ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ
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SUM RULES FOR $1/m$ CORRECTIONS TO THE FORM FACTORS OF SEMILEPTONIC B DECAYS

BUDKERINP 92-80

НОВОСИБИРСК

Sum rules for $1/m$ corrections to the form factors of semileptonic B decays

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ABSTRACT

Semileptonic B decays are described by the Isgur-Wise form factor to the leading order in $1/m$; four new functions appear in the first order [5]. Values of these functions are crucial for the applicability of the whole approach to processes involving c quark. We obtain the sum rules for three subleading form factors from the QCD sum rules with finite masses by expanding to the first order in $1/m$. The results respect the pattern of the first $1/m$ corrections established in HQET, and obey the Luke's theorem. The numerical estimates show that $1/m_c$ corrections are sizable but not catastrophic.

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1 Introduction

Recently a significant progress has been achieved in the heavy quark physics in the framework of the Heavy Quark Effective Theory (HQET) [1], see also the reviews [2] and references cited therein. The matrix elements of the vector current $V_\mu = \bar{c}\gamma_\mu b$ for $B \to D^{(*)}$ decays

$$\begin{aligned}\langle D|V_\mu|B\rangle &= \sqrt{m_B m_D}\,(\xi_+(v+v')_\mu + \xi_-(v-v')_\mu)\,,\\ \langle D^*|V_\mu|B\rangle &= \sqrt{m_B m_{D^*}}\,\xi_V\, i\varepsilon_{\mu\nu\alpha\beta}e^*_\nu v'_\alpha v_\beta,\end{aligned} \tag{1}$$

to the leading order in $1/m$ are expressed via the Isgur-Wise form factor: $\xi_+ = \xi_V = \xi(\mathrm{ch}\,\varphi)$, $\xi_- = 0$, where $\mathrm{ch}\,\varphi = vv'$, v, v' are 4-velocities of B, $D^{(*)}$, $\xi(1) = 1$ [3, 4]. First $1/m_c$ corrections involve four new functions [5]; $1/m_b$ corrections contain no new elements [6]:

$$\begin{aligned}\xi_+ &= \xi\left[1+\left(\frac{1}{m_c}+\frac{1}{m_b}\right)\rho_1\right],\\ \xi_- &= \xi\left(\frac{1}{m_c}-\frac{1}{m_b}\right)\left(-\frac{\varepsilon}{2}+\rho_4\right),\\ \xi_V &= \xi\left[1+\left(\frac{1}{m_c}+\frac{1}{m_b}\right)\frac{\varepsilon}{2}+\frac{\rho_2}{m_c}+\frac{\rho_1-\rho_4}{m_b}\right],\end{aligned} \tag{2}$$

where the HQET ground state meson's energy $\varepsilon = m_B - m_b = m_D - m_c = m_{D^*} - m_c$ (these differences are equal up to $1/m$ corrections). The Luke's theorem [5–7] states that $\rho_1(1) = 0$, $\rho_2(1) = 0$.

The most interesting physical applications of HQET are those to $b \to c$ weak transitions. Therefore the applicability of the whole approach to processes involving c quark (i. e. the size of $1/m_c$ corrections) is a crucial question of HQET. If these corrections are large, HQET has only a purely academic interest; if they are modest, it can be applied to real-world processes.

A nonperturbative method is needed to calculate form factors. QCD sum rules [8, 9] were used for investigation of the form factors (1) at finite m_b, m_c in [10, 11]. HQET sum rule for the Isgur-Wise form factor was considered in [12, 13]. It coincides with the limit $m_{b,c} \to \infty$ of the QCD sum rules. Results at finite and infinite masses are compared in [14]. There are two alternative ways to obtain sum rules for the subleading form factors ρ_i. One can expand the known finite-mass QCD results to the first order in $1/m$. Alternatively, one can start from the HQET lagrangian and currents in the first order in $1/m$. The second way gives more insight into the sources of the heavy quark symmetry breaking (e. g. the heavy quark chromomagnetic moment vertex); the general theorems of HQET (like (2) and the Luke's theorem) can be traced in the calculation. But the first way is less labour-consuming provided that the QCD results are already known, and allows to consider easily also higher $1/m$ corrections. It also gives a strong check of the QCD results. Here we use this way.

The situation is similar in a simpler case of 2-point sum rules. Here the QCD (Borel-transformed) sum rules [15–19] coincide with the leading-order HQET sum rules [20, 21] in the limit $m_{b,c} \to \infty$. The first $1/m$ correction was obtained by expanding the QCD results [22], and also in the framework of HQET [23].

2 Sum rules

We consider three-point correlators

$$\begin{aligned} K_\mu(p_b,p_c) &= \int dx_b dx_c e^{-ip_b x_b + ip_c x_c} \langle T j_B(x_b) V_\mu(0) j_D^+(x_c)\rangle \\ &= K_+(p_b^2,p_c^2,t)p_\mu + K_-(p_b^2,p_c^2,t)q_\mu, \qquad (3) \\ K_{\mu\nu}(p_b,p_c) &= K_V(p_b^2,p_c^2,t)i\varepsilon_{\mu\nu\alpha\beta}p_\alpha q_\beta, \end{aligned}$$

where $K_{\mu\nu}$ is similar to K_μ with $j_{D^*\nu}$ instead of j_D; $j_B = \overline{q}\gamma_5 b$, $j_D = \overline{q}\gamma_5 c$, $j_{D^*\nu} = \overline{q}\gamma_\nu c$ are the currents with the quantum numbers of B, D, D^*; $p = p_b + p_c$, $q = p_b - p_c$, $t = q^2$. We have calculated perturbative spectral densities $\rho_i(s_b,s_c,t)$ and quark condensates' contributions $K_i^q(p_b^2,p_c^2,t)$ up to

dimension 6 for the invariant functions $K_i(p_b^2,p_c^2,t)$ using REDUCE [24]; part of the results was published in [10].

In order to consider the limit $m_{b,c} \to \infty$, we proceed the new variables $p_{b,c}^2 = m_{b,c}^2 + 2m_{b,c}\omega_{b,c}$, $t = m_b^2 + m_c^2 - 2m_b m_c \operatorname{ch}\varphi$. In these variables the support of perturbative spectral densities [10] is the wedge

$$e^{-\varphi} < \omega_b/\omega_c < e^{\varphi}. \qquad (4)$$

At $t > 0$ spectral densities are singular at the parabola $s_b^2 + s_c^2 + t^2 - 2s_b s_c - 2s_b t - 2s_c t = 0$ which touches the boundary of the physical region (4) at

$$\omega_b^* = \frac{m_b m_c \operatorname{sh}\varphi}{t}\left(m_b - m_c e^{-\varphi}\right), \quad \omega_c^* = \frac{m_b m_c \operatorname{sh}\varphi}{t}\left(-m_c + m_b e^{\varphi}\right). \qquad (5)$$

The double dispersion representation should be modified above this point [25]. In the limit $m_{b,c} \to \infty$, this area goes to infinity and hence lies outside the lowest mesons' duality region essential for the sum rules.

Making the double Borel transform from the variables $\omega_{b,c}$ to $E_{b,c}$, we obtain QCD sum rules (for finite $m_{b,c}$)

$$\begin{aligned} &\frac{f_B m_B^2}{m_b}\frac{f_D m_D^2}{m_c}\xi_\pm(\operatorname{ch}\varphi)e^{-(\varepsilon_B+\varepsilon_D)/(2E)} = \frac{2m_b m_c}{\sqrt{m_B m_D}} \\ &\left\{\int [(m_B \pm m_D)\rho_+ + (m_B \mp m_D)\rho_-]\, e^{-(\omega_b+\omega_c)/(2E)} d\omega_b d\omega_c \right. \\ &\left. +4E^2\widehat{B}\left[(m_B \pm m_D)K_+^q + (m_B \mp m_D)K_-^q\right]\right\}, \qquad (6) \\ &\frac{f_B m_B^2}{m_b} f_{D^*} m_{D^*} \xi_V(\operatorname{ch}\varphi) e^{-(\varepsilon_B+\varepsilon_{D^*})/(2E)} \\ &= 8m_b m_c\sqrt{m_B m_D}\left\{\int \rho_V e^{-(\omega_b+\omega_c)/(2E)} d\omega_b d\omega_c + 4E^2\widehat{B}K_V^q\right\}, \end{aligned}$$

where we have taken $E_b = E_c = 2E$; $m_B^2 = m_b^2 + 2m_b\varepsilon_B$, and similarly for $D^{(*)}$. The integrals are calculated over the part of the wedge (4) dual to the lowest mesons in both channels, i. e. the wedge minus the higher states' continuum region. It is convenient to introduce the variables ω, η instead of $\omega_{b,c} = \omega(1 \pm \eta \operatorname{th}^2\frac{\varphi}{2})$ so that the physical region (4) is $-1 < \eta < 1$, and to measure quantities with the dimension of energy (such as E) in the units k, $k^3 = \frac{\pi^2}{6}|\langle\overline{q}q\rangle|$ ($m_0 = 4kE_0$). Expanding to the first order in $1/m_{b,c}$, we

obtain

$$\frac{f_B m_B^2 f_D m_D^2}{(m_b m_c)^{3/2}|\langle\bar{q}q\rangle|}e^{-(\varepsilon_B+\varepsilon_D)/(2E)}\xi_+(\mathrm{ch}\,\varphi) = A(\varphi,E)+\left(\frac{1}{m_c}+\frac{1}{m_b}\right)kB_1(\varphi,E),$$

$$\frac{f_B m_B^2 f_D m_D^2}{(m_b m_c)^{3/2}|\langle\bar{q}q\rangle|}e^{-(\varepsilon_B+\varepsilon_D)/(2E)}\xi_-(\mathrm{ch}\,\varphi) = \left(\frac{1}{m_c}-\frac{1}{m_b}\right)\left[-\frac{\varepsilon}{2}A(\varphi,E)+kB_4(\varphi,E)\right], \qquad (7)$$

$$\frac{f_B m_B^2 f_{D^*} m_{D^*}}{m_b^{3/2}m_c^{1/2}|\langle\bar{q}q\rangle|}e^{-(\varepsilon_B+\varepsilon_{D^*})/(2E)}\xi_V(\mathrm{ch}\,\varphi) = \left[1+\left(\frac{1}{m_c}+\frac{1}{m_b}\right)\frac{\varepsilon}{2}\right]A(\varphi,E) + \frac{kB_2(\varphi,E)}{m_c}+\frac{k(B_1(\varphi,E)-B_4(\varphi,E))}{m_b}.$$

Here

$$A(\varphi,E) = \frac{1}{4\,\mathrm{ch}^4\frac{\varphi}{2}}\int\omega^2 e^{-\omega/E}d\eta d\omega + 1 - \frac{2\,\mathrm{ch}\,\varphi+1}{3}\frac{E_0^2}{E^2}+\frac{\alpha_s\,\mathrm{ch}\,\varphi}{27\pi E^3},$$

$$B_1(\varphi,E) = \frac{1}{16\,\mathrm{ch}^6\frac{\varphi}{2}}\int(\mathrm{ch}\,\varphi-5-\eta^2\,\mathrm{ch}\,\varphi+\eta^2)\omega^3 e^{-\omega/E}d\eta d\omega + 2\frac{E_0^2}{E}+\frac{\alpha_s(4\,\mathrm{ch}\,\varphi-1)}{27\pi E^2}, \qquad (8)$$

$$B_2(\varphi,E) = -\frac{1}{8\,\mathrm{ch}^6\frac{\varphi}{2}}\int(3-\eta^2)\omega^3 e^{-\omega/E}d\eta d\omega - \frac{8\alpha_s}{27\pi E^2},$$

$$B_4(\varphi,E) = \frac{1}{8\,\mathrm{ch}^4\frac{\varphi}{2}}\int(1-\eta^2)\omega^3 e^{-\omega/E}d\eta d\omega + \frac{2E_0^2}{3E}+\frac{\alpha_s(4\,\mathrm{ch}\,\varphi+1)}{27\pi E^2}.$$

Similar expansion of 2-point sum rules gives

$$\frac{f_D^2 m_D^4}{m_c^3|\langle\bar{q}q\rangle|}e^{-\varepsilon_D/E} = A(0,E)+\frac{2kB_1(0,E)}{m_c}, \qquad (9)$$

$$\frac{f_{D^*}^2 m_D^2}{m_c|\langle\bar{q}q\rangle|}e^{-\varepsilon_D/E} = A(0,E)+\frac{2kB_2(0,E)}{m_c}$$

(the sum rule for f_B is similar to the one for f_D). From (7) and (9) we obtain

the sum rules for $\xi(\mathrm{ch}\,\varphi)$ [12, 13] and for $\rho_{1,2,4}(\mathrm{ch}\,\varphi)$

$$\xi(\mathrm{ch}\,\varphi) = \frac{A(\varphi,E)}{A(0,E)}, \quad \rho_1(\mathrm{ch}\,\varphi)/k = \frac{B_1(\varphi,E)}{A(\varphi,E)}-\frac{B_1(0,E)}{A(0,E)}, \qquad (10)$$

$$\rho_2(\mathrm{ch}\,\varphi)/k = \frac{B_2(\varphi,E)}{A(\varphi,E)}-\frac{B_2(0,E)}{A(0,E)}, \quad \rho_4(\mathrm{ch}\,\varphi)/k = \frac{B_4(\varphi,E)}{A(\varphi,E)}.$$

The results of $1/m$ expansion of the QCD sum rules obey the structure (2) and the Luke's theorem. It is easy to explain why $\rho_1(1)=0$. At finite $m_b = m_c$, $q\to 0$ the 3-point correlator is related to the 2-point one by the Ward identity $K_+(p^2,p^2) = \frac{d\Pi(p^2)}{dp^2}$ [10], and hence the QCD sum rule reproduces the exact result $\xi_+(t=0)=1$. At infinite $m_{b,c}$, the Ward identity is $K(\omega,\omega) = \frac{d\Pi(\omega)}{d\omega}$, and hence the HQET sum rule reproduces the exact result $\xi(1)=1$. Then we obtain from (2) $\rho_1(1)=0$. Therefore it is not accidental that the $1/m$ correction to the sum rule for f_D (9) is described by the same function B_1 as to the sum rule for ξ_+ (7) but at $\varphi=0$. The similar fact for the sum rule for f_{D^*} (9) and $1/m_c$ correction to ξ_V (7) looks like a miracle in the framework of QCD sum rules, but it leads to the Luke's theorem for ρ_2 in (10). The dependence between $1/m_b$ correction to ξ_V and $1/m$ corrections to $\xi_\pm$ in (7) is one more miracle. From (9) we obtain (compare with [22])

$$f_D = \frac{\tilde{f}}{\sqrt{m_c}}\left(1+\frac{c_1}{m_c}\right), \quad f_{D^*} = \frac{\tilde{f}}{\sqrt{m_c}}\left(1+\frac{c_2}{m_c}\right), \qquad (11)$$

where the coefficients are given by the sum rules

$$\frac{\tilde{f}^2}{|\langle\bar{q}q\rangle|}e^{-\varepsilon/E} = A(0,E), \qquad (12)$$

$$c_1/k = \frac{B_1(0,E)}{A(0,E)}-2\varepsilon/k, \quad c_2/k = \frac{B_2(0,E)}{A(0,E)}-\varepsilon/k.$$

3 Results

At the standard values of the condensates, $k = 260$–280MeV, $E_0 = \frac{m_0}{4k} = 0.85$. The 2-point sum rule (12) for $\tilde{f}^2$ was analyzed in [20]. The continuum threshold is $\omega_0 = 3$ and the meson energy is $\varepsilon = 1.65$ in the Borel parameter plato $E = 1.7$–2.5 (all in k units). Now we know that the perturbative correction to these results is large [21]. But this correction is unknown in

the 3-point sum rules. We may hope that it will be partially compensated in the ratio of 3-point to 2-point sum rules. For consistency, we use the 2-point results without perturbative corrections as an input for the 3-point sum rules analysis. In any case, even a rough estimate of ρ_i is currently valuable.

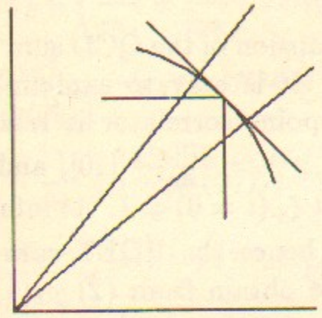

Figure 1: Continuum models

In the case of 3-point sum rules, e. g. for $\xi(\mathrm{ch}\,\varphi)$ (10), the higher states' continuum spectral density is modelled by the perturbative spectral density with some smooth curve as a continuum threshold (Fig. 1). At $\varphi \to 0$ the wedge becomes infinitesimally narrow, and the continuum starts at ω_0 for consistency with the 2-point sum rules. The wedge area is $\sim \varphi$, therefore the spectral density must be $\sim 1/\varphi$ in order to yield the perturbative contribution ~ 1 (of course, the explicit calculations confirm it). The simplest continuum model is that with the straight line threshold. If one chooses any smooth curve instead, the difference area is $\sim \varphi^3$ (Fig. 1), and the variation of $\xi(\mathrm{ch}\,\varphi)$ is $\sim \varphi^2$. It influences the slope of $\xi(\mathrm{ch}\,\varphi)$ at 1; this freedom is analogous to the freedom of choosing the continuum threshold in the 2-point sum rule (12) for $\widetilde{f}^2$. But if one chooses a continuum threshold with a cusp in the physical region (as was done in [12]), the difference area is $\sim \varphi^2$ (Fig. 1), and the variation of $\xi(\mathrm{ch}\,\varphi)$ is $\sim \varphi$. This gives an infinite slope of $\xi(\mathrm{ch}\,\varphi)$ at 1, what contradicts to its general analytical properties. Besides that, there are no physical reasons for the continuum threshold to be non-smooth. Here we restrict ourselves to the simplest continuum model with the straight-line threshold; dependence of $\xi(\mathrm{ch}\,\varphi)$ on the threshold curvature was investigated in [13].

The sum rule for $\xi(\mathrm{ch}\,\varphi)$ (10) is well known [12, 13]. It contains two main terms: the perturbative contribution and the quark condensate one. The quark condensate contribution is constant (up to nonlocality effects discussed

in [12]). The perturbative contribution contains the light quark propagator from x_c to x_b, and falls as $1/(x_b - x_c)^4$. The Euclidean distances from 0 to x_b and from 0 to x_c are $1/(2E)$; the angle between these lines is φ. Hence the distance from x_b to x_c is $\mathrm{ch}\,\frac{\varphi}{2}/E$. The perturbative contribution is suppressed as $1/\,\mathrm{ch}^4\,\frac{\varphi}{2}$ compared to the 2-point (straight-line) case. This leads to the decreasing of $\xi(\mathrm{ch}\,\varphi)$ with increasing φ. Graphs for $\xi(\mathrm{ch}\,\varphi)$ at several Borel parameters are shown at Fig. 2; the remarkable stability is seen.

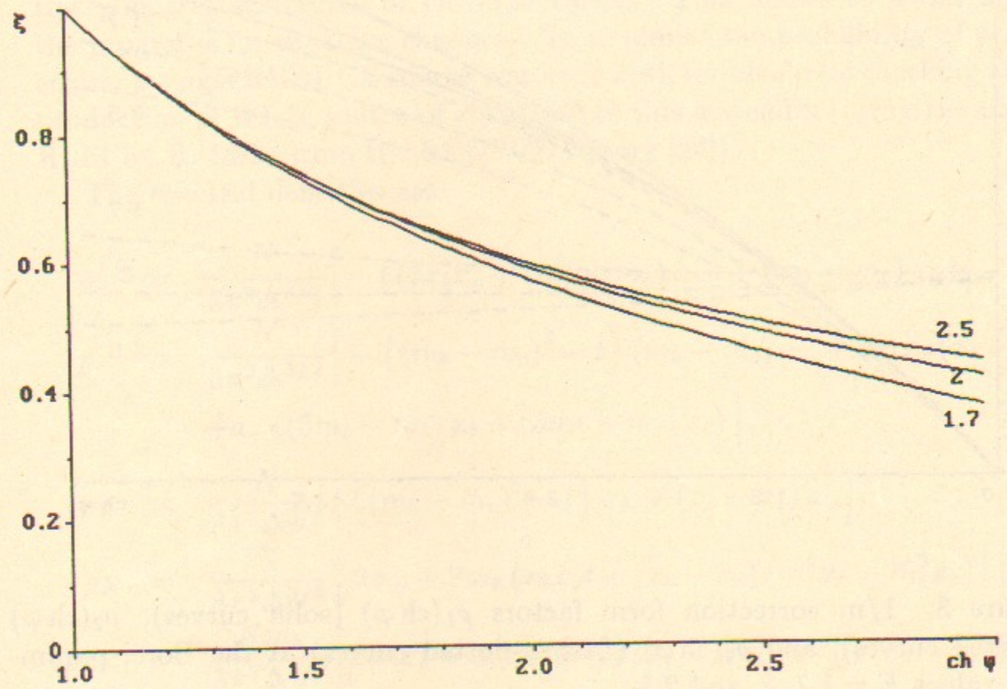

Figure 2: Isgur-Wise form factor $\xi(\mathrm{ch}\,\varphi)$ at the Borel parameter values $E =$ 1.7, 2, and 2.5

Now we present the new results for the $1/m$ correction form factors $\rho_{1,2,4}(\mathrm{ch}\,\varphi)$. It is seen from the sum rules (10) that their scale is set by the unit $k = \left(\frac{\pi^2}{6}|\langle\overline{q}q\rangle|\right)^{1/3} = 260$–$280$MeV. The dimensionless functions in the sum rules don't contain large numerical factors. Therefore the unit k determines the scale of $1/m_{b,c}$ corrections in the semileptonic B decays form factors. The numerical analyses of the sum rules confirms this conclusion; the results are shown at Fig. 3. The curves $\rho_{1,2}(\mathrm{ch}\,\varphi)$ are pinned at the origin by the Luke's theorem; at $\mathrm{ch}\,\varphi > 1$ they grow and reach the values of order

k. On the other hand, $\rho_4(\mathrm{ch}\,\varphi)$ is small and nearly constant. Variation of the results with the Borel parameter E allows to estimate their accuracy.

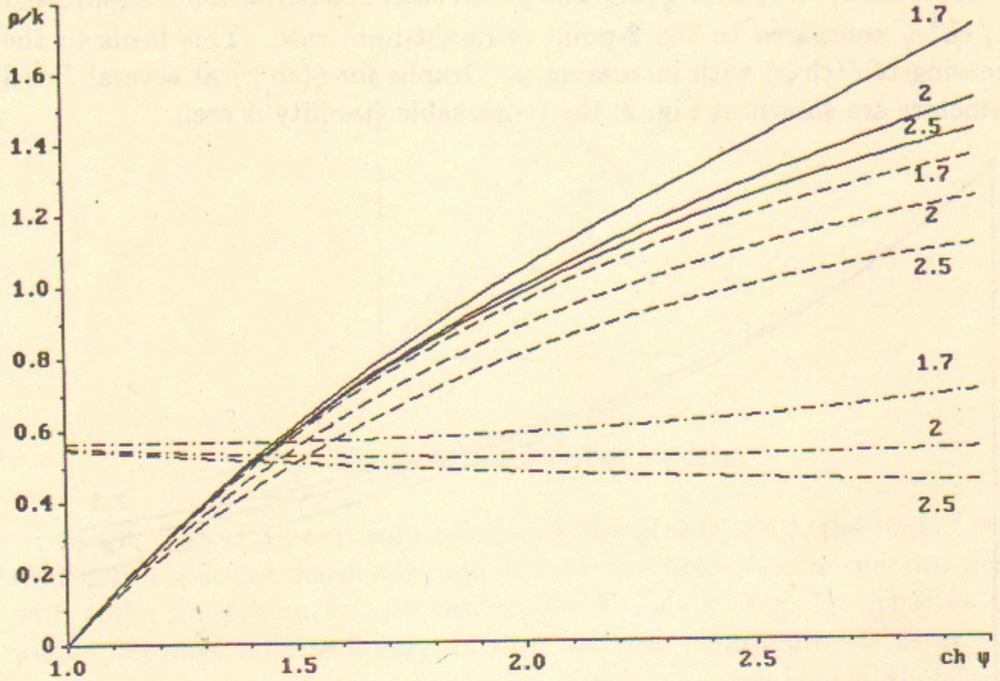

Figure 3: $1/m$ correction form factors $\rho_1(\mathrm{ch}\,\varphi)$ (solid curves), $\rho_2(\mathrm{ch}\,\varphi)$ (dashed curves), and $\rho_4(\mathrm{ch}\,\varphi)$ (dashed-dotted curves) at the Borel parameter values $E = 1.7$, 2, and 2.5

In this work we have investigated the sum rules for $\rho_{1,2,4}(\mathrm{ch}\,\varphi)$. It is sufficient for the decay $B \to D$. Form factors of the decay $B \to D^*$ also contain $\rho_3(\mathrm{ch}\,\varphi)$. In order to obtain sum rules for it, it is necessary to consider an axial correlator in addition to (3). Such an analysis will be presented elsewhere.

When this work was completed and presented at the seminar at SLAC, M. Neubert informed us about his preprint [26]. In this work the sum rules for the subleading $B \to D^{(*)}$ form factors were obtained in the framework of HQET. The results are similar to our ones. We are grateful to M. Neubert for giving us the preprint [26] and for the useful discussion.

A Correlators in QCD

Here we present the spectral densities and quark condensates' contributions up to dimension 6 for the correlators (3) and the correlator similar to $K_{\mu\nu}$ but with the axial current $\bar{c}\gamma_\mu\gamma_5 b$ (it has the structure $K_A g_{\mu\nu} + K_{++} p_\mu p_\nu + K_{+-} p_\mu q_\nu + K_{-+} q_\mu p_\nu + K_{--} q_\mu q_\nu$). The results for all correlators have been produced by a single REDUCE [24] program in which only few lines with the γ-matrix structures of currents varied. This allows to avoid bugs in the programs for separate channels. To minimize the probability of printing errors, a single REDUCE source was used both for algebraic checking and for production of LaTeX source of equations in this appendix (using the package RLFI by R. Liska from the REDUCE library [24]).

The spectral densities are

$$\begin{aligned}
\rho_+ &= \frac{N}{8\pi^2\Delta^{3/2}}\Big[-t\,(2x_bx_c + (m_b+m_c)\,a_+) + (m_b-m_c)\,a_+ b\Big],\\
\rho_- &= \frac{N}{8\pi^2\Delta^{3/2}}\Big[-\Big((m_b-m_c)^2 - t\Big)\,(m_b-m_c)\,a_+ + 2x_bx_c\,(x_b-x_c)\\
&\quad + a_-\,((3m_c-m_b)\,x_b + (3m_b-m_c)\,x_c)\Big],\\
\rho_V &= \frac{N}{8\pi^2\Delta^{3/2}}\Big[\Big((m_b-m_c)^2 - t\Big)\,a_+ + (x_b-x_c)\,a_-\Big],\\
\rho_A &= \frac{N}{8\pi^2\Delta^{3/2}}\Big[\Delta a_+ + 2m_b\,(x_bx_ct + (x_b-x_c)\,(m_c^2x_b - m_b^2x_c))\Big],\\
\rho_{++} &= \frac{Nm_b}{8\pi^2\Delta^{5/2}}\Big[\Delta t\,(x_b+x_c) + \Delta\,(6x_bx_c - (m_b^2-m_c^2)\,(x_b-x_c))\\
&\quad + 6t\,(x_b+x_c)\,(2x_bx_c + m_c^2x_b + m_b^2x_c) - \mathbf{6x_bx_cb^2}\Big],\\
\rho_{+-} &= \frac{N}{8\pi^2\Delta^{5/2}}\Big[-\Delta t x_b\,(m_b-m_c)\\
&\quad -\Delta\,((x_b-x_c)\,a_+ - (m_b-m_c)\,((m_b^2+m_c^2)\,x_b - 2m_b^2x_c))\\
&\quad -6m_b\,(x_bx_ct + (x_b-x_c)\,(m_c^2x_b - m_b^2x_c))\,b\Big],\\
\rho_{-+} &= \frac{N}{8\pi^2\Delta^{5/2}}\Big[-\Delta t x_b\,(m_b+m_c)\\
&\quad +\Delta\,((x_b-x_c)\,a_- + (m_b+m_c)\,((m_b^2+m_c^2)\,x_b - 2m_b^2x_c))\\
&\quad -6m_b\,(x_bx_ct + (x_b-x_c)\,(m_c^2x_b - m_b^2x_c))\,b\Big],\\
\rho_{--} &= \frac{Nm_b}{8\pi^2\Delta^{5/2}}\Big[\Delta t\,(x_b-x_c)
\end{aligned}$$

$$
\begin{aligned}
&+\Delta\left(\left(-2x_c\left(2x_b-x_c\right)\right)-\left(3m_c^2+m_b^2\right)x_b+\left(3m_b^2+m_c^2\right)x_c\right)\\
&-6t\left(x_b-x_c\right)\left(m_c^2x_b-m_b^2x_c\right)\\
&-6\left(x_b\left(x_b-x_c\right)+\left(m_b^2+m_c^2\right)x_b-2m_b^2x_c\right)\\
&\left(x_c\left(x_c-x_b\right)+\left(m_c^2+m_b^2\right)x_c-2m_c^2x_b\right)\Big],
\end{aligned}
$$

where $x_{b,c}=s_{b,c}-m_{b,c}^2$, $a_\pm=m_cx_b\pm m_bx_c$, $b=x_b-x_c+m_b^2-m_c^2$.

The results for $\overline{q}(\gamma_5,\gamma_\nu)c\to\overline{q}(1,\gamma_\nu\gamma_5)c$, $\overline{b}(\gamma_\mu,\gamma_\mu\gamma_5)c\to\overline{b}(\gamma_\mu\gamma_5,\gamma_\mu)c$ can be easily obtained by

$$K_i'=-K_i(m_c\to-m_c). \tag{13}$$

There is an interesting check of these formulae. If we multiply a correlator with $\overline{q}\gamma_\nu c$ by $p_{c\nu}$ and use the identity $S_q(k)\gamma_\nu S_c(p_c+k)p_{c\nu}=m_cS_q(k)S_c(p_c+k)+S_q(k)-S_c(p_c+k)$, we obtain the corresponding correlator with $\overline{q}c$ plus terms having no double discontinuity. Therefore

$$
\begin{aligned}
\rho_A+(s_b+3s_c-t)\rho_{++}+(s_b-s_c-t)\rho_{+-} &= -2\rho_+(m_c\to-m_c),\\
-\rho_A+(s_b+3s_c-t)\rho_{-+}+(s_b-s_c-t)\rho_{--} &= -2\rho_-(m_c\to-m_c).
\end{aligned}
$$

The quark condensates' contributions are

$$
\begin{aligned}
\frac{K_+^q}{|\langle\overline{q}q\rangle|} &= -\frac{m_b+m_c}{2x_bx_c}+\frac{m_0^2}{4}\Bigg[(m_b+m_c)\left(\frac{m_b^2}{x_b^3x_c}+\frac{m_c^2}{x_c^3x_b}+\frac{c}{3x_b^2x_c^2}\right)\\
&\quad+\frac{2}{3}\left(\frac{2m_b+m_c}{x_b^2x_c}+\frac{2m_c+m_b}{x_c^2x_b}\right)\Bigg]\\
&\quad+\frac{m_1^3}{3}\Bigg[-(m_b+m_c)\,a_+\left(\frac{m_b^2}{x_b^4x_c^2}+\frac{m_c^2}{x_c^4x_b^2}+\frac{2}{3}\frac{(m_b-m_c)^2-t}{x_b^3x_c^3}\right)\\
&\quad+\frac{2}{3}\left(\frac{m_b\left(m_b+2m_c\right)}{x_b^3x_c}+\frac{m_c\left(m_c+2m_b\right)}{x_c^3x_b}+\frac{c-2t}{x_b^2x_c^2}\right)\\
&\quad+4\left(\frac{1}{x_b^2x_c}+\frac{1}{x_c^2x_b}\right)\Bigg]\\
\frac{K_-^q}{|\langle\overline{q}q\rangle|} &= \frac{m_b-m_c}{2x_bx_c}+\frac{m_0^2}{4}\Bigg[-(m_b-m_c)\left(\frac{m_b^2}{x_b^3x_c}+\frac{m_c^2}{x_c^3x_b}+\frac{c}{3x_b^2x_c^2}\right)\\
&\quad+\frac{2}{3}\left(-\frac{3m_b-m_c}{x_b^2x_c}+\frac{3m_c-m_b}{x_c^2x_b}\right)\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{m_1^3}{3}\Bigg[(m_b-m_c)\,a_+\left(\frac{m_b^2}{x_b^4x_c^2}+\frac{m_c^2}{x_c^4x_b^2}+\frac{2}{3}\frac{(m_b-m_c)^2-t}{x_b^3x_c^3}\right)\\
&\quad-\frac{4}{3}\frac{(m_bx_b+m_cx_c)\,a_-}{x_b^3x_c^3}-\frac{8}{3}\left(\frac{1}{x_b^2x_c}-\frac{1}{x_c^2x_b}\right)\Bigg]\\
\frac{K_V^q}{|\langle\overline{q}q\rangle|} &= -\frac{1}{2x_bx_c}+\frac{m_0^2}{4}\left[\frac{m_b^2}{x_b^3x_c}+\frac{m_c^2}{x_c^3x_b}+\frac{c}{3x_b^2x_c^2}+\frac{2}{3x_b^2x_c}\right]\\
&\quad+\frac{m_1^3}{3}\Bigg[-a_+\left(\frac{m_b^2}{x_b^4x_c^2}+\frac{m_c^2}{x_c^4x_b^2}+\frac{2}{3}\frac{(m_b-m_c)^2-t}{x_b^3x_c^3}\right)\\
&\quad-\frac{4}{3}\left(-\frac{m_b}{x_b^3x_c}+\frac{2m_c}{x_c^3x_b}+\frac{m_b+m_c}{x_b^2x_c^2}\right)\Bigg]\\
\frac{K_A^q}{|\langle\overline{q}q\rangle|} &= -\frac{(m_b+m_c)^2-t}{2x_bx_c}-\frac{1}{2x_b}-\frac{1}{2x_c}\\
&\quad+\frac{m_0^2}{4}\Bigg[\left((m_b+m_c)^2-t\right)\left(\frac{m_b^2}{x_b^3x_c}+\frac{m_c^2}{x_c^3x_b}+\frac{c}{3x_b^2x_c^2}\right)\\
&\quad+\frac{m_b^2}{x_b^3}+\frac{m_c^2}{x_c^3}+\frac{3m_b^2+4m_c^2+9m_bm_c-4t}{3x_b^2x_c}\\
&\quad+\frac{2m_b^2+3m_c^2+3m_bm_c-2t}{3x_c^2x_b}+\frac{2}{3}\left(\frac{1}{x_b^2}-\frac{1}{x_bx_c}\right)\Bigg]\\
&\quad+\frac{m_1^3}{3}\Bigg[-\left((m_b+m_c)^2-t\right)a_+\left(\frac{m_b^2}{x_b^4x_c^2}+\frac{m_c^2}{x_c^4x_b^2}\right.\\
&\quad\left.+\frac{2}{3}\frac{(m_b-m_c)^2-t}{x_b^3x_c^3}\right)-\frac{m_b^3}{x_b^4}-\frac{m_c^3}{x_c^4}\\
&\quad+\frac{m_b\left(3m_b^2+2m_c^2+3m_bm_c-2t\right)}{3x_b^3x_c}\\
&\quad-\frac{m_c\left(10m_b^2+11m_c^2+19m_bm_c-10t\right)}{3x_c^3x_b}\\
&\quad-\frac{6m_b^3+5m_b^2m_c+7m_bm_c^2-2m_c^3-2\left(3m_b-m_c\right)t}{3x_b^2x_c^2}
\end{aligned}
$$

$$+\frac{4}{3}\left(\frac{m_b}{x_b^3}-\frac{2m_c}{x_c^3}+\frac{m_b+4m_c}{x_b^2x_c}+\frac{m_b-2m_c}{x_c^2x_b}\right)\Bigg]$$

$$\frac{K_{++}^q}{|\langle\bar{q}q\rangle|} = \frac{1}{2x_bx_c}-\frac{m_0^2}{4}\left[\frac{m_b^2}{x_b^3x_c}+\frac{m_c^2}{x_c^3x_b}+\frac{m_b^2+2m_c^2-2t}{3x_b^2x_c^2}\right]$$
$$+\frac{m_1^3}{3}\left[m_b\left(\frac{m_b^2}{x_b^4x_c}+\frac{m_c^2}{x_c^4x_b}+\frac{m_b^2+2m_c^2-2t}{3x_b^3x_c^2}+\frac{m_c^2+2m_b^2-2t}{3x_c^3x_b^2}\right)\right.$$
$$\left.+2\left(-\frac{m_b}{x_b^3x_c}+\frac{m_c}{x_c^3x_b}+\frac{m_c}{x_b^2x_c^2}\right)\right]$$

$$\frac{K_{+-}^q}{|\langle\bar{q}q\rangle|} = -\frac{m_0^2}{12}\left[\frac{m_b(m_b-m_c)}{x_b^2x_c^2}-\frac{2}{x_b^2x_c}\right]$$
$$-\frac{m_1^3}{9}\left[(m_b-m_c)\left(-\frac{m_b^2}{x_b^3x_c^2}+\frac{3m_c^2}{x_c^4x_b}+2\frac{m_b^2+m_c^2-t}{x_c^3x_b^2}\right)\right.$$
$$\left.+2\left(\frac{m_b}{x_b^3x_c}-\frac{m_c}{x_c^3x_b}+\frac{2m_b-5m_c}{x_b^2x_c^2}\right)\right]$$

$$\frac{K_{-+}^q}{|\langle\bar{q}q\rangle|} = -\frac{m_0^2}{12}\left[\frac{m_b(m_b+m_c)}{x_b^2x_c^2}-\frac{2}{x_b^2x_c}+\frac{2}{x_c^2x_b}\right]$$
$$-\frac{m_1^3}{9}\left[(m_b+m_c)\left(-\frac{m_b^2}{x_b^3x_c^2}+\frac{3m_c^2}{x_c^4x_b}+2\frac{m_b^2+m_c^2-t}{x_c^3x_b^2}\right)\right.$$
$$\left.+2\frac{(x_b-x_c)\,a_-}{x_b^3x_c^3}\right]$$

$$\frac{K_{--}^q}{|\langle\bar{q}q\rangle|} = -\frac{1}{2x_bx_c}+\frac{m_0^2}{4}\left[\frac{m_b^2}{x_b^3x_c}+\frac{m_c^2}{x_c^3x_b}+\frac{3m_b^2+2m_c^2-2t}{3x_b^2x_c^2}\right.$$
$$\left.+\frac{2}{3}\left(\frac{2}{x_b^2x_c}+\frac{1}{x_c^2x_b}\right)\right]$$
$$+\frac{m_1^3}{3}\left[m_b\left(-\frac{m_b^2}{x_b^4x_c}+\frac{m_c^2}{x_c^4x_b}-\frac{3m_b^2+2m_c^2-2t}{3x_b^3x_c^2}\right.\right.$$
$$\left.\left.+\frac{3m_c^2+2m_b^2-2t}{3x_c^3x_b^2}\right)+\frac{2}{3}\left(\frac{m_b}{x_b^3x_c}-\frac{3m_c}{x_c^3x_b}+\frac{m_b+3m_c}{x_b^2x_c^2}\right)\right]$$

where $c = 2m_b^2+2m_c^2-m_bm_c-2t$; $m_0^2 = i\langle\bar{q}gG_{\mu\nu}^q t^a\sigma_{\mu\nu}q\rangle/\langle\bar{q}q\rangle$, $m_1^3 = -\frac{1}{4}\langle\bar{q}gJ_\mu^a t^a\gamma_\mu q\rangle/\langle\bar{q}q\rangle = \frac{C_F}{N}\pi\alpha_s\langle\bar{q}q\rangle$ in the factorization approximation ($J_\mu^a = D_\nu^{ab}G_{\mu\nu}^b = g\sum_{q'}\overline{q'}t^a\gamma_\mu q$, $C_F = \frac{N^2-1}{2N}$, $N=3$ is the number of colours).

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V.N. Baier and A.G. Grozin

Sum rules for $1/m$ corrections to the form factors of semileptonic B decays

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BUDKERINP 92-80

Ответственный за выпуск С.Г. Попов

Работа поступила 23 октября 1992 г.
Подписано в печать 3.XI. 1992 г.
Формат бумаги 60×90 1/16 Объем 1,3 печ.л., 1,0 уч.-изд.л.
Тираж 290 экз. Бесплатно. Заказ N 80

Обработано на IBM PC и отпечатано
на ротапринте ИЯФ им. Г.И. Будкера СО РАН,
Новосибирск, 630090, пр. академика Лаврентьева, 11.