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BUNCH LENGTHENING — IS IT INEVITABLE?

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НОВОСИБИРСК

Bunch Lengthening — Is It Inevitable?

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ABSTRACT

Should the bunch inevitably extend with the current increase, or, on the contrary, can one manage to shorten it? In other words, is it possible to design such a storage ring, where with an increase in the number of particles the bunch will not lengthen, but, on the contrary, only shorten?

1. QUESTION

The effect of a bunch lengthening with an increase in the number of stored particles has been observed in many high energy physics experiments. An explanation to this phenomenon is found out in models taking into account particle interaction due to the inhomogeneity of a vacuum chamber (cavities, plates etc.) [1, 2]. So, the possible way of bunch lengthening becomes clear. Though, there is no answer yet to the question whether this lengthening is inevitable. Should the bunch inevitably extend with the current increase, or, on the contrary, can one manage to shorten it?

In the experiments there was observed some shortening, though a weak one and on the threshold of an effect, and then with the current increase it was changed by lengthening. Is it a result of fundamental limitations or of some particular peculiarities of the facility?

In other words, is it possible to design such a storage ring, where with an increase in the number of particles the bunch will not lengthen, but, on the contrary, only shorten?

2. MODEL

Let us assume, that the fields, induced by a bunch in the vacuum chamber structure are damped during a single revolution, i. e. the storage ring provides no multirevolution memory. The longitudinal field of a particle, integrated over its closed trajectory can be described by a wake function $W(x)$ (where x is the distance between particles) or by its Fourier transform-impedance:

$$Z_k = \int_0^\infty W(x)\, e^{-ikx}\, dx\,. \tag{1}$$

Let us consider a model system with a step-like wake function:

$$W(x) = W_0\, \theta(x)\,. \tag{2}$$

The choice of a sign $W_0 > 0$ corresponds to coherent energy loss. Let us show, that for such an interaction the bunch shorten with the number of particles.

3. STEADY STATE

In this part we follow [3]. The linear density $\lambda(x)$ of a bunch in the thermodynamic equilibrium satisfies the Haissinski equation [4] which can be presented in the following form:

$$\frac{d\lambda}{dx} = -\frac{\lambda}{\Delta_T^2}\left\{ x - \frac{Ne^2}{\varkappa}\int_x^\infty dx'\, \lambda(x')\, W(x'-x)\right\}. \tag{3}$$

The second term in the braces takes accounts of the action of the bunch self-consistent field. Here N is the number of particles, Δ_T is the r.m.s. deviation at a zero current, $\varkappa$ — RF rigidity, defined in such a way, that $\varkappa x$ is the energy transfer per revolution of a particle, advancing the equilibrium one at a distance x. The density is normalized to unity:

$$\int_{-\infty}^{\infty} \lambda(x)\, dx = 1\,. \tag{4}$$

For kernel (2) the nonlinear integral system (3, 4) allows an analytic solution. In case of a strong own field, when

$$\frac{Ne^2 W_0}{\varkappa \Delta_T} \gg 1\,, \tag{5.0}$$

$$n(x) = \frac{1}{1 + \exp\left\{ \frac{1}{\Delta}(x - x_0)\right\}}\,, \tag{5a}$$

$$\lambda(x) = \frac{dn}{dx}\,. \tag{5b}$$

The shift of a bunch as a whole

$$x_0 = \frac{Ne^2 W_0}{2\varkappa}\,. \tag{5c}$$

The bunch length

$$\Delta = \frac{2\Delta_T^2 \varkappa}{Ne^2 W_0} = \frac{\Delta_T^2}{x_0}\,. \tag{5d}$$

No other solutions for system exist.

4. HAMILTONIANNESS OF THE SYSTEM

Generally speaking, a relativistic bunch is a non-Hamiltonian system: the head particles act on the tail ones, while the tail do not affect the head ones. In this case Newton's Third Law does not work. Therefore, the law of energy conservation cannot be applied to this case, the energy of relativistic motion of bunch particles can increase infinitely at the expense of the bunch motion as a whole. The system with a wake function (2) has a remarkable property — it is equivalent to the **Hamiltonian** one, its energy of particles relative motion is conserved.

Let us prove this statement.

The motion of particles in a bunch can be described by a system of N equations:

$$\ddot{x}_i + \Omega^2 x_i = \sum_j F(x_j - x_i)\,,$$

$$F(x) = \frac{e^2}{\varkappa}\, \Omega^2 W(x)\,, \tag{6}$$

where Ω is the frequency of one-particle synchrotron oscillations. In this case, the force F is convenient to be presented as a sum of two terms:

$$F(x) = \frac{F_0}{2} + \frac{F_0}{2}\,\mathrm{sign}\,(x)\,,$$

$$F_0=\frac{e^2\Omega^2 W_0}{\varkappa}. \qquad (7)$$

For the center of masses coordinate $\bar{x}$ it may be found from (6), that

$$\ddot{\bar{x}}+\Omega^2\bar{x}=\frac{NF_0}{2} \qquad (8)$$

oscillations with a frequency Ω with respect to the equilibrium position (5c)

Substracting (8) from each equation of system (6), for relative deviations $\xi_i=x_i-\bar{x}$ we obtain:

$$\ddot{\xi}_i+\Omega^2\xi_i=\frac{F_0}{2}\sum_j \operatorname{sign}(\xi_j-\xi_i). \qquad (9)$$

In this variables the acting force is equal to the counteracting force and we have a many-particle Hamiltonian system with a Hamiltonian

$$H=\sum_i\left(\frac{\pi_i^2}{2}+\frac{\Omega^2\xi_i^2}{2}\right)+\frac{F_0}{2}\sum_{i<j}|\xi_i-\xi_j| \qquad (10)$$

π_i, ξ_i are canonically conjugated values.

A canonical momentum π_i has a dimension of velocity, proportional to the relative energy deviation ε:

$$\pi_i=\alpha c\varepsilon_i, \qquad (11)$$

where c is the light, $\alpha=\frac{E}{\omega}\frac{d\omega}{dE}$ — momentum compaction factor.

The relative motion energy of particles is conserved, $H=\text{const}$, hence, we obtain an estimation for the bunch length Δ for the case of a nonweak self-field:

$$\Delta\simeq\frac{2H}{F_0N^2}. \qquad (12)$$

In particular, when $H=0$ all the particles are in the same point, $\Delta=0$.

Due to the influence of a thermostat (synchrotron friction and noise) the system comes to a thermodynamic equilibrium, which is

described by Gibbs N-particle distribution. This state is sure to be stable. By integrating N-particle density over $N-1$ pare of variables we obtain a single-particle phase density, by integrating it over momentum, we obtain a linear density, $\lambda(x)$, satisfying the Haissinski's equation with

$$\Delta_T^2=\frac{c^2\alpha^2\overline{\varepsilon^2}}{\Omega^2} \qquad (13)$$

$\overline{\varepsilon^2}$ is the root-mean-square spread of relative energy deviations determined by the synchrotron radiation friction and noise.

So, the state of system (5) is stable, its coherent oscillations do not increase.

For further analysis it is important to estimate the decrements of oscillations.

5. COHERENT OSCILLATIONS

In a stationary state (5) the attraction of particles due to their own fields is balanced by their thermal motion. The condition of this balance can be written as an equality in the order of magnitude of a coherent longest wavelength oscillations frequency ω_c to an average frequency of the thermal motion of a particle along the bunch $\bar{\omega}_T$:

$$\omega_c\simeq\bar{\omega}_T. \qquad (14)$$

At a considerable shortening (of a bunch) the movement of particles is absolutely unharmonic, the spread of thermal frequencies $\delta\omega_T$ is in an order of ω_T:

$$\delta\omega_T\simeq\bar{\omega}_T. \qquad (15)$$

Relations (14), (15) correspond to a strong interaction between a wave and resonant particles, which in case of stable stationary state results in the Landau damping with a decrement of an order of the oscillation frequency. With a decrease in the length of the wave of the coherent motion $1/k$ the decrement increases $\simeq kv_T$, $v_T=\pi_T$ is the spread of velocities of the movement along the bunch.

6. SENSITIVITY TO WAKE FIELD VARIATIONS

In previous sections it was shown that for an infinite-step wake field (2) (the sign here is significant) the bunch shortens more and more with the increase of the number of particles, tends to collapse into an infinitely thin disk.

To what extend is this result sensitive to the wake function variations? It's no doubt, that every real function will differ from this idealization by some features at short distances and by turning into zero at large ones. A real step differs from the ideal one by $\delta W(x)$ at $x \lesssim \Delta$; besides, it has some finite length a, decreasing noticeably, or, perhaps, changing its sign at $x \gtrsim a$. It is obvious that the smallness of changes of the stationary state requires that the influence of perturbation δW on the self-consistent field of the bunch should be small, which is equivalent to insignificance of the corresponding correction in the impedance δZ:

$$\left|\delta Z\left(\frac{1}{\Delta}\right)\right| \ll \left|Z\left(\frac{1}{\Delta}\right)\right|, \tag{16}$$

where

$$Z(k) = \frac{W_0}{ik} \tag{17}$$

is the Fourier representation of a step function. This is necessary but not a sufficient condition of the smallness of the stationary state change. If a general shift of a bunch x_0 (5c) exceeds the step length a, then, as shown in [3] the bunch is devided into a succession of shortened sub-bunches, following one another at an interval $\simeq a$. Therefore, the condition

$$a > x_0 \tag{18}$$

is also a necessary one and together with (16) they constitute the sufficient condition for the weakness of a change of the stationary state by deviations of the wake function from the ideal step.

The addition of δZ to the impedance results in the shift of oscillation frequencies in the complex plane, which in case of its big enough value can result in the bunch instability. The stability of long-wave oscillations $k\Delta = 1$ is guaranteed by condition (16). The stability in the short-wave region needs a special consideration. In this case, the instability is possible only when a coherent shift in the frequency $\delta\omega_k$ originating from the impedance δZ_k, exceeds the

Landau's damping decrement:

$$|\delta\omega_k| \gtrsim k v_T, \tag{19}$$

where

$$\delta\omega_k^2 = -ik\delta Z_k \frac{Ne^2\Omega^2}{\sqrt{2\pi}\,\Delta\varkappa}. \tag{20}$$

Hence, for the impedance δZ_k, growing with k not faster than linearly requirement (18) appears sufficient to provide the stability. In an opposite case the stability condition becomes more rigid:

$$\left|\frac{\delta Z\left(\frac{1}{\Delta}\right)}{Z\left(\frac{1}{\Delta}\right)}\right| \max_k \left\{\frac{\delta Z_k}{k\,\delta Z\left(\frac{1}{\Delta}\right)\Delta}\right\} < 1. \tag{21}$$

7. ON THE POSSIBILITY OF A STABLE COLLAPSE FOR OTHER WAKE FUNCTIONS

The distortion of a potential well due to a bunch self-field, which is taken into account by the Haissinski's equation (3), may provide qualitively different results, depending on the type of the wake function $W(x)$.

For a weak self-consistent field, when

$$\frac{Ne^2\left|Z\left(\frac{1}{\Delta_T}\right)\right|}{\sqrt{2\pi}\,\varkappa\Delta_T^2} \ll 1 \tag{22}$$

its influence on the bunch density can be evaluated via a perturbation method [3, 5]. In this case, the sign of the effect (lengthening or shortening) is determined by the sign of an imaginary part of the impedance. The effect in this region is weak and consequently of little interest.

An opposite to (22) situation has been considered in [3]. Summing up the results obtained there, one can arrive to a conclusion that with respect to a corresponding solutions kernels are divided into 4 types:

i) with a finite length solution;
ii) with a single-collapse solution;
iii) with a succession-of-collapses solution;

iv) with no solution (corresponds to lengthening via widening).

The latter comprises, for instance, a δ-function derivative with a negative sign. The functions $W(x)$ localized at small distances comparatively to the bunch length, having there singularities with an substantial contribution to the impedance, correspond either to the iv- or to the i-type.

Solutions of the ii-type correspond to the positive functions $W(x)$ nondecreasing over the distance of a bunch shift as a whole, as well as to the functions with a slight integral difference from them.

We did not manage to give a rigorous proof concerning the problem of a collapse stability for the general case of kernels ii. But some considerations in favour of the stability of these states though not proved rigorously, seem quite plausible.

The impedance $Z(k)$ corresponding to the ii-type of nucleus decreases with the growth of argument as $1/k$ or faster. Hence, the parameter $\frac{kv_T}{|\omega_k|}$ (ω_k is determined by (20)), which characterizes the value of Landau damping, increases with k not slower than linearly. Therefore, longest wavelength disturbances, i. e. dipole and quadrupole ones, seem most dangerous concerning instabilities. Dipole oscillations can be considered as a motion of one macroparticle, quadrupole ones — as a motion of two macroparticles. It is easy to show that both these systems are equivalent to Hamiltonian ones (in the sense of the 4-th section). Consequently, the energy is conserved here, and a build-up of oscillations is impossible. Estimation of the thermal motion of particles, which make up macroparticles, leads to a conclusion on the Landau damping of these oscillations. And since most long-wave oscillations are damped, others will be more so.

8. SHORTENING-PRODUCING IMPEDANCE

So, to make the bunch shorten it is necessary to make a shortening-producing wake function, shortening-producing impedance — the ii-type ones in the storage ring. Is it possible? If possible, how? Unfortunately, the author failed to find in the references dealing with the calculation of wake fields at least one suitable example.

On the other hand, the nonexistence of such objects is not proved either. The question still remains open.

The report about bunch lengthening on ADONE [6] contains some encouraging information, where based on the measurement of the length and scaling theory [2, 7] the conclusion is made that the impedance is independent of the frequency (of k in our terms) in the long-wave region, and is inversely proportional to the frequency in the short-wave one with a boundary of approximately 10 cm. Measurements concern the region 6 — 16 cm.

Such an impedance corresponds to a step wake function with a step length of about 10 cm. Bunches several times shorter than step length should shorten, if the wake function remains shortening-producing at smaller distances.

The lengthening observed in these experiments may be understand by taking into considerations that the bunch had a length comparable with the boundary one.

9. QUESTION

It is clear, what the impedance should be to make the bunch shorten to a single stable collapse. Is it possible, and if possible, how can this impedance be created on the storage ring orbit?

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