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Abstract

We demonstrate that a simple model successfully describing experimental data for the process $e^+e^- \to 4\pi$ can also qualitatively account for the data of CLEOII and ALEPH obtained recently for the decay $\tau \to 4\pi\nu_\tau$. The model is based on the assumption of the $a_1(1260)\pi$ and $\omega\pi$ dominance as intermediate states. Our observation is in contrast with the claim by ARGUS that the ρ-meson signal in the $\tau \to 4\pi\nu_\tau$ decay can not be explained by the $a_1\pi$ intermediate state.

1 Introduction

At present hadron processes at low energies (below J/ψ) can not be satisfactorily described by QCD. Therefore, in order to understand the phenomena in this energy range it is necessary to apply different phenomenological models and compare their predictions with the accessible experimental data. Investigation of exclusive channels in e^+e^- annihilation into hadrons at low energies and semihadronic decays of τ lepton is very attractive for this purpose. It provides the important information on the bound states of light quarks and elucidates the role of different mechanisms in these processes.

Production of four pions is one of the dominant processes of e^+e^- annihilation into hadrons in the energy range from 1.05 to 2.5 GeV. Due to the conservation of vector current (CVC) the cross section of this process is related to the probability of $\tau \to 4\pi\nu_\tau$ decay [1]. Therefore, all realistic models describing the first process, should also be appropriate for the description of the other one.

One of the main difficulties in the study of four pion production is caused by the existence of different intermediate states via which the final state could be produced, such as $\omega\pi$, $\rho\sigma$, $a_1(1260)\pi$, $h_1(1170)\pi$, $\rho^+\rho^-$, $a_2(1320)\pi$, $\pi(1300)\pi$. The relative contributions of the mentioned processes can't be obtained without the detailed analysis of the process dynamics. Some information on the process $e^+e^- \to 4\pi$ has been obtained in [2-8], where the investigation of e^+e^- annihilation into hadrons was restricted by measurements of the cross sections only. However, the abundance of various possible mechanisms and their complicated interference results in the necessity of simultaneous analysis of two possible final states ($2\pi^+2\pi^-$ and $\pi^+\pi^-2\pi^0$) which requires a general purpose detector capable of measuring energies and angles of both charged and neutral particles.

In the energy range below 1.4 GeV the detector of this type CMD-2 is operating at VEPP-2M collider in Novosibirsk [9]. During a few last years very large data samples have been obtained which open qualitatively new possibilities for the investigation of the multihadronic production in e^+e^- annihilation.

Very recently, a model-dependent analysis was performed [10] of both possible channels in e^+e^- annihilation into four pions at energies 1.05–1.38 GeV, using the data collected with the CMD-2 detector. The discussion of previous works devoted to this subject is also present in [10].

The detailed analysis of the process $e^+e^- \to 4\pi$ unambiguously demonstrated that the main contribution to the cross section in the energy range 1.05 – 1.38 GeV, in addition to previously well-studied $\omega\pi^0$ [11, 12], is given by the $\rho\pi\pi$ intermediate state. Moreover, the latter is completely saturated by the $a_1\pi$ mechanism. The contribution of other intermediate states was estimated to be less than 15 % .

In this paper we use the assumption of the $a_1\pi$ dominance for comparison of the available experimental data for $\tau \to 4\pi\nu_\tau$ decay [13, 14, 15, 16] with the prediction of our model successfully describing the data of $e^+e^- \to 4\pi$.

2 Results and discussion

The initial hadron state (referred to as $\tilde{\rho}$), which decays into four pions has the ρ-meson quantum numbers. Due to the conservation of vector current the probability $d\Gamma_1$ and $d\Gamma_2$ of $\tau^- \to \pi^-\pi^+\pi^-\pi^0\nu_\tau$ and $\tau^- \to \pi^-\pi^0\pi^0\pi^0\nu_\tau$ decays, respectively, can be written as

$$\frac{d\Gamma_i}{ds} = \frac{G^2\cos^2\theta}{96\pi^3 m_\tau^3}(m_\tau^2 + 2s)(m_\tau^2 - s)^2 R_{4\pi}\frac{dW_i}{W_1 + W_2} \qquad (1)$$

where $R_{4\pi}$ is the ratio of the cross section $e^+e^- \to 4\pi$ and $e^+e^- \to \mu^+\mu^-$, dW_1 and dW_2 are the probabilities of $\tilde{\rho}^-$ decays into $\pi^-\pi^+\pi^-\pi^0$ and $\pi^-\pi^0\pi^0\pi^0$, respectively. Let dW_3 and dW_4 be the probabilities of $\tilde{\rho}^0$ decays into $\pi^+\pi^-\pi^+\pi^-$ and $\pi^+\pi^-\pi^0\pi^0$, then due to the isospin invariance, we have

$$W_1 = \frac{1}{2}W_3 + W_4 \quad , \quad W_2 = \frac{1}{2}W_3 \,. \qquad (2)$$

The explicit forms of the matrix elements, corresponding to W_i, are presented in the Appendix. In order to get the predictions for τ decay we neglect the interference between the $\omega\pi$ and $a_1\pi$ amplitudes, and rewrite (1) in the following form:

$$\begin{aligned} \frac{d\Gamma_1}{ds} &= \frac{G^2\cos^2\theta}{96\pi^3 m_\tau^3}(m_\tau^2+2s)(m_\tau^2-s)^2\left[R_{\omega\pi}\frac{dW_\omega}{W_\omega}+R_{2\pi^+2\pi^-}\frac{dW_1}{W_3}\right] \\ \frac{d\Gamma_2}{ds} &= \frac{G^2\cos^2\theta}{96\pi^3 m_\tau^3}(m_\tau^2+2s)(m_\tau^2-s)^2 R_{2\pi^+2\pi^-}\frac{dW_2}{W_3}\ , \end{aligned} \qquad (3)$$

where dW_ω is the probability of $\tilde{\rho}^- \to \omega\pi^-$ decay, $R_{\omega\pi}$ is the ratio of the cross section $e^+e^- \to \omega\pi$ and $e^+e^- \to \mu^+\mu^-$.

To get $R_{2\pi^+2\pi^-}$ and $R_{\omega\pi}$ we used the cross section of e^+e^- annihilation in the energy range 1–2 GeV obtained by several experimental groups and presented in Figs. 1 and 2 together with our fit. Our approximation of the experimental data took into account possible uncertainties in the data. Assuming that the total cross section $e^+e^- \to 2\pi^+2\pi^-$ is saturated by the $a_1\pi$ mechanism, we calculated the cross section $e^+e^- \to \pi^+\pi^-2\pi^0$. The comparison of our approximation with the experimental data is shown in Fig. 3. One can see some difference between our expectation and data at energies above 1.6 GeV. We suppose that this difference can be explained by the possible systematic errors or by contributions of other intermediate states which become more important at higher energies. However, we expect that this difference does not affect very much our predictions for τ since this energy region has the additional suppression factor due to the limited phase space in case of τ decay (see (1)). In order to check this we compared the distribution of the four-pion invariant mass with the recent data of CLEOII [16] and obtained that our interpolation of e^+e^- data was in acceptable agreement with the τ decay data.

In order to fix the parameters of our model we have used the data of $e^+e^- \to 4\pi$ [10] and $\tau^- \to 2\pi^0\pi^-2\nu_\tau$ [18]. Since the data in [10] were obtained below the threshold of $a_1\pi$ production it was difficult to extract independently both the mass and width of a_1. The mass of a_1 was taken from the PDG table [19] and the width was obtained as a result of optimal description of $e^+e^- \to \pi^+\pi^-2\pi^0$ data. Here we use the same a_1 mass and the width we get as a result of the optimal description of the three pion invariant mass distribution in

$\tau^- \to 2\pi^0\pi^-\nu_\tau$ decay by our matrix element (see Appendix) [18]. We used this approach since the data on τ decay are more sensitive to the parameters of the a_1 meson. The comparison of our description of the three pion invariant mass distribution with the data is presented in Fig. 4. We obtained the value of a_1 width which also provides a good description of $e^+e^- \to 4\pi$ data (see Figs. 5, 6). In [18] evidence was obtained that the a_1 meson has significant probability to decay into three pions through the $\sigma\pi$ intermediate state. The analysis of $e^+e^- \to 4\pi$ data also confirmed this observation. Therefore, here we take into account the admixture of $\sigma\pi$ to the a_1 decay amplitude with the parameters extracted from $e^+e^- \to 4\pi$ data. For comparison we also present the results obtained without the $\sigma\pi$ contribution.

When we get the parameters of our model, we can pass to the description of $\tau^- \to \pi^-\pi^+\pi^-\pi^0\nu_\tau$ decay. The most interesting information on the mechanism of the four pion channel can be obtained from two-pion mass distributions. Fig. 7 shows the comparison of our predictions with the data of CLEOII detector [15] obtained without subtraction of the $\omega\pi^-$ contribution. One can see rather good agreement with this data. The same comparison with the ARGUS data [13] is made in Fig. 8. In this case our predictions are not in rough contradiction with the data, but the agreement is essentially worse than with that of CLEOII, especially in the invariant mass range where the enhancement due to the ρ meson can be seen.

The data obtained with the subtraction of $\omega\pi^-$ contribution allows one to make a more detailed comparison of the differential distributions predicted within the assumption of the $a_1\pi$ dominance. For this purpose we have used the data obtained by ALEPH [14] (see Fig. 9) and very recent high-statistics data of CLEOII [16] (see Fig. 10). In the first case we see good agreement although we can not take into account the detector efficiency and energy resolution. For the data of CLEOII, which have essentially higher statistics, the agreement is a little bit worse. Unfortunately, in this data the contributions of background events (such as $K^0\pi^-\pi^0\nu_\tau$ and $K^\pm\pi^\mp\pi^-\pi^0\nu_\tau$) were not subtracted, though their fraction was significant (about 8%). The biggest discrepancy can be seen for the $\pi^+\pi^-$ invariant mass distribution. However, the data of CLEOII in this case systematically differ from data obtained by ALEPH (see Fig. 9c).

In spite of some disagreement between the data of different groups we can conclude that the assumption of a_1 dominance is in qualitative agreement with all available data. The quantitative comparison of our model can be made only by full simulation which takes into account the energy resolution and detector efficiency as well as all possible background sources. On the base of our analysis we come to conclusion that the previously obtained upper limit (less than 44% at 95% CL) [13] for $a_1\pi\nu_\tau$ contribution to the $\tau \to 4\pi\nu_\tau$ decay is hardly correct. Note that in some theoretical works based on the effective chiral theory of mesons [17] the predictions of $a_1\pi$ dominance were made but without detail comparison with experimental data.

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3 Appendix: A model for $e^+e^- \to 4\pi$ and $\tau \to 4\pi\nu_\tau$ processes

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To describe four pion production we use a simple model assuming quasitwoparticle intermediate states and taking into account the important effects of the identity of the final pions as well as the interference of all possible amplitudes. We use the notation $\tilde{e}^a_\mu$ for the polarization vector of the initial hadron state ($\tilde{\rho}$), superscript corresponds to isospin. Here we consider the contributions of $a_1(1260)\pi$ and $\omega\pi$) intermediate states to the amplitudes $\tilde{\rho} \to 4\pi$ in the energy range under study. Other contributions to these amplitudes from broad resonances having the masses close to the threshold of $\rho\pi$ production ($a_2(1320)$, $\pi(1300)$ etc.) are discussed in [10].

As known, the form of the propagators $1/D(q)$ is very important for analysing the data. It was found in [10] that it is necessary to take into account the dependence of the imaginary part of the propagators (width) on virtuality while the corresponding corrections to the real part which can be expressed through the imaginary part by dispersion relations are not so important. We represent the function $D(q)$ in the form

$$D(q) = q^2 - M^2 + iM\Gamma \frac{g(q^2)}{g(M^2)}\,, \tag{4}$$

where M and Γ are the mass and width of the corresponding particle, and the function $g(s)$ describes the dependence of the width on the virtuality. If $q^2 = M^2$ then $D = iM\Gamma$, in accordance with the usual definition of mass and width of the resonance. In the case of the ρ-meson we used the following representation for the function $g_\rho(q)$:

$$g_\rho(s) = s^{-1/2}(s - 4m^2)^{3/2}\,, \tag{5}$$

where m is the pion mass.

We include into consideration two the most important channels of $a_1 \to 3\pi$ decay : $a_1 \to \rho\pi \to 3\pi$ and $a_1 \to \sigma\pi \to 3\pi$. Taking into account the quantum numbers of pion and a_1-meson, we can write the matrix elements corresponding to the processes $\tilde{\rho}(P) \to a_1(q)\pi(p)$, $a_1(q) \to \rho(P')\pi(p)$, and $a_1(q) \to \sigma(P')\pi(p)$ as

$$\begin{aligned} T(\tilde{\rho} \to a_1\pi) &= F_{\tilde{\rho}a_1\pi}\varepsilon^{abc}(P_\mu \tilde{e}^a_\nu - P_\nu \tilde{e}^a_\mu)q_\mu A^{b*}_\nu \phi^{c*}\,, \\ T(a_1 \to \rho\pi) &= F_{a_1\rho\pi}\varepsilon^{abc} q_\mu A^a_\nu (P'_\mu e^{b*}_\nu - P'_\nu e^{b*}_\mu)\phi^{c*}\,, \\ T(a_1 \to \sigma\pi) &= F_{a_1\sigma\pi}(q_\mu A^a_\nu - q_\nu A^a_\mu)P'_\mu p_\nu \phi^{a*}\,, \end{aligned} \tag{6}$$

where a, b, c are isospin indices, A^a_μ and e^a_μ are the polarization vectors of a_1- and ρ-mesons, ϕ^b is the pion wave function, $F_{a_1\rho\pi}$, $F_{a_1\sigma\pi}$ and $F_{\tilde{\rho}a_1\pi}$ are form factors depending on the virtuality of initial and final particles. The explicit form of these form factors in the energy range considered is not very essential (it contributes to the theoretical uncertainty of the model). The matrix element of the transition $T(\rho \to \pi\pi)$

and $T(\sigma\to\pi\pi)$ reads

$$\begin{aligned} T(\rho\to\pi\pi) &= F_{\rho\pi\pi}\varepsilon^{abc}e^a_\mu(p^{(b)}_\mu - p^{(c)}_\mu)\phi^{b*}\phi^{c*}\,, \\ T(\sigma\to\pi\pi) &= F_{\sigma\pi\pi}\phi^{a*}\phi^{a*}\,, \end{aligned} \tag{7}$$

where $p^{(b)}_\mu$ and $p^{(c)}_\mu$ are 4-momenta of the corresponding pions. The matrix element $T(\tilde\rho\to 4\pi)$ in the rest frame of $\tilde\rho$ can be written as $T=\tilde{\mathbf{e}}\mathbf{J}$, where $\tilde{\mathbf{e}}$ is the polarization vector of $\tilde\rho$. The corresponding probability of $\tilde\rho$ decay is given by the usual formula

$$dW(s)=\frac{|T|^2}{2E}\,(2\pi)^4\delta^{(4)}(\sum_{i=1}^{4}p_i-P)\,\prod_{i=1}^{4}\frac{d\mathbf{p}_i}{2\varepsilon_i(2\pi)^3}\,, \tag{8}$$

where P is the initial 4-momentum of $\tilde\rho$ ($P^0=E=\sqrt{s}$, $\mathbf{P}=0$), $p_i=(\varepsilon_i,\mathbf{p}_i)$ are the momenta of pions. Note that due to the helicity conservation in e^+e^- annihilation only transverse space components (with respect to electron and positron momenta) of the 4-vector $\tilde e_\mu$ are not zero. Using (6) and (7) we obtain the following expressions for the contributions of the $a_1(1260)$-meson to the current $\mathbf{J}_{a_1}=\mathbf{J}_{a_1\to\rho\pi}+\mathbf{J}_{a_1\to\sigma\pi}$ in the process
$\tilde\rho^0\to\pi^+(p_1)\pi^+(p_2)\pi^-(p_3)\pi^-(p_4)$:

$$\begin{aligned} \mathbf{J}^{++--}_{a_1\to\rho\pi} &= G\,[\mathbf{t}_1(p_1,p_2,p_3,p_4)+\mathbf{t}_1(p_1,p_4,p_3,p_2)+\mathbf{t}_1(p_2,p_1,p_3,p_4) \\ &\quad +\mathbf{t}_1(p_2,p_4,p_3,p_1)+\mathbf{t}_1(p_1,p_2,p_4,p_3)+\mathbf{t}_1(p_1,p_3,p_4,p_2) \\ &\quad +\mathbf{t}_1(p_2,p_1,p_4,p_3)+\mathbf{t}_1(p_2,p_3,p_4,p_1)]\,, \\ \mathbf{J}^{++--}_{a_1\to\sigma\pi} &= G\,[\mathbf{t}_2(p_1,p_2,p_3,p_4)-\mathbf{t}_2(p_1,p_4,p_3,p_2)+\mathbf{t}_2(p_2,p_1,p_3,p_4) \\ &\quad -\mathbf{t}_2(p_2,p_4,p_3,p_1)+\mathbf{t}_2(p_1,p_2,p_4,p_3)-\mathbf{t}_2(p_1,p_3,p_4,p_2) \\ &\quad +\mathbf{t}_1(p_2,p_1,p_4,p_3)-\mathbf{t}_2(p_2,p_3,p_4,p_1)]\,, \end{aligned} \tag{9}$$

where

$$\begin{aligned} \mathbf{t}_1(p_1,p_2,p_3,p_4) &= \frac{F^2_{a_1}(P-p_4)}{D_{a_1}(P-p_4)D_\rho(p_1+p_3)}\times \\ &\times\{(E-\varepsilon_4)[\mathbf{p}_1(E\varepsilon_3-p_4p_3)-\mathbf{p}_3(E\varepsilon_1-p_4p_1)] \\ &\quad -\mathbf{p}_4[\varepsilon_1(p_4p_3)-\varepsilon_3(p_4p_1)]\}\,, \end{aligned} \tag{10}$$

$$\mathbf{t}_2(p_1,p_2,p_3,p_4) = \frac{z\,F_{a_1}^2(P-p_4)}{D_{a_1}(P-p_4)D_\sigma(p_1+p_3)} \times$$
$$\times (P-p_4)^2\left[(E-\varepsilon_4)\mathbf{p}_2 + \varepsilon_2\mathbf{p}_4\right]$$

Here $1/D_A(q)$, $1/D_\rho(q)$ and $1/D_\sigma(q)$ are propagators of a_1 , ρ and σ mesons, $F_{a_1}(q)$ is the form factor, G is some constant, z is the dimensionless complex constant. Similarly to (9), we obtain for the contributions of $a_1(1260)$ to the current $\mathbf{J}_{a1}$ in the process $\tilde{\rho}^0 \to \pi^+(p_1)\pi^-(p_4)\pi^0(p_2)\pi^0(p_3)$:

$$\begin{aligned} \mathbf{J}^{+-00}_{a_1\to\rho\pi} &= G\left[-\mathbf{t}_{a_1}(p_4,p_2,p_3,p_1) + \mathbf{t}_{a_1}(p_1,p_2,p_3,p_4)\right. \\ &\quad \left. - \mathbf{t}_{a_1}(p_4,p_3,p_2,p_1) + \mathbf{t}_{a_1}(p_1,p_3,p_2,p_4)\right] , \\ \mathbf{J}^{+-00}_{a_1\to\sigma\pi} &= G\left[\mathbf{t}_2(p_2,p_1,p_3,p_4) - \mathbf{t}_2(p_2,p_4,p_3,p_1)\right. . \end{aligned} \tag{11}$$

The contributions of the $a_1(1260)$-meson to the currents $\mathbf{J}_{a_1}$ in the process
$\tilde{\rho}^- \to \pi^+(p_1)\pi^0(p_2)\pi^-(p_3)\pi^-(p_4)$ reads :

$$\begin{aligned} \mathbf{J}^{+0--}_{a_1\to\rho\pi} &= -G\left[\mathbf{t}_1(p_1,p_3,p_4,p_2) + \mathbf{t}_1(p_1,p_4,p_3,p_2) + \mathbf{t}_1(p_2,p_1,p_3,p_4)\right. \\ &\quad + \mathbf{t}_1(p_2,p_1,p_4,p_3) - \mathbf{t}_1(p_1,p_3,p_2,p_4) - \\ &\quad \left. -\mathbf{t}_1(p_1,p_4,p_2,p_3)\right] , \\ \mathbf{J}^{+0--}_{a_1\to\sigma\pi} &= G\left[\mathbf{t}_2(p_1,p_3,p_4,p_2) + \mathbf{t}_2(p_1,p_4,p_3,p_2) - \right. \\ &\quad \left. -\mathbf{t}_2(p_1,p_2,p_3,p_4) - \mathbf{t}_2(p_1,p_2,p_4,p_3)\right] , \end{aligned} \tag{12}$$

and in the process $\tilde{\rho}^- \to \pi^0(p_1)\pi^0(p_2)\pi^0(p_3)\pi^-(p_4)$ it reads :

$$\begin{aligned} \mathbf{J}^{0---}_{a_1\to\rho\pi} &= G\left[\mathbf{t}_1(p_4,p_1,p_2,p_3) + \mathbf{t}_1(p_4,p_1,p_3,p_2) + \mathbf{t}_1(p_4,p_2,p_1,p_3)\right. \\ &\quad + \mathbf{t}_1(p_4,p_2,p_3,p_1) + \mathbf{t}_1(p_4,p_3,p_1,p_2) + \\ &\quad \left. +\mathbf{t}_1(p_4,p_3,p_2,p_1)\right] , \end{aligned} \tag{13}$$

$$\begin{aligned} \mathbf{J}^{0---}_{a_1\to\sigma\pi} &= G\left[\mathbf{t}_2(p_1,p_4,p_2,p_3) + \mathbf{t}_2(p_1,p_4,p_3,p_2) + \mathbf{t}_2(p_2,p_4,p_3,p_1) - \right. \\ &\quad \left. -\mathbf{t}_2(p_1,p_2,p_3,p_4) - \mathbf{t}_2(p_1,p_3,p_2,p_4) - \mathbf{t}_2(p_2,p_3,p_1,p_4)\right] . \end{aligned} \tag{14}$$

The function $g_{a_1}(s)$ in the propagator of a_1 has the form:

$$\begin{aligned} g_{a_1}(s) &= F^2_{a_1}(Q) \int \left\{ \left| \frac{\varepsilon_2 \mathbf{p}_1 - \varepsilon_1 \mathbf{p}_2}{D_\rho(p_1+p_2)} + \frac{\varepsilon_2 \mathbf{p}_3 - \varepsilon_3 \mathbf{p}_2}{D_\rho(p_2+p_3)} + \frac{z\sqrt{s}\mathbf{p}_2}{D_\sigma(p_1+p_3)} \right|^2 + \right. \\ & \left. + \frac{|z|^2 s}{3!} \left| \frac{\mathbf{p}_1}{D_\sigma(p_2+p_3)} + \frac{\mathbf{p}_2}{D_\sigma(p_1+p_3)} + \frac{\mathbf{p}_3}{D_\sigma(p_1+p_2)} \right|^2 \right\} \times \\ & \times \frac{d\mathbf{p}_1\, d\mathbf{p}_2\, d\mathbf{p}_3\, \delta^{(4)}(p_1+p_2+p_3-Q)}{2\varepsilon_1 2\varepsilon_2 2\varepsilon_3 (2\pi)^5}, \end{aligned} \qquad (15)$$

where $Q^0 = \sqrt{s}$ and $\mathbf{Q} = 0$. The first term in the braces corresponds to the decay $a_1 \to \pi^+\pi^-\pi^0$, and the second one to $a_1 \to 3\pi^0$. The function $g_\sigma(s)$ in the propagator of σ is equal to

$$g_\sigma(s) = (1 - 4m^2/s)^{1/2}. \qquad (16)$$

As a form factor, we used the function $F(q) = (1+m^2_{a_1}/\Lambda^2)/(1+q^2/\Lambda^2)$ with $\Lambda \sim 1$ GeV. We found that in the energy region under discussion the amplitudes are not very sensitive to a value of Λ.

The amplitude of the process $\tilde{\rho}(\mathcal{P}) \to \omega(q)\pi(p)$ has the form:

$$T(\tilde{\rho} \to \omega\pi) = F_{\tilde{\rho}\omega\pi} \varepsilon_{\mu\nu\alpha\beta} \mathcal{P}_\mu q_\nu \tilde{e}^a_\alpha e^*_\beta \phi^{a*}, \qquad (17)$$

where e_β is the polarization vector of the ω-meson. The matrix element of the transition $\omega \to \rho\pi$ can be written in the similar form. The ω-meson contributes to channels $\tilde{\rho}^0 \to \pi^+(p_1)\pi^0(p_2)\pi^0(p_3)\pi^-(p_4)$, $\tilde{\rho}^- \to \pi^+(p_1)\pi^-(p_2)\pi^-(p_3)\pi^0(p_4)$ and $\tilde{\rho}^+ \to \pi^-(p_1)\pi^+(p_2)\pi^+(p_3)\pi^0(p_4)$ The corresponding current is equal to

$$\begin{aligned} \mathbf{J}_\omega &= G_\omega \left[\mathbf{t}_\omega(p_2,p_4,p_1,p_3) - \mathbf{t}_\omega(p_2,p_1,p_4,p_3) \right. \\ & \left. - \mathbf{t}_\omega(p_2,p_3,p_1,p_4) \right] + (p_2 \leftrightarrow p_3), \end{aligned} \qquad (18)$$

where

$$\begin{aligned} \mathbf{t}_\omega(p_1,p_2,p_3,p_4) &= \frac{F^2_\omega(P-p_1)}{D_\omega(P-p_1)D_\rho(p_3+p_4)} \qquad (19) \\ & \times \{ (\varepsilon_4 \mathbf{p}_3 - \varepsilon_3 \mathbf{p}_4)(\mathbf{p}_1\mathbf{p}_2) - \mathbf{p}_2(\varepsilon_4 \mathbf{p}_1 \mathbf{p}_3 - \varepsilon_3 \mathbf{p}_1 \mathbf{p}_4) \\ & - \varepsilon_2 [\mathbf{p}_3(\mathbf{p}_1\mathbf{p}_4) - \mathbf{p}_4(\mathbf{p}_1\mathbf{p}_3)] \}, \end{aligned}$$

ε_i is the energy of the corresponding pion, $F_\omega(q)$ is the form factor. Since the width of the ω is small, we set $g_\omega(s) = 1$ in the propagator $D_\omega(q)$.

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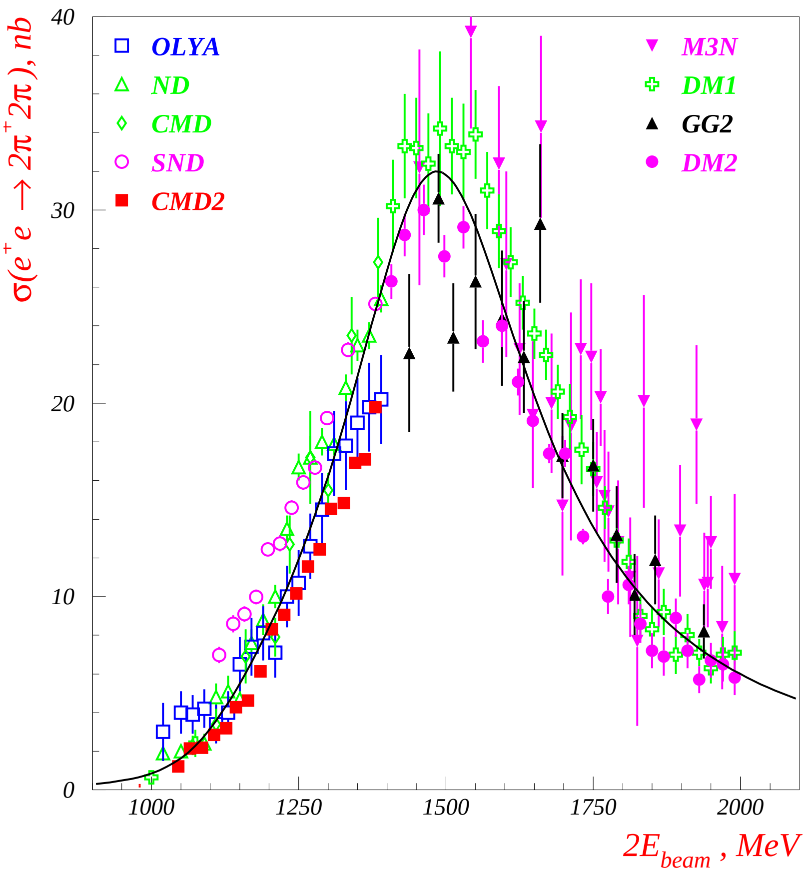

Figure 1: Energy dependence of the $e^+e^- \to 2\pi^+2\pi^-$ cross section

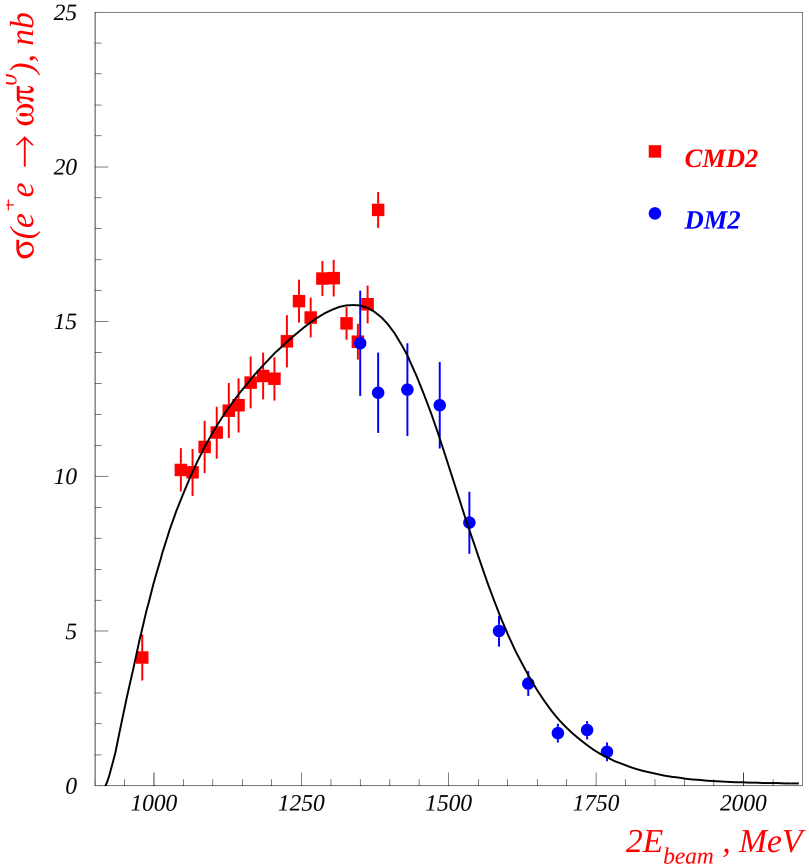

Figure 2: Energy dependence of the $e^+e^- \to \omega\pi^0$ cross section

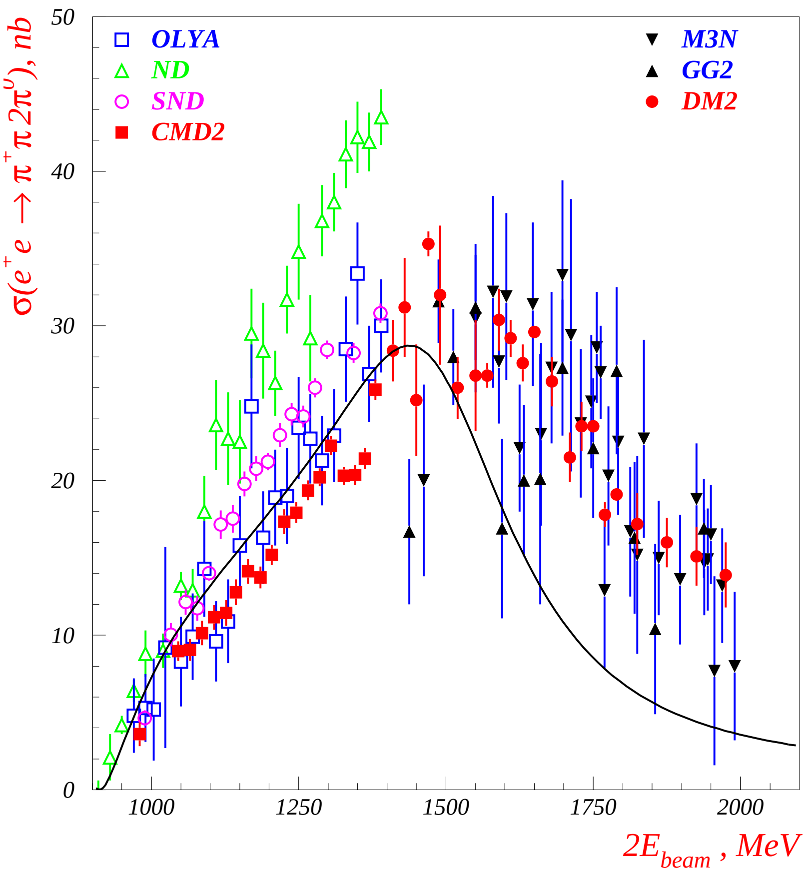

Figure 3: Energy dependence of the $e^+e^- \to \pi^+\pi^-2\pi^0$ cross section

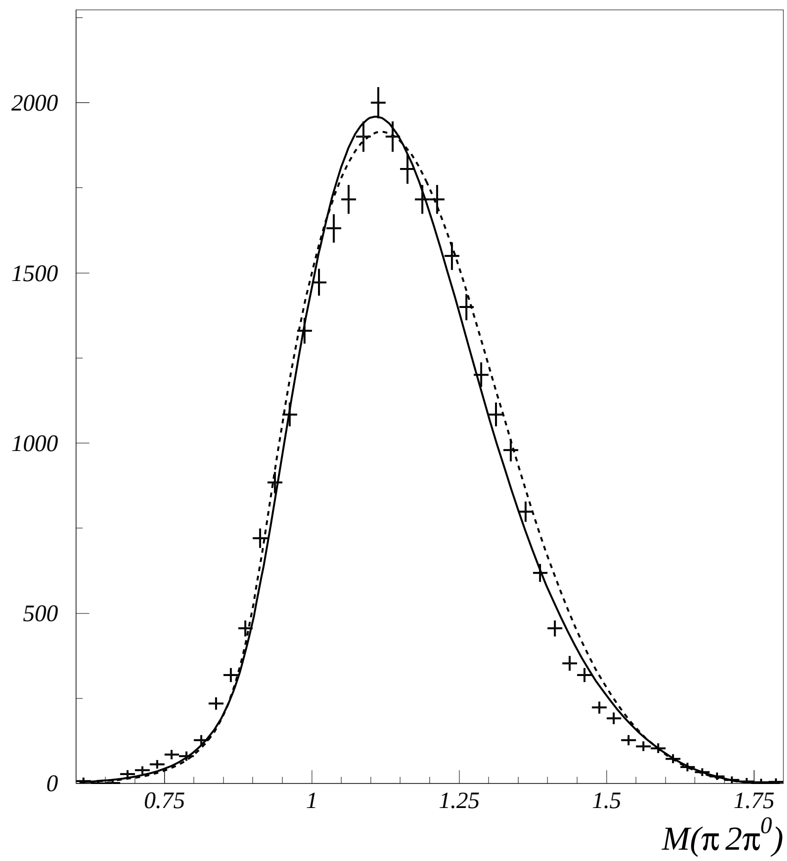

Figure 4: Three pion invariant mass distribution in the $\tau^- \to \pi^- 2\pi^0 \nu_\tau$ decay. The solid curve is obtained taking into account the $a_1 \to \sigma\pi$ decay, the dashed one corresponds to the $a_1 \to \rho\pi$ decay only.

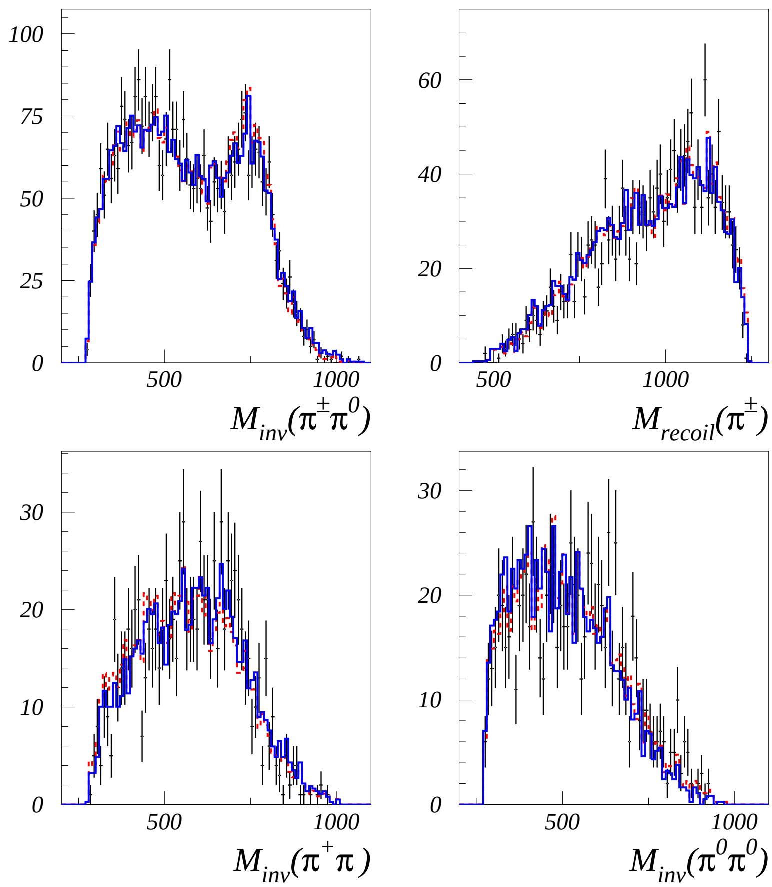

Figure 5: Distributions over invariant mass $M_{inv}(\pi^{\pm}\pi^0)$, $M_{recoil}(\pi^{\pm})$, $M_{inv}(\pi^+\pi^-)$, $M_{inv}(\pi^0\pi^0)$ for $e^+e^- \to \pi^+\pi^-2\pi^0$ process after $\omega\pi^0$ events subtraction. The solid curve is obtained taking into account the $a_1 \to \sigma\pi$ decay, the dashed one corresponds to the $a_1 \to \rho\pi$ decay only.

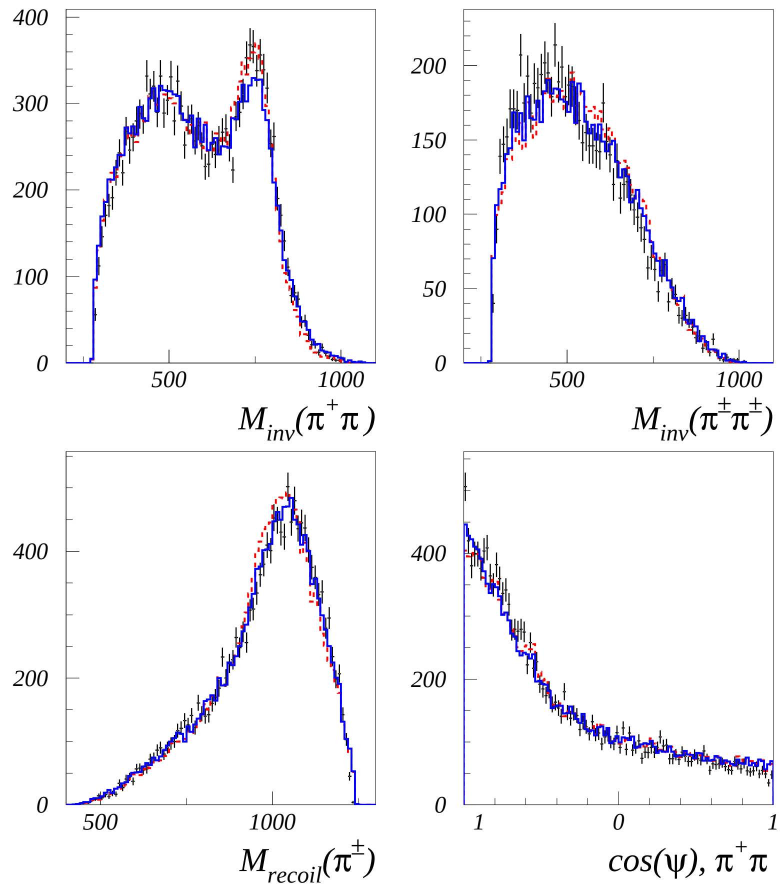

Figure 6: Distributions over invariant mass $M_{inv}(\pi^+\pi^-)$, $M_{inv}(\pi^\pm\pi^\pm)$, $M_{recoil}(\pi^\pm)$, and the distribution over the angle between the momenta of π^+ and π^- for $e^+e^- \to 2\pi^+2\pi^-$ process. The solid curve is obtained taking into account the $a_1 \to \sigma\pi$ decay, the dashed one corresponds to the $a_1 \to \rho\pi$ decay only.

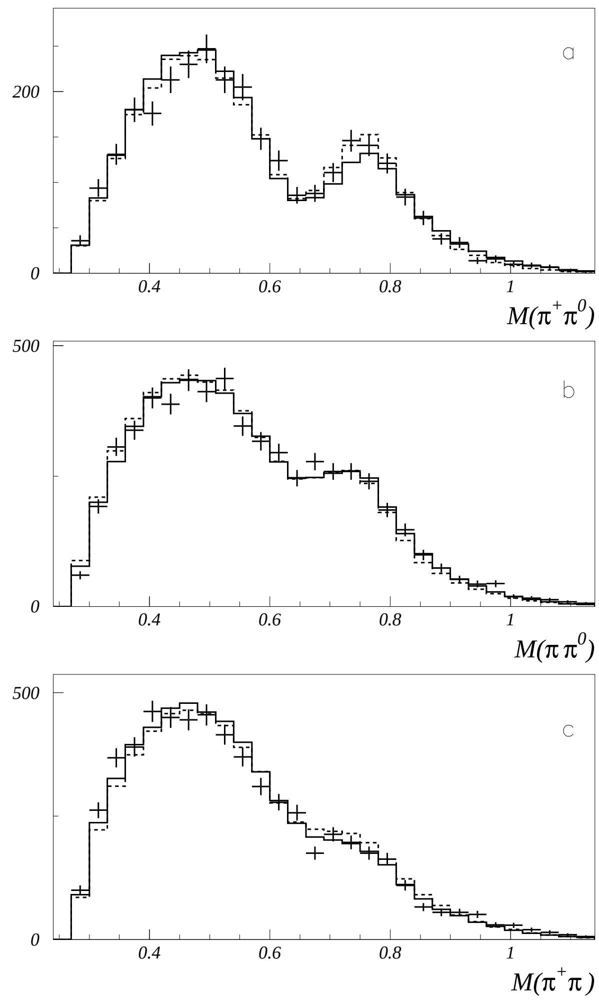

Figure 7: Distributions over invariant mass $M_{inv}(\pi^+\pi^0)$, $M_{inv}(\pi^-\pi^0)$, and $M_{inv}(\pi^+\pi^-)$ for $\tau^- \to 2\pi^-\pi^+\pi^0\nu_\tau$ decay obtained by CLEOII [15]. The solid curve is the prediction obtained with the $a_1 \to \sigma\pi$ decay taken into account, the dashed one corresponds to the $a_1 \to \rho\pi$ decay only.

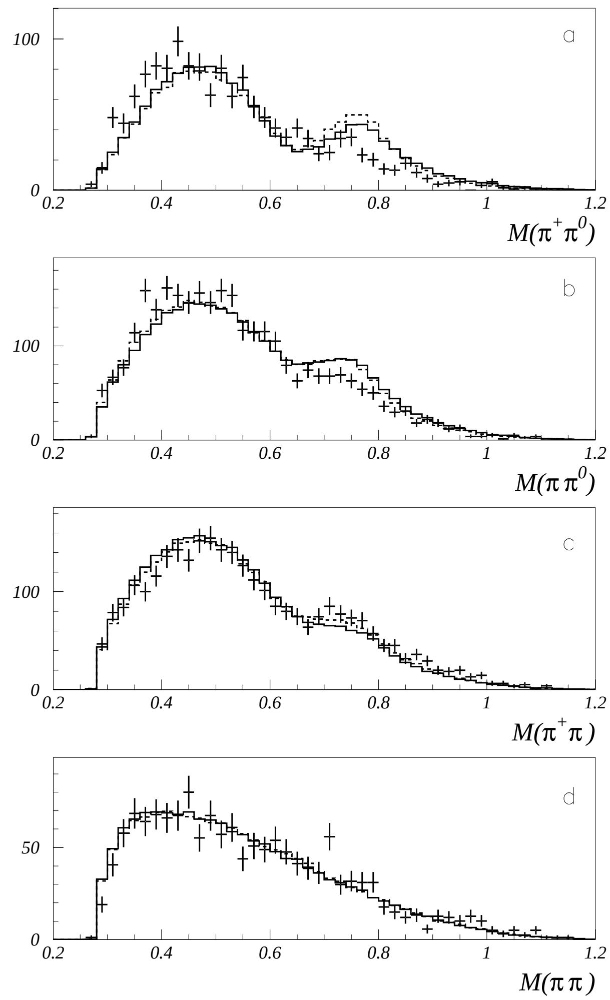

Figure 8: Distributions over invariant mass $M_{inv}(\pi^+\pi^0)$, $M_{inv}(\pi^-\pi^0)$, $M_{inv}(\pi^+\pi^-)$, and $M_{inv}(\pi^-\pi^-)$ for $\tau^- \to 2\pi^-\pi^+\pi^0\nu_\tau$ decay obtained by ARGUS [13]. The solid curve is the prediction obtained with the $a_1 \to \sigma\pi$ decay taken into account, the dashed one corresponds to the $a_1 \to \rho\pi$ decay only.

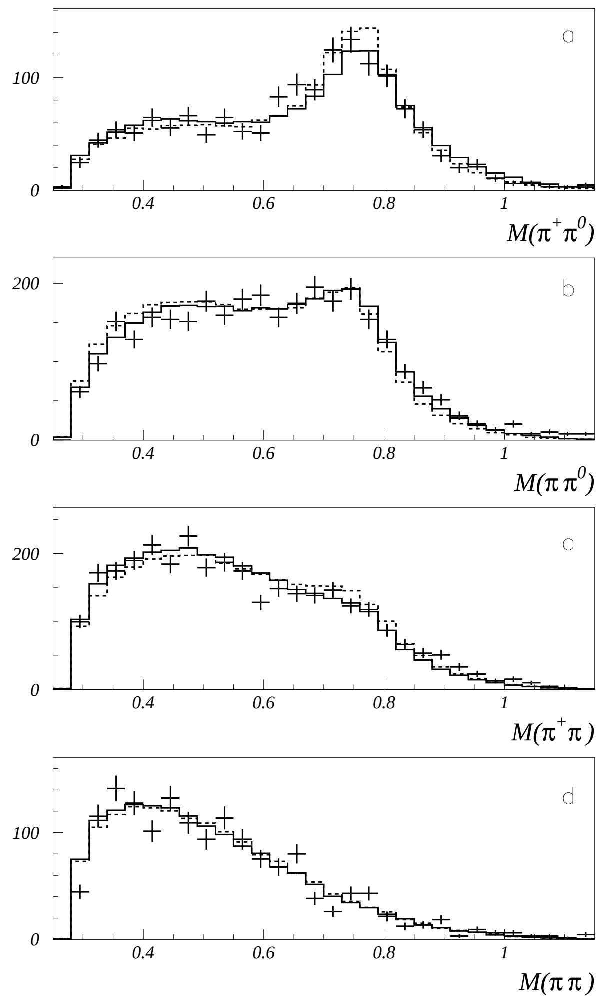

Figure 9: Distributions over invariant mass $M_{inv}(\pi^+\pi^0)$, $M_{inv}(\pi^-\pi^0)$, $M_{inv}(\pi^+\pi^-)$, and $M_{inv}(\pi^-\pi^-)$ for $\tau^- \to 2\pi^-\pi^+\pi^0\nu_\tau$ decay after $\omega\pi^-$ events subtraction obtained by ALEPH [14]. The solid curve is the prediction obtained with the $a_1 \to \sigma\pi$ decay taken into account, the dashed one corresponds to the $a_1 \to \rho\pi$ decay only.

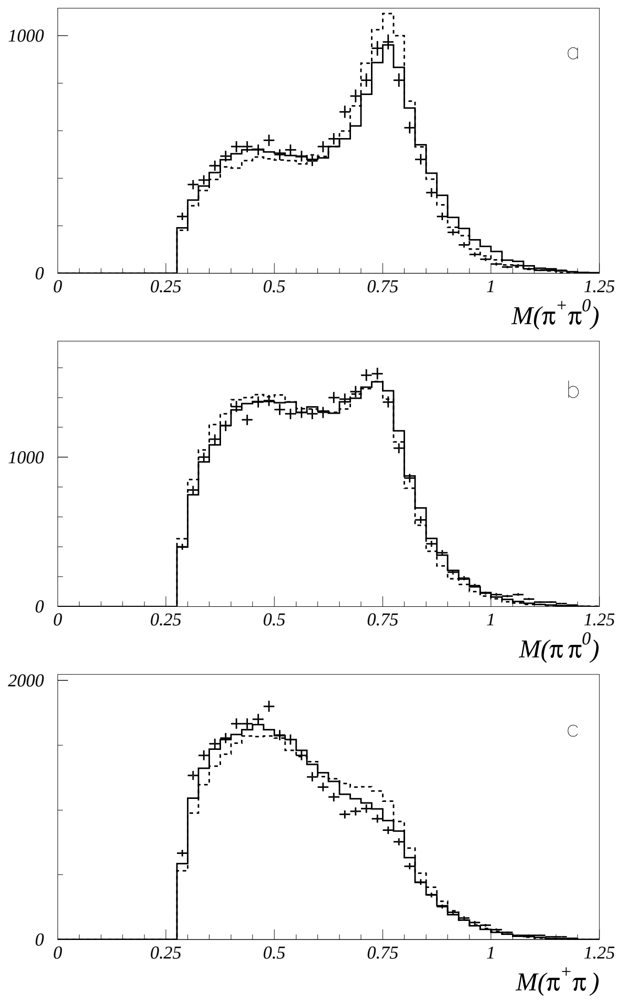

Figure 10: Distributions over invariant mass $M_{inv}(\pi^+\pi^0)$, $M_{inv}(\pi^-\pi^0)$, and $M_{inv}(\pi^+\pi^-)$ for $\tau^- \to 2\pi^-\pi^+\pi^0\nu_\tau$ decay after $\omega\pi^-$ events subtraction obtained by CLEOII [16]. The solid curve is the prediction obtained with the $a_1 \to \sigma\pi$ decay taken into account, the dashed one corresponds to the $a_1 \to \rho\pi$ decay only.