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НОВОСИБИРСК

RADIATION INTENSITY OF ULTRARELATIVISTIC PARTICLES AT AXIAL CHANNELING AND ITS ORIENTATION DEPENDENCE

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A b s t r a c t

The total radiation intensity I of ultrarelativistic electrons and positrons at axial channeling in thin crystals is calculated using a realistic model of axial potential. The approach avoids a cumbersome analysis of radiation from concrete trajectories. A general expression for intensity $I(\vartheta_0)$ as a function of incident angle ϑ_0 of particles which is valid for any axial symmetric potential $U(\rho)$ is derived. Explicit formulas for the potential (29) are written. Intensities $I(\vartheta_0)$ for axial and planar channeling are compared.

1. Introduction

The radiation at channeling of relativistic particles in single crystals has been widely discussed in recent years (see /1-3/ and the references there). At present a theoretical analysis of the radiation at planar channeling (one-dimensional problem) permits one to obtain a quite satisfactory quantitative description of experiment. We have recently obtained such a description /4,5/, which is based on the authors' earlier papers /1,6/. The radiation at axial channeling (two-dimensional problem) is studied mainly on a qualitative level (see*, e.g., /7,8/). The present paper is devoted to a calculation of the total radiation intensity at axial channeling of positrons and electrons in thin crystals and its orientation dependence using a realistic approximation of the potential of a chain of atoms in the continuous model.

If one is interested in such characteristics of the radiation, which depend only on the instant values of coordinates and momenta of particles, then one must know only a distribution of particles in phase space which is transverse to the direction of axes. The problem is substantially simplified for this case, since it is possible to avoid a very cumbersome analysis of the radiation from concrete trajectories. Such characteristics are the total radiation intensity, as well as the radiation spectrum in the limit, when one can consider the radiation at channeling in the frame of magnetic bremsstrahlung theory (see, e.g. /9/). The conditions of applicability of this consideration were discussed in Ref. /10/.

In the present paper, we will use a statistical description of the axial channeling. The particle distribution can be written in the form /11/

$$dN(\varepsilon_\perp,\vec{\rho},l) = F(\varepsilon_\perp,\vec{\rho},l)\,d\varepsilon_\perp d^2\rho = g(\varepsilon_\perp,l)\,f(\varepsilon_\perp,\vec{\rho})\,d^2\rho\, d\varepsilon_\perp \tag{1}$$

where

$$f(\varepsilon_\perp,\vec{\rho}) = \frac{1}{S(\varepsilon_\perp)}\vartheta(\varepsilon_\perp - U(\vec{\rho})) \tag{2}$$

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* In the recent paper /8/ the radiation at axial channeling has been considered using the Coulomb type potential. It will be shown below that this type of the potential is unadequate for this problem.

with normalization conditions

$$\int F(\varepsilon_\perp,\vec{\rho},l)\,d\varepsilon_\perp d^2\rho = \int g(\varepsilon_\perp,l)\,d\varepsilon_\perp = \int f(\varepsilon_\perp,\vec{\rho})\,d^2\rho = 1 \qquad (3)$$

Here $\varepsilon_\perp = \varepsilon v_\perp^2/2 + U(\vec{\rho})$ is the transverse particle energy, $\vec{\rho}$ ($\vec{v}_\perp$) is transverse coordinate (velocity) of the particle, $U(\vec{\rho})$ is the potential of the transverse motion, $\vartheta(x) = \begin{cases} 1 & x>0 \\ 0 & x<0 \end{cases}$, $S(\varepsilon_\perp)$ is an available area of the transverse motion at a fixed value of $\varepsilon_\perp$ within one elementary cell, l is the penetration depth of the particle inside the crystal.

By definition, a distribution function $g(\varepsilon_\perp,l)$ is determined, in thin crystal (see Refs. /1,10/), by the initial conditions for incident particles, when one can neglect the change of distribution function $g(\varepsilon_\perp,l)$ with l due to multiple scattering*. Moreover, one can consider the distribution over transverse coordinates as an uniform one, since the transverse dimensions of incident beam are much larger than the distances between axes. Then the initial distribution function over transverse energy $\varepsilon_\perp$ at a fixed incident angle ϑ_0 is

$$g(\varepsilon_\perp)\,d\varepsilon_\perp = n_\perp d^2\rho_0 \qquad (4)$$

where $n_\perp$ is a density of the chains of atoms. For the axially symmetric potential, one has

$$g(\varepsilon_\perp)\,d\varepsilon_\perp = 2\pi n_\perp \rho_0\, d\rho_0 = 2\pi n_\perp \rho_0(\varepsilon_\perp,\vartheta_0)\left|\frac{dU(\rho_0)}{d\rho_0}\right|^{-1} d\varepsilon_\perp \qquad (5)$$

where $\rho_0(\varepsilon_\perp,\vartheta_0)$ is a solution of equation

$$\varepsilon_\perp(\rho_0) = U(\rho_0) + \frac{\varepsilon\vartheta_0^2}{2} \equiv U(\rho_0) + \varepsilon_0 \qquad (6)$$

II. General approach to the radiation at axial channeling

An instantaneous radiation intensity at motion of ultrarelativistic particle in the potential $U(\vec{\rho})$ is determined

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* There is also a limitation of the crystal thickness from below, which is connected with an establishment of equilibrium distribution. This means that the crystal thickness L is much larger that the free path length $\lambda_\perp$ /11/.

by the known equation* (see e.g. /12/)

$$I(\vec{\rho}) = \frac{2}{3}e^2\vec{w}^2\gamma^4 = \frac{2}{3}\frac{e^2\gamma^2}{m^2}(\vec{\nabla}U)^2 \equiv A(\vec{\nabla}U)^2 \qquad (7)$$

where $\vec{w}$ is the instantaneous acceleration, $\gamma = \varepsilon/m$ is the Lorentz factor. Using the distribution function (1), one obtains for radiation intensity of particles at the axial channeling in a single crystal

$$I(l) = A\int d\varepsilon_\perp\, g(\varepsilon_\perp,l)\frac{1}{S(\varepsilon_\perp)}\int(\vec{\nabla}U)^2\vartheta(\varepsilon_\perp - U(\rho))\,d^2\rho \qquad (8)$$

The integrals over $d^2\rho$ in (8) are taken within one elementary cell. The contribution to the radiation of the particles moving above a barrier (when $\varepsilon_\perp \geq U_0$, U_0 = max U, U $\geq$ 0) is given by

$$I_{nc} = An_\perp\int(\vec{\nabla}U)^2 d^2\rho \int_{\varepsilon_\perp\geq U_0} g(\varepsilon_\perp,l)\,d\varepsilon_\perp \qquad (9)$$

As far as at $\varepsilon_\perp \geq U_0$ the whole transverse coordinate space is accessible, then $S(\varepsilon_\perp)=1/n_\perp$. In the case $\varepsilon_0 \geq U_0$, all the particles are moving above the barrier; then using the normalization condition for the function $g(\varepsilon_\perp,l)$ one has for the radiation intensity

$$I_{as} = An_\perp\int(\vec{\nabla}U)^2 d^2\rho \qquad (10)$$

Thus, the radiation intensity at $\varepsilon_0 \geq U_0$ ($\vartheta_0 \geq \vartheta_c$, $\vartheta_c = \sqrt{2U_0/\varepsilon}$ is the Lindhard angle) does not depend on the incident angle and charge sign of the particle for any potential**. This situation differs from the case of the planar channeling, where the difference between radiation intensities of the particles with positive and negative charge sign and their dependence on ε_0 vanish in the limit $\varepsilon_0 \gg U_0$ only.

For the electrons moving in the axial channel, the above consideration is based on the fact that for initial trajectories which give the main contribution to the radiation, an angular momentum fails to be a good integral of motion. Due

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* Here and below we put c = 1.

** Let us remark, that the curves in Fig. 2 of Ref. /8/ do not follow such behaviour.

to this, averaging over momenta has been carried out in the distribution function $g(\varepsilon_\perp)$. Generally speaking, this approach is valid for electrons with not very high energies ($\varepsilon \lesssim 10$ GeV).

Let us discuss some general properties of the radiation intensity at the axial channeling. One can restrict oneself to a consideration of one cell in the plane transverse to the axes, which contains a projection of one chain of atoms forming the axis. The main contribution to integrals (8)-(10) is given by the region, where the gradient of the potential $U(\vec{\rho})$ is large. This region has some characteristic scale a_s which is a screening radius. Inside this region the potential is axially symmetric with a good accuracy. Let us represent the elementary cell as a circle with the radius $r_0 = 1/\sqrt{\pi n_\perp}$. For real crystals, $r_0 >> a_s$. This means that one can spread to ∞ the upper limit of integration over ρ in Eq. (10), then

$$I_{as} \simeq 2\pi A n_\perp \int_0^\infty \left(\frac{dU}{d\rho}\right)^2 \rho\, d\rho = 2\pi A n_\perp U_0^2 \int_0^\infty f'^2(\rho)\rho\, d\rho \qquad (11)$$

where $f(\rho) = U(\rho)/U(0)$. If f is a function of ρ/a_s $f(\rho/a_s)$, then after a substitution $\rho \to \rho a_s$ one can see that I_{as} does not depend on a specific value of the screening radius and depends on a shape of the potential only*. This result differs essentially from the case of planar channeling, where the intensity of the radiation is increasing when $a_s \to 0$. So, with an accuracy up to a numerical coefficient, the radiation intensity at $\varepsilon_0 \gtrsim U_0$ has the form

$$I_{as} \backsim 2\pi A n_\perp U_0^2$$

Let us consider now qualitative features of an orientation dependence of the radiation at the axial channeling in thin crystals. At $\vartheta_0 = 0$ a portion of positrons ΔN^+ which approach to the axis at a distance $r \leqslant a_s$, where the radiation mainly takes place, $\Delta N^+ \sim a_s^2/r_0^2$, so

$$I^+(\vartheta_0 = 0) \sim (a_s^2/r_0^2) I_{as} \qquad (12)$$

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* We will consider below a more complicated case, where actually there are two characteristic scales.

For electrons there is a different situation. The portion of electrons, which has an impact parameter $\vec{\rho}_0$ inside $d^2\rho$, is given by

$$n_\perp \frac{d^2\rho\, d^2\rho_0}{\pi \rho_0^2} \vartheta(\rho_0 - \rho)$$

Averaging I (7) with this distribution and taking into account that in the integral over ρ the main contribution is given by the region $\rho \gtrsim a_s$, we obtain up to logarithmic accuracy,

$$I^-(\vartheta_0 = 0) \simeq I_{as} \int_{a_s}^{r_0} \frac{d^2\rho_0}{\pi \rho_0^2} = I_{as} \ln \frac{r_0^2}{a_s^2} \qquad (13)$$

Thus, at zero incident angle the radiation intensity of electrons exceeds considerably the radiation intensity of positrons.

For an axially symmetric potential we obtain the radiation intensity, substituting (5) into Eq. (8):

$$I(\vartheta_0) = 4\pi^2 A n_\perp \int_0^{r_0} \rho_0\, d\rho_0 \frac{1}{s(\varepsilon_\perp(\rho_0))} \int \left(\frac{dU}{d\rho}\right)^2 \rho\, d\rho\, \vartheta(\varepsilon_\perp(\rho_0) - U(\rho)) \qquad (14)$$

where $s(\varepsilon_\perp(\rho_0)) = 2\pi \int \rho\, d\rho\, \vartheta(\varepsilon_\perp(\rho_0) - U(\rho))$, the value of $\varepsilon_\perp(\rho_0)$ is given by Eq. (6). Going over to the variables $x = \rho^2/a_s^2$, $y = \rho_0^2/a_s^2$, and after some manipulations we have for radiation intensity of positrons

$$I^+(\vartheta_0) = 4\pi A n_\perp \Big\{ Q\Big[\vartheta(\varepsilon_0 - U_0) + \frac{y^+}{x_0}\vartheta(U_0 - \varepsilon_0)\Big] + \vartheta(U_0 - \varepsilon_0) \int_{y_+}^{x_0} \frac{dy}{x_0 - x_+(y)} \int_{x_+(y)}^{x_0} x U'^2(x)\, dx \Big\} \qquad (15)$$

where $\varepsilon_0 = \varepsilon\vartheta_0^2/2$, $x_0 = r_0^2/a_s^2$, $U_0 = U(0) > 0$, $U(x_0) = 0$ the quantities y_+, $x_+(y)$ are determined by equations

$$U(x_+(y)) = U(y) + \varepsilon_0 \;, \quad U(y_+) = U_0 - \varepsilon_0$$
$$Q = \int_0^{x_0} dx\, x\, U'^2(x) \qquad (16)$$

and for radiation intensity of electrons

$$I^-(\vartheta_0) = 4\pi A n_\perp \Big\{ Q\Big[1 - \frac{y_-}{x_0}\vartheta(U_0 - \varepsilon_0)\Big] + \vartheta(U_0 - \varepsilon_0) \int_0^{y_-} \frac{dy}{x_-(y)} \int_0^{x_-(y)} x U'^2(x)\, dx \Big\} \qquad (17)$$

where

$$\mathcal{U}(x_-(y)) = \mathcal{U}(y) - \varepsilon_0 \quad , \quad \mathcal{U}(y_-) = \varepsilon_0 \tag{18}$$

In Ref. /8/ the potential was used

$$\mathcal{U}(\rho) = \frac{\alpha}{\rho}\vartheta(\rho - u_1) + \frac{\alpha}{u_1}\vartheta(u_1 - \rho) \tag{19}$$

In this case there is one characteristic scale (it is the thermal vibration amplitude u_1), so in accordance with the above results the radiation intensity of the particles moving above the barrier ($\varepsilon_\perp > \mathcal{U}_0 = \alpha / u_1$) does not depend on the amplitude of thermal vibrations.

Substituting (19) in Eq. (11), we have

$$I_{as}^{(1)} = 2\pi A n_\perp \mathcal{U}_0^2 \int_0^\infty f'^2(z)\,dz = 2\pi A n_\perp \mathcal{U}_0^2 \int_1^\infty \frac{dz}{z^3} = \frac{2\pi e^2 \mathcal{U}_0^2 \gamma^2}{3m^2} n_\perp \tag{20}$$

Moreover, for the more general form of the potential

$$\mathcal{U}(\rho) = \frac{\alpha}{\rho^\nu}\vartheta(\rho - u_1) + \frac{\alpha}{u_1^\nu}\vartheta(u_1 - \rho) \tag{21}$$

we obtain

$$I_{as}^{(\nu)} = \nu \frac{2\pi e^2 \mathcal{U}_0^2 \gamma^2}{3m^2} n_\perp \tag{22}$$

The main contribution to the intensity is given by the region $\rho \sim u_1$ (at $\rho \gtrsim \rho_1 >> u_1$ this contribution is $\backsim u_1^{2\nu}/\rho_1^{2\nu}$). One can readily obtain a simple analytical expression for the radiation intensity in the potential (19) using developed approach. In particular, at $\vartheta_0 = 0$ we have from (15) - (18)

$$I^+ = \frac{u_1^2}{\tau_0^2}\left(\ln\frac{\tau_0^2}{u_1^2} + 1\right) I_{as}^{(1)} \; , \quad I^- = \left(\ln\frac{\tau_0^2}{u_1^2} - 1\right) I_{as}^{(1)} \tag{23}$$

where $I_{as}^{(1)}$ is given by Eq. (20). These results agree naturally with the performed qualitative analysis (cf. (12) - (13)). However, one should bear in mind, that in a realistic potential of the chain of atoms the region of integral convergence is $\rho \sim a_s >> u_1$. This means, in particular, that Eq. (23) diminishes substantially the radiation intensity of positrons since $a_s^2/u_1^2 \gtrsim 10$. Moreover, one can obtain an erroneous notion of a spectrum of the radiation using the potential (19),

because the main contribution in this case is given by the trajectories with $\rho \sim u_1$, while actually the whole interval from u_1 to a_s contributes. For these reasons it seems unadequate an utilization of the potential (19) for the description of the radiation at axial channeling.

III. Potential for axial channeling

In further calculations we are in need of an explicit form of the potential $\mathcal{U}(\vec{\rho})$. We will use the Moliere potential for the isolated atom (see e.g. /9/) and take into account the thermal (zero) vibrations of atoms. In the case, when in plane (x,y) transverse to the channeling axis, the chains of atoms, of which the axes consist, form a rectangular lattice with coordinates $2n\vec{a} + 2m\vec{b}$; $(\vec{a}\vec{b}) = 0$; n, m are integers, the potential in this plane has the form

$$\mathcal{U}(x,y) = \frac{V_0}{\pi a b}\sum_{i=1}^{3}\sum_{n,m=-\infty}^{\infty}\alpha_i \frac{\exp\left\{-\frac{\pi^2 u_1^2}{2}\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right) + i\pi\left(\frac{mx}{a} + \frac{ny}{b}\right)\right\}}{\frac{m^2}{a^2} + \frac{n^2}{b^2} + \frac{1}{\pi^2 b_i^2}} \tag{24}$$

where $b_i = a_{s0}/\beta_i$; a_{s0} is the Thomas-Fermi screening radius, $a_{s0} = 0.8853\, a_0 Z^{-1/3}$; a_0 is the Bohr radius; $\alpha_i = (0.1;0.55;0.35)$; $\beta_i = (6.0;1.2;0.3)$; $V_0 = Ze^2/d$, d is an average distance between atoms in the chain, u_1 is the thermal vibration amplitude.

For the square lattice, where a = b = Δ , the expression (24) can be written in the form (for usually used crystals, this case is realized for the axis <100>)

$$\mathcal{U}(x,y) = \frac{V_0}{\pi}\sum_{i=1}^{3}\sum_{n,m=-\infty}^{\infty}\alpha_i \frac{e^{-\varkappa^2(m^2+n^2)}}{m^2 + n^2 + \gamma_i^2} e^{i\frac{\pi}{\Delta}(mx+ny)} \tag{25}$$

where

$$\varkappa^2 = \frac{\pi^2 u_1^2}{2\Delta^2} \; , \qquad \gamma_i^2 = \frac{\Delta^2}{\pi^2 b_i^2}$$

In the limit $\gamma_i^2 >> 1$, $\varkappa^2 << 1$, $x^2 + y^2 << \Delta^2$ the expression (25) can be represented as

$$U(\vec{\rho}) \simeq \sum_{i=1}^{3} \frac{\alpha_i}{\pi} V_0 \int d^2 z \, \frac{\exp\left\{-\varkappa^2 z^2 + \frac{i\pi}{\Delta}\vec{z}\vec{\rho}\right\}}{z^2 + \gamma_i^2} \tag{26}$$

This limit corresponds to an approximation of the isolated chain of atoms, which potential in the continuous model is given by (26). After rather simple manipulations we obtain from (26) the following integral representation

$$U(\vec{\rho}) = \sum_{i=1}^{3} \frac{\alpha_i}{\pi} V_0 e^{u_1^2/2b_i^2} \int_{u_1^2/2b_i^2}^{\infty} dt \int d^2 z \exp\left\{-(1+z^2)t + i\vec{\rho}\vec{z}/b_i\right\} =$$
$$= \sum_{i=1}^{3} \alpha_i V_0 e^{u_1^2/2b_i^2} \int_0^1 \frac{dt}{t} \exp\left\{-\frac{u_1^2}{2b_i^2 t} - \frac{\rho^2 t}{2u_1^2}\right\} \tag{27}$$

This form of the potential of the isolated chain of atoms is presented in Ref. /8/. However, one should bear in mind that if parameter $\varkappa^2$ is always actually small, then the values of the coefficients γ_i^2 are very different. So one has γ_i^2 = (22; 0.9;0.05) for diamond* and γ_i^2 = (90;3.6;0.2) for silicon). The contribution to the potential of term with i = 1 (where $\gamma_1^2 \gg 1$) is relatively small as α_1 = 0.1. The parameter γ_2^2 is increasing with Z and, for instance, for wolframium γ_2^2 = 14.6. However, parameter γ_3^2 is never large ($\gamma_3^2 < 1$ for the all used crystals), so that for complete calculations it is necessary to use the potential (25), but not its limiting representation (26) - (27). This is especially important for diamond and wolframium.

As it was stressed, the main contribution to the radiation intensity is given by the region where the gradient of the potential is large. This situation takes place when the distance from axis is rather small and the potential is axially symmetric with a good accuracy. For this reason one can avoid calculations with the very complicate potential (25) and it is possible to use a simple approximation of the potential (25) (in some sence this is the so called standard potential

* All the parameters are given for axis <100>.

/11/)

$$U(\rho) = \frac{Ze^2}{d} \ln\left(1 + \xi \frac{a_0^2 Z^{-1/3}}{\rho^2 + 2\beta u_1^2}\right) + \text{const} \tag{28}$$

The parameters ξ and β are fitted comparing equation (28) with numerical calculations of the potential (25). The difference between the potentials (28) and (25) does not exceed 10% for the parameters ξ and β listed in the Table in the region of the main contribution to the radiation. The parameters ξ and β only slightly differs from 1 for the number of crystals. Diamond and wolframium are an exception due to an anomalously small thermal vibration amplitude.

IV. Radiation in the realistic potential

In this section we will obtain the total radiation intensity of electrons and positrons at axial channeling for the potential (28) as well as its dependence on incident angle ϑ_0 . It is convenient to go over to the variable $x^2 = \rho^2/a_s^2$ (cf. Eqs. (15) - (18)), where $a_s^2 = \xi a_0^2 Z^{-1/3}$. Let us introduce also $\eta = 2\beta u_1^2 / a_s^2$ containing the thermal vibration amplitude (the values $V_0 = Ze^2/d$, $x_0 = \tau_0^2/a_s^2$, η are also listed in the Table). Then the potential (28) can be written in the form

$$U(x) = V_0\left[\ln\left(1 + \frac{1}{x+\eta}\right) - \ln\left(1 + \frac{1}{x_0+\eta}\right)\right] \tag{29}$$

Substituting the potential (29) into Eq. (15), we obtain the total radiation intensity of positrons at axial channeling, depending on the incident angle ϑ_0

$$I^{+}(\vartheta_0) = I_0\Big\{\vartheta(\varepsilon_0 - U_0)\varphi(\eta) + \vartheta(U_0 - \varepsilon_0)\Big[(1+2\eta)\ln\lambda + (1-\lambda)\left(\eta + \frac{1+\eta}{\lambda}\right) +$$
$$+ \frac{1}{x_0}\Big[\left(\lambda\eta - \frac{1+2\eta}{1-\lambda}\right)\ln\lambda - \eta(1+\lambda+2\eta)\ln\frac{1+\eta}{\eta} -$$
$$- \frac{1-\lambda}{\lambda}\left(1+\eta(1-\lambda)\right)\left(\ln\frac{x_0(1+\eta(1-\lambda))}{1+\eta} + 1\right)\Big]\Big]\Big\} \tag{30}$$

where

$$I_0 = \frac{8\pi e^2 V_0^2 \gamma^2}{3m^2} n_\perp, \quad \lambda = e^{\varepsilon_0/V_0}, \quad \varepsilon_0 = \varepsilon \vartheta_0^2/2$$

$$\varphi(\zeta) = (1+2\zeta)\ln\frac{1+\zeta}{\zeta} - 2 \tag{31}$$

The values of I_0/γ^2 are listed in the Table. We retain, in Eq. (30), a term proportional to $1/x_0$, since at $\varepsilon_0 = 0$ ($\lambda = 1$) all other terms vanish. The quantity $I^+(\vartheta_0)$ increases monotonically from the small value at $\varepsilon_0 = 0$

$$I^+(0) = \frac{I_0}{x_0}\left[1+2\zeta - 2\zeta(1+\zeta)\ln\frac{1+\zeta}{\zeta}\right] \tag{32}$$

and reaches its maximal value at $\varepsilon_0 = u_0$

$$I^+(\varepsilon_0 = u_0) = I_{as} = I_0\varphi(\zeta) \tag{33}$$

and with further increasing of angle ϑ_0 remains constant. Note that $dI^+/d\varepsilon_0\big|_{\varepsilon_0=u_0} = 0$. The result obtained is in agreement with the qualitative analysis, which was made in Sect. 2. The function $I^+(\vartheta_0)$ for W is shown in Fig. 1 (curve 1) and for Si in Fig. 2 (curve 1).

Substituting the potential (29) into Eq. (17) we obtain the total radiation intensity of electrons at axial channeling and its dependence on ϑ_0

$$\begin{aligned} I^-(\vartheta_0) = I_0\Big\{ & \vartheta(\varepsilon_0 - u_0)\varphi(\zeta) + \vartheta(u_0-\varepsilon_0)\Big[\mu(1+\lambda)\big(\ln(\mu(1+\zeta)) - \zeta\big) - \\ & -\mu + \frac{\ln\lambda}{\lambda} + \frac{1}{2\lambda}\ln\left[\frac{1}{\zeta(1+\zeta)(\mu+1/x_0)^2}\right]\left(\frac{(1+2\zeta)\ln\frac{1+\zeta}{\zeta}}{(1+\zeta\mu)^2} - 2\right) + \\ & + \frac{1+2\zeta}{(1+\mu\zeta)^2}\left[\mu(1+\zeta\mu)\left((1+\zeta)\ln\frac{1+\zeta}{\zeta} - \ln(\mu(1+\zeta)) + \zeta\ln\lambda\right) - \right. \\ & \left. - \frac{\ln\lambda}{\lambda}\left(\frac{1}{2}\ln\lambda + \ln(\mu(1+\zeta))\right)\right]\Big]\Big\} \end{aligned} \tag{34}$$

where $\mu = 1 - 1/\lambda$ and the remaining definitions are given in

Eq. (30). The quantity $I^-(\vartheta_0)$ reaches its maximal value at $\vartheta_0 = 0$

$$I^-(0) = I_{as}\ln\frac{x_0}{\sqrt{\zeta(1+\zeta)}} \tag{35}$$

(cf. Eq. (13)) and when ε_0 increases from 0 to u_0, $I^-(\vartheta_0)$ decreases monotonically from $I^-(0)$ to I_{as} at $\varepsilon_0 = u_0$ and then remains constant. The function $I^-(\vartheta_0)$ for W is shown in Fig. 1 (curve 2) and for Si in Fig. 3 (curve 1).

V. Comparison of the total intensities of radiation at axial and planar channeling

In this paper the radiation of relativistic particles at axial channeling with use of the realistic potential (see Eqs. (25), (28)) is considered for the first time. The developed approach permits one to obtain the radiation intensity at axial channeling without analysis of the contributions of concrete trajectories both for thin and thick crystals. So the thin crystals Eqs. (15), (17) give one an opportunity to obtain easily radiation intensity for any axially symmetric potential. We would remind that consideration was carried out in frame of classical theory. This means that the quasiclassical nature of the particle motion was assumed (in real conditions this is valid starting from energy a few tens MeV) and a recoil at the radiation was neglected (this is possible up to energy $\sim$ TeV). The results obtained are valid within this energy interval, which is most important from the experimental point of view.

It is of obvious interest to compare obtained radiation intensity at axial channeling with a radiation intensity at planar channeling /1/. The radiation intensity of positrons in Si is shown in Fig. 2 as a function of incident angle ϑ_0 with respect to axis <100> (curve 1) for axial channeling and with respect to plane (110) (curve 2) for planar channeling. The quantities u_0 and I_0 being taken from present paper. In Fig. 3 the same is plotted for electrons. One can see in figures, that the radiation intensity of electrons at axial channeling $I^{-ax}(\vartheta_0)$ exceeds appreciably that at planar channeling $I^{-pl}(\vartheta_0)$ for the all angles ϑ_0. For positrons, the ratio between $I^{+ax}(\vartheta_0)$ and $I^{+pl}(\vartheta_0)$ depends on the incident angle ϑ_0 and, only at $\vartheta_0 \gtrsim \vartheta_c$ $I^{+ax}(\vartheta_0)$, exceeds appreciably $I^{+pl}(\vartheta_0)$. For wide enough (in ϑ_0) beams, the ratio between axial and planar channeling can be described by the ratio of the asymptotic intensities (see Eq. (33) and Eq. (2.16) in Ref. /1/)

$$R_{as} \equiv \frac{I_{as}^{ax}}{I_{as}^{pl}} = 2\pi \left(\frac{V_0}{u_0^{pl}}\right)^2 \left(\frac{d_{pl}}{2\Delta}\right)^2 \frac{\varphi(\zeta)}{\delta^2} \left(\frac{\mathrm{sh}\, 2\delta}{2\delta} - 1\right)^{-1} (\mathrm{ch}\,\delta - 1)^2 \quad (36)$$

where d_{pl} is a distance between planes, the values of δ are listed in the Table in Ref. /1/, the u_0^{pl} are listed in the Table with the thermal vibrations taken into account. For the axis $\langle 100 \rangle$, the parameters Δ, ζ, V_0 and the gain are also listed in the Table. One can see that, though the quantities I^{ax} and I^{pl} are proportional to the squares of the corresponding U_0 and for axial channeling the value of U_0 is several times larger than for the planar channeling, but actually $R_{as} \sim u_0^{ax}/u_0^{pl}$ in a rough estimation, i.e. the gain R_{as} increases only linearly. The reason is that for the above barrier particles at planar channeling the particle is in the region of large gradient of the potential relatively longer time than at axial channeling.

For radiation of electrons, the gain is maximal at $\vartheta_0 = 0$, i.e. for the beams with small angular spread. One can obtain an estimate of $R(\vartheta_0 = 0)$ taking into account that $I^{-pl}(0)$ for all crystals is nearly twice as much as I_{as}^{-pl} and for axial channeling it is necessary to use Eq. (35).

Table

Parameters of crystals at temperature T = 293°K (axis $\langle 100 \rangle$, plane (110)) and some characteristics of the radiation

Crystal	2Δ (Å)	u_1 (Å) (T=293°K)	V_0 (eV)	U_0 (eV)	U_0^{pl} (eV)	χ	β	x_0	$1/\eta$	$\frac{I_0}{\gamma^2}$ (eV/cm)	R_{as}
C(d)	1.26	0.40	24	90	27.5	0.75	0.75	4.4	48	17	3.3
Si(d)	1.92	0.075	37	86	25	1	1	10	10	17	3.8
Ge(d)	2.00	0.085	81	153	44	1	1	14	6.1	76	3.7
W	2.24	0.050	336	761	140	0.8	1.2	30	8.8	1039	7

Δ is a distance between axes $\langle 100 \rangle$ (planes (110)), u_1 is a thermal vibration amplitude. V_0, β, η, χ, x_0 are parameters characterizing the potential (Eqs. (26),(29)); U_0 is a depth of the potential well, I_0 is a characteristic of the radiation intensity (Eq. (31)), R is a gain in the radiation at axial channeling comparing with planar channeling

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Figure Captions

Fig. 1. Dependence of radiation intensity on $\varepsilon_o/u_o=(\vartheta_o/\vartheta_c)^2$ at axial channeling in W (axis $<100>$) for positrons (curve 1) and electrons (curve 2). Quantity I_o is given by Eq. (31).

Fig. 2. Dependence of radiation intensity on ε_o/u_o in Si for positrons at planar chaneling (plane (110)), curve 2 and at axial channeling (axis $<100>$), curve 1. Quantities ϑ_c, U_o are taken for axial channeling (see Table), I_o is given by Eq. (31).

Fig. 3. The same as in fig. 2, but for electrons.

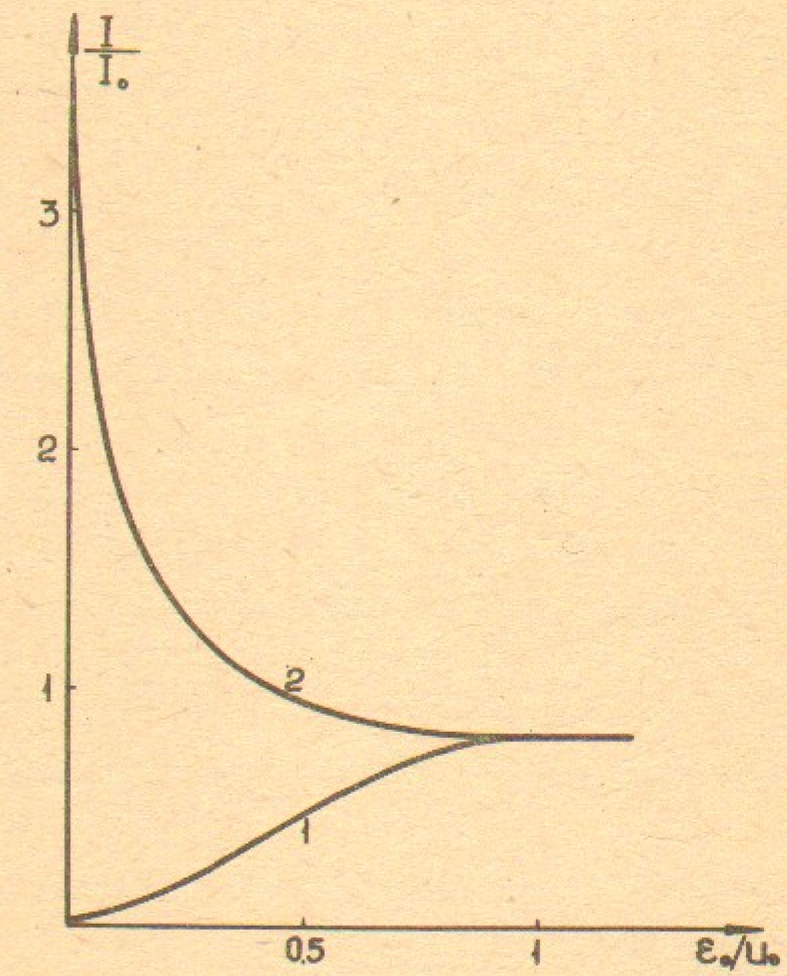
$\frac{I}{I_0}$
3
2
1
2
1
0.5
1
ε_0/u_0

Fig. 1.

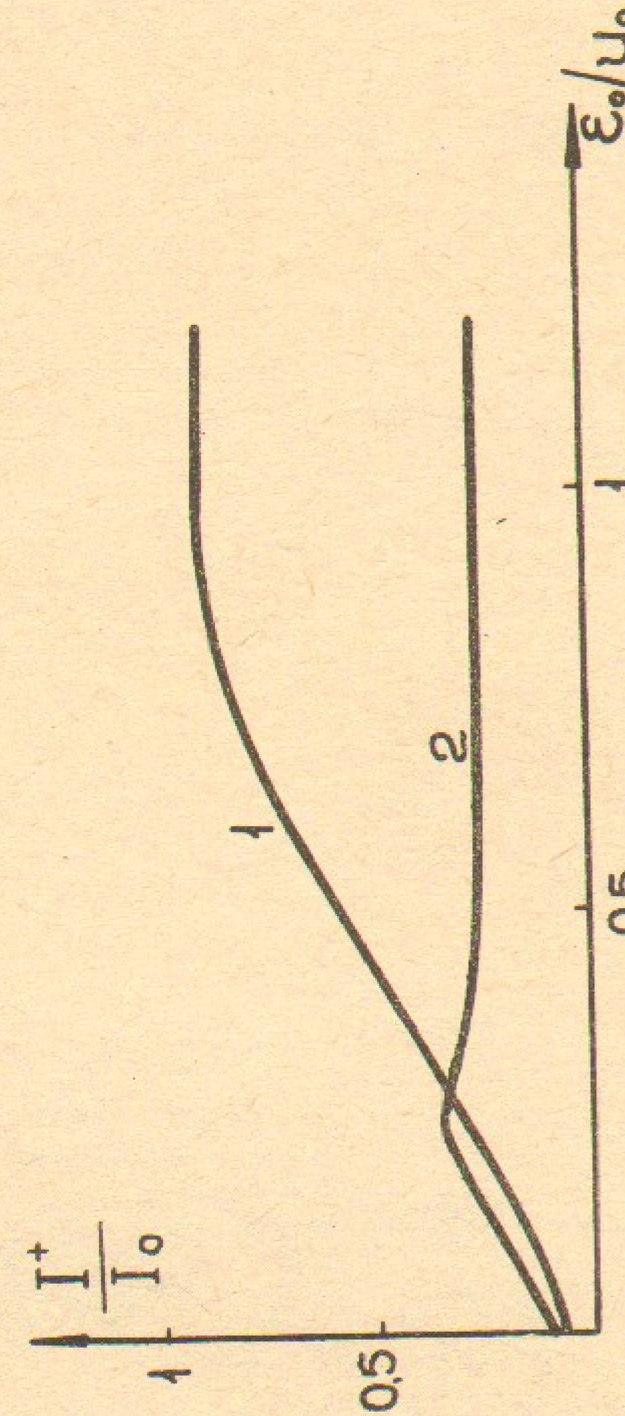
$\frac{I^+}{I_0}$
1
0,5
1
2
0,5
1
ε_0/u_0

Fig.2.

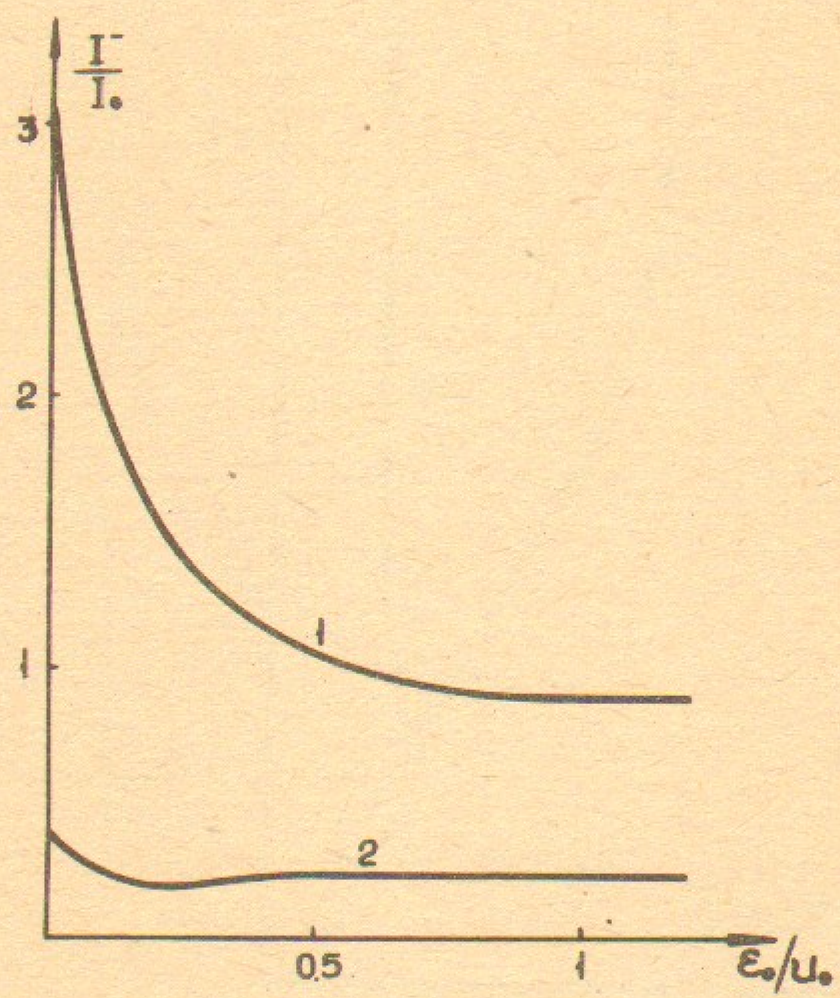

Fig. 3.

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ОРИЕНТАЦИОННАЯ ЗАВИСИМОСТЬ ИНТЕНСИВНОСТИ
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