qquad (6.18)$$

Summarizing, we can write the contribution to the quark impact factors from quark-gluon intermediate state in the compact form

$$\Phi^{cc'(1)\{QG\}}_{AA'}(\vec{q}_1,\vec{q},s_0) + \text{counterterm} =$$

$$g^4\left[-\delta_{A'A}\delta^{cc'}\frac{\tilde{I}^{(+)}_C(\vec{q}_1,\vec{q})}{4}+\frac{1}{2N}(t^{c'}t^c)_{A'A}\left(-N^2\tilde{I}^{(+)}_A(\vec{q}_1,\vec{q})+I^{(+)}_B(\vec{q}_1,\vec{q})\right)\right]\,, \tag{6.19}$$

with $\tilde{I}^{(+)}_A(\vec{q}_1,\vec{q})$, $\tilde{I}^{(+)}_C(\vec{q}_1,\vec{q})$ and $I^{(+)}_B(\vec{q}_1,\vec{q})$ given in Eqs. (6.11), (6.15), (6.18), respectively.

In the special case of forward scattering, the expression for the quark impact factor at 1-loop order greatly simplifies. We have indeed

$$\Phi^{cc'(1)}_{A'A}(\vec{q}_1,0;s_0) = \Phi^{cc'(1)(Q)}_{AA'}(\vec{q}_1,0;s_0)+\Phi^{cc'(1)\{QG\}}_{AA'}(\vec{q}_1,0,s_0)+\text{counterterm}|_{t=0}\ .$$

The contribution from the one-quark intermediate state, $\Phi^{cc'(1)(Q)}_{AA'}(\vec{q}_1,0;s_0)$, is given in the explicit form for arbitrary ϵ by the R.H.S. of Eq. (6.8) evaluated at $t=0$. In order to determine the contribution from the quark-gluon intermediate state, $\Phi^{cc'(1)\{QG\}}_{AA'}(\vec{q}_1,0,s_0)$, and from the counterterm at $t=0$, it is necessary to calculate the integrals in the R.H.S. of Eq. (6.19) at $t=0$.. This can be done for arbitrary ϵ yielding the following results

$$\tilde{I}^{(+)}_A(\vec{q}_1,0) = \delta_{\lambda_{A'},\lambda_A}\frac{2}{(4\pi)^{2+\epsilon}}\,\Gamma(-\epsilon)\frac{[\Gamma(1+\epsilon)]^2}{\Gamma(1+2\epsilon)}(\vec{q}_1^{\,2})^\epsilon\left[2\ln\left(\frac{s_0}{\vec{q}_1^{\,2}}\right)\right.$$

$$\left.+2\psi(1-\epsilon)-2\psi(1+2\epsilon)+2\psi(1)-2\psi(\epsilon)-\frac{1}{\epsilon}-\frac{3}{1+2\epsilon}\right]\,, \tag{6.20}$$

$$\tilde{I}^{(+)}_C(\vec{q}_1,0) = \delta_{\lambda_{A'},\lambda_A}\frac{2}{(4\pi)^{2+\epsilon}}\,\Gamma(-\epsilon)\frac{[\Gamma(1+\epsilon)]^2}{\Gamma(1+2\epsilon)}(\vec{q}_1^{\,2})^\epsilon\left[2\ln\left(\frac{s_0}{\vec{q}_1^{\,2}}\right)\right.$$

$$\left.+2\psi(1-\epsilon)+2\psi(1+2\epsilon)-2\psi(1)-2\psi(\epsilon)-\frac{1}{\epsilon}-3+\epsilon\right] \tag{6.21}$$

and

$$I^{(+)}_B(\vec{q}_1,0) = \delta_{\lambda_{A'},\lambda_A}\frac{2}{(4\pi)^{2+\epsilon}}\,\Gamma(-\epsilon)\frac{[\Gamma(1+\epsilon)]^2}{\Gamma(1+2\epsilon)}(\vec{q}_1^{\,2})^\epsilon\left[1+\frac{2}{\epsilon(1+2\epsilon)}\right]\,, \tag{6.22}$$

Let us present in the explicit form the color singlet impact factor. Performing the projection on the color singlet with the help of the operator (6.1), we obtain

$$\Phi^{(0)(1)}_{A'A}(\vec{q}_1, 0; s_0)$$

$$= \delta_{A',A}\delta_{\lambda_{A'},\lambda_A} g^4 \frac{2\,\Gamma(-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{[\Gamma(1+\epsilon)]^2}{\Gamma(1+2\epsilon)} (\vec{q}_1^{\,2})^\epsilon \frac{\sqrt{N^2-1}}{2N} \left[-n_f \frac{1+\epsilon}{(1+2\epsilon)(3+2\epsilon)} \right.$$

$$\left. +N\left(-\ln\left(\frac{s_0}{\vec{q}_1^{\,2}}\right) + \psi(1) - \psi(1-\epsilon) + \frac{3}{2} + \frac{15}{8(1+2\epsilon)} - \frac{1}{8(3+2\epsilon)} - \frac{\epsilon}{2} \right) \right]$$

$$\approx \delta_{A',A}\delta_{\lambda_{A'},\lambda_A}\, g^2 \frac{\sqrt{N^2-1}}{2N} \left[-g^2\, N \frac{\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \frac{[\Gamma(\epsilon)]^2}{\Gamma(2\epsilon)} (\vec{q}_1^{\,2})^\epsilon \right]$$

$$\times \left[-\ln\left(\frac{s_0}{\vec{q}_1^{\,2}}\right) + \left(\frac{10}{3} - \frac{1}{3}\frac{n_f}{N} \right) + \epsilon \left(-\frac{38}{9} + \frac{\pi^2}{6} + \frac{5}{9}\frac{n_f}{N} \right) \right] . \qquad (6.23)$$

The quark impact factor in the forward case ($t = 0$ and color singlet in the t-channel) was considered in Refs. [11, 24, 25]. In Refs. [24, 25] it was calculated for massless quarks with accuracy up to terms finite in the $\epsilon \to 0$ limit. In this particular case our result (6.23) is in agreement with the corresponding result of Ref. [25], though the comparison is not straightforward because of the different definitions adopted. For details see Ref. [20].

7 Discussion

In this paper we have obtained an integral representation for the NLA non-forward quark impact factors with singlet and octet representation in the t-channel in QCD with massive quarks for arbitrary space-time dimension $D = 4 + 2\epsilon$. Using this integral representation, we have explicitly verified the fulfillment of the "second" bootstrap condition derived in Ref. [16] for the gluon Reggeization at the NLA in perturbative QCD. This check is very important since the gluon Reggeization plays the most relevant role in the derivation of the BFKL equation at the NLA. Moreover, it represents a test of correctness of the calculations performed so far to determine the NLA corrections to the BFKL equation. Subsequently, we have carried out explicitly the integrations in the case of massless quarks, using the ϵ-expansion when necessary.

We finally note that throughout the paper we have used the unrenormalized coupling constant g and the same parameter ϵ to regularize both infrared and ultraviolet divergences. The final result for the quark impact factors is affected by both kind of divergences. The ultraviolet ones are easily removed by introducing the renormalized charge in the $\overline{MS}$ scheme

$$g = g(\mu)\mu^{-\epsilon}\left[1 + \left(\frac{11}{3} - \frac{2}{3}\frac{n_f}{N}\right)\frac{g^2(\mu)N\Gamma(1-\epsilon)}{2\epsilon(4\pi)^{2+\epsilon}}\right] . \qquad (7.1)$$

The infrared divergences, however, are not canceled. This is expected for the quark impact factor, since the quark is a colored object, whereas impact factors of colorless particles only must be infrared safe. Recall that for the colorless particles the infrared safety of the impact factors is guaranteed [26] by their definition given in [16] and used in this paper.

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