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EFFECTIVE NARROWING OF THE SPECTRAL DISTRIBUTION AT MULTIPLE PASSAGE OF PHOTONS THROUGH A FREE ELECTRON LASER (FEL)

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A b s t r a c t

It is shown that the spectral density at the centre of the radiation line is effectively increased if the gain of a FEL is of the order of unity and when the signal repeatedly passes through an optical cavity because of the nonlinear interaction between harmonics. The action of a FEL is studied for the case of a non-monoenergetic electron beam.

1. Introduction

In a series of papers by the authors /1-4/ the theory of the action of free electron lasers is developed. The case of small gains, $G \ll 1$, is considered in the paper /1/. The papers /2,4/ are devoted to the action of FELs at arbitrary gains in the linear regime. As the ondulator length, and hence the signal, grows, nonlinear phase oscillations begin determine the action of a FEL. The system of nonlinear equations describing these oscillations is derived and their dynamics during a single passage of the photon beam is analysed in the papers /3,4/.

In the present paper this system of equations is applied to solve 2 problems. Section 2 treats the action of a FEL in the linear regime when the incident electron beam is a non-monoenergetic one. Section 3 deals with the kinetics of building up a photon bunch during its repeated passages through an optical cavity.

2. The Action of a FEL in the Case of a Non-Monoenergetic Beam

Since the beams used in a FEL are not monoenergetic, of interest is, undoubtedly, how a FEL acts in the case when the energy of an incident beam is described by a certain distribution function. In order to analyse this question, it is convenient to use the system of equations (15)-(16) in Ref./3/

$$\frac{\partial z}{\partial s} - \frac{\partial z}{\partial \tau_1} = - \int_0^{2\pi} \frac{d\alpha_0}{2\pi} \int_{-\infty}^{+\infty} d\sigma_0 \, e^{-i\alpha(s,\alpha_0,\sigma_0)} f(\sigma_0) ,$$
$$\frac{d\alpha}{ds} = \sigma , \quad \frac{d\sigma}{ds} = -\left(z e^{i\alpha} + z^* e^{-i\alpha}\right) \tag{2.1}$$

with the initial conditions

$$\alpha(0) = \alpha_0 , \quad \sigma(0) = \sigma_0 \tag{2.2}$$

The quantity z determines the field intensity of the wave:

$$W = \frac{H_0^2 a^{4/3}|z|^2}{4\pi} \qquad (2.3)$$

where H_0 is the field strength of the lattice (ondulator), $a = \omega_p^2/2\nu_0\Omega_0$, ω_p is the plasma frequency, $\omega_p^2 = \frac{4\pi e^2 n}{m}$, n is the electron beam density, ν_0 the lattice frequency, s is the dimensionless coordinate (the axis of the lattice is directed along the x-axis): $s = \frac{\Omega_0 a^{1/3}}{\gamma_0} x$, $\Omega_0 = \frac{eH_0}{m}$, γ_0 is the Lorentz factor of incident particles, $T_1 = \Gamma T = \Gamma \nu_R (t-x)$, $\nu_R = \frac{2\gamma^2 \nu_0}{1+\Omega_0^2/\nu_0^2}$ is the resonance frequency, $\Gamma = a^{1/3}\Omega_0/\gamma_0\nu_0 \simeq \frac{\Delta\nu}{\nu}$ is a relative width of the signal in case of a monochromatic incident beam; the component of the particle's 4-velocity is the following: $u^x = \gamma_0\left(1 - \frac{\Gamma\sigma}{2}\right)$, i.e. σ is the dimentionless longitudinal particle momentum counted off the average momentum in the beam; $f(\sigma_0)$ is the initial distribution in the beam.

Let us consider the generation process in the linear regime, $\left|z/z(0)\right| \ll 1$, when the different harmonics of the outgoing signal do not interact with each other, and one can put $z(s,T) = e^{i\varkappa T_1} z(s)$, where $\varkappa$ is the harmonic's frequency counted off the resonance one in terms of Γ (see Ref./3/). Linearizing the system (2.1) we obtain

$$\frac{dz}{ds} - i\varkappa z = -i\int_0^s ds'(s-s')z(s')\int_{-\infty}^{+\infty} d\sigma_0 f(\sigma_0) e^{-i\sigma_0(s-s')} \qquad (2.4)$$

If $f(\sigma_0) = \delta(\sigma_0)$, equation (2.4) is converted to equation (9) in Ref./2/. Equation (2.4) is convenient to solve using the Laplace transformation, and we have as a result

$$\frac{z(s)}{z(0)} = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \frac{e^{\sigma s}\, d\sigma}{\sigma - i\varkappa + i\int_{-\infty}^{+\infty} d\sigma_0 \frac{f(\sigma_0)}{(\sigma + i\sigma_0)^2}} \qquad (2.5)$$

This result is a solution of the initial problem for an arbitrary distribution function $f(\sigma_0)$.

For studying the limiting cases the distribution function may be written as follows:

$$f(\sigma_0) = \frac{1}{\beta}\varphi\left(\frac{\sigma_0}{\beta}\right) \qquad (2.6)$$

where β is the characteristic width of the distribution: $\varphi(x)$ differs a great deal from 0 only at $x \lesssim 1$. For the case $\beta \gg 1$ it is convenient to return to equation (2.4). Making the substitution

$$z = e^{i\varkappa s} e^{S} \qquad (2.7)$$

and assuming that the rate of S phase variation is low, we obtain

$$S = -i\int_{-\infty}^{+\infty} d\sigma_0 f(\sigma_0)\int_0^s (s-t)\, t\, e^{-it(\sigma_0+\varkappa)}\, dt \qquad (2.8)$$

After taking the integral in (2.8) we have

$$\mathrm{Re}\, S = -s^3\int_{-\infty}^{+\infty} d\sigma_0 f(\sigma_0) R\left(\frac{\sigma_0+\varkappa}{2}s\right) \qquad (2.9)$$

where $R(x) = -\frac{1}{4}\frac{d}{dx}\left(\frac{\sin^2 x}{x^2}\right)$ is the characteristic function of the problem at small gains (see formulas (11) and (12) in Ref. /1/). Formulas (2.9) holds at $s \ll 1$ for any values of β as well as at $\beta \gg 1$ and $\beta s \gg 1$. In the last case the function $R(x)$ is reduced (with an accuracy of up to a factor) to the derivative in the δ-function. Then we find from (2.9) and (2.7)

$$\left|\frac{z(s)}{z(0)}\right|^2 = e^{-\pi s f'(-\varkappa)} \qquad (2.10)$$

Let us discuss the results obtained. Formula (2.9) has a simple physical meaning: to find a gain (if it is small) in the case when the incident beam is non-monoenergetic, one should average the gain (dependent on the longitudinal momentum) over the distribution function of the initial particles. In the case of a narrow (compared to the variation scale of the spectral curve) distribution we have the result for a monoenergetic beam with small corrections. For a wide distribution, the destructive interference holds which leads to a substantial decrease of the gain.

For illustration of these arguments let us consider the Lorentz's distribution:

$$f(\sigma_0) = \frac{1}{\pi}\,\frac{\beta}{\beta^2+\sigma_0^2} \qquad (2.11)$$

Then at $\beta s \ll 1$ we have from (2.9) and (2.7)

$$\left|\frac{Z(s)}{Z(0)}\right|^2 = 1 + 2s^3 R\left(-\frac{\varkappa s}{2}\right) + 2\beta s^4 R_1\left(-\frac{\varkappa s}{2}\right) \qquad (2.12)$$

where $R_1(x) = -\frac{1}{4}\frac{d}{dx}\left(\frac{\cos x}{x^2}\left(\cos x - \frac{\sin x}{x}\right)\right)$. The first two terms in (2.12) are the result for a monoenergetic beam and the last term with a small factor, $\beta s \ll 1$, is the correction. At $\beta s \gg 1$ and $\beta \gg 1$ we have from (2.10)

$$\left|\frac{Z(s)}{Z(0)}\right|^2 = e^{-\frac{2\delta\lambda\beta s}{(1+\lambda^2)^2}} \qquad (2.13)$$

where $\lambda = \varkappa/\beta$, $\delta = \frac{1}{\beta^3}$. It is seen that for a wide beam the gain turns out to be strongly suppressed compared to that for a monoenergetic beam when $S \gg 1$ (see Refs./2,4/). Moreover, a maximum of the spectral curve is shifted to the point $\varkappa = -\frac{\beta}{\sqrt{3}}$, $|\varkappa| \gg 1$, while for a monoenergetic beam this maximum is reached at $\varkappa \simeq -1$.

3. Kinetics of Building up a Photon Beam During Repeated Passage

If the magnetic lattice (ondulator) is long enough, then the photon beam generated in a FEL attains a maximum (just after the phase oscillations become important) during a single passage. The kinetics for this case is described in Refs./2-4/. However, at a relatively short length of the ondulator it may turn out to be desirable to equip a FEL with an optical cavity and to use multiple passage of the photon beam. To describe the action of a FEL in this situation, the system (15)-(16) in Ref./3/ may be used also:

$$\frac{\partial Z_n}{\partial s} - \frac{\partial Z_n}{\partial \tau_L} = -\int_0^{2\pi}\frac{d\alpha_0}{2\pi}\, e^{-i\alpha(s,\alpha_0)}$$

$$\frac{d\alpha}{ds} = \sigma, \quad \frac{d\sigma}{ds} = -\left(Z_n e^{i\alpha} + Z_n^* e^{-i\alpha}\right) \qquad (3.1)$$

with the boundary conditions

$$\alpha(0,\alpha_0) = \alpha_0, \quad \sigma(0) = 0, \quad Z_{n+1}(0) = (1-\eta)Z_n(s_0) \qquad (3.2)$$

where η is the total loss coefficient, n is the passage number, $s_0 = \frac{\Omega_0 a^{1/3}}{\gamma_0}L$, L is the length of the ondulator; the remaining notation is given in the foregoing section. The system (3.1)-(3.2) is a dimensionless one and describes the universal behaviour of the system. It has been solved numerically for various values of s_0 and η. As found out, at $\eta \ll 1$ the change of η does not lead to any qualitative effects. In connection with this, η was chosen to be equal to 0.05. The calculated results for s_0 = 3 are given in Figs. 1 and 2 and for s_0 = 5 in Figs. 3 and 4. Fig.1 shows the evolution of the total signal (the integral over all harmonics) as a function of the number of passages n . The quantity $\sqrt{\bar{I}(n)}$ is plotted along the ordinate axis ($\bar{I}(n)$ is the dimensionless intensity, $\bar{I}(n)=|Z_n|^2$, see formula (2.3)); in this case $\sqrt{\bar{I}(0)}$ = 0.01. Fig.2 shows the evolution of the signal for a definite harmonic (whose frequency in terms of $\varkappa=\gamma\frac{\nu_0}{\varrho_0 a^{1/3}}\left(\frac{\nu}{\nu_R}-1\right)$ is given near each curve). The quantity $I(\varkappa)$ is along the ordinate axis; note that $I(\varkappa)$ is the dimensionless spectral density of the intensity: $\bar{I}=\int\frac{d\varkappa}{2\pi}I(\varkappa)$, and $\sqrt{\bar{I}(0)}$ = 0.01. Analogous results for S_0 = 5 are shown in Figs. 3 and 4.

The analysis of Figs. 1-4 allows the following conclusions to be made:

1. The total intensity of the signal grows slowly (diffusively) with the number of passages.

2. Because of the nonlinear interaction between harmonics, the signal intensity, at frequencies far from the resonance ones, begin vanish. It follows that the spectral density at the centre of the radiation line (band) substantially grows.

R e f e r e n c e s

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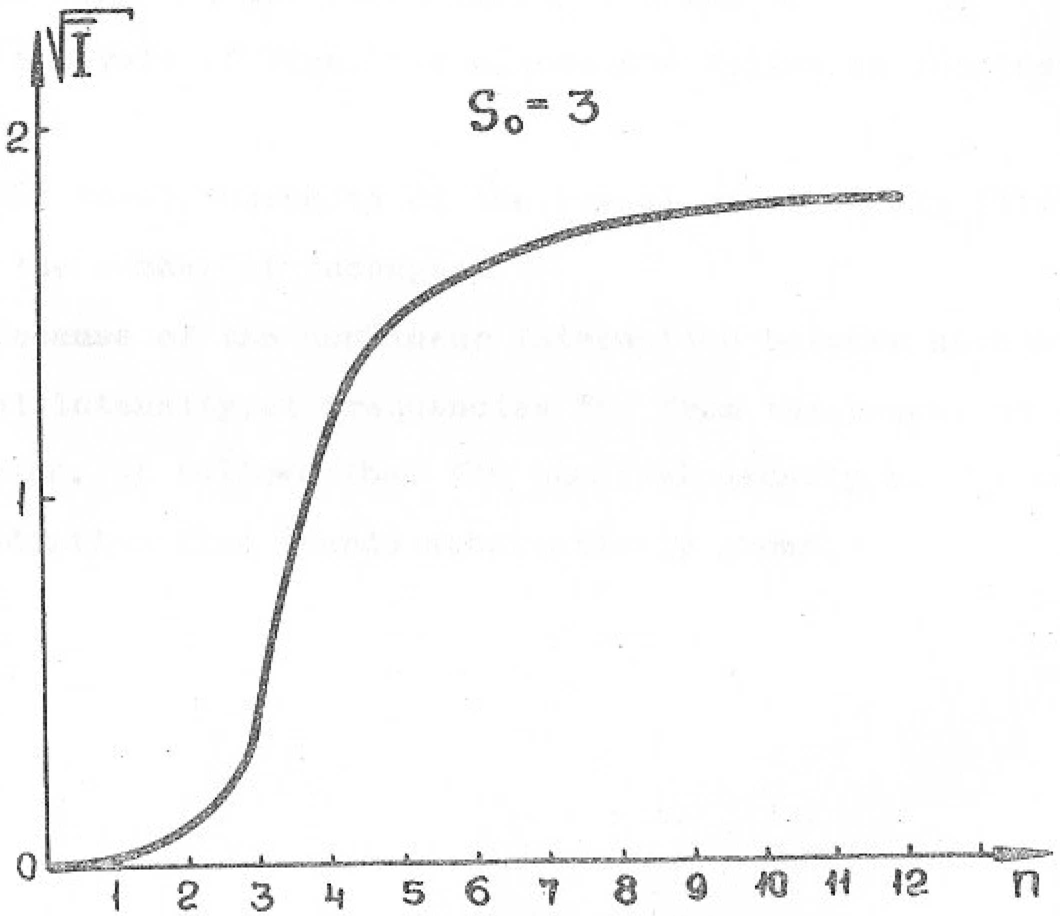

Fig.1. The average outgoing signal (the root from the mean intensity) in dimensionless units, $\overline{I}(n) = \overline{|z_n|^2}$, as a function of the number of passages n at S_o = 3 and η = 0.05.

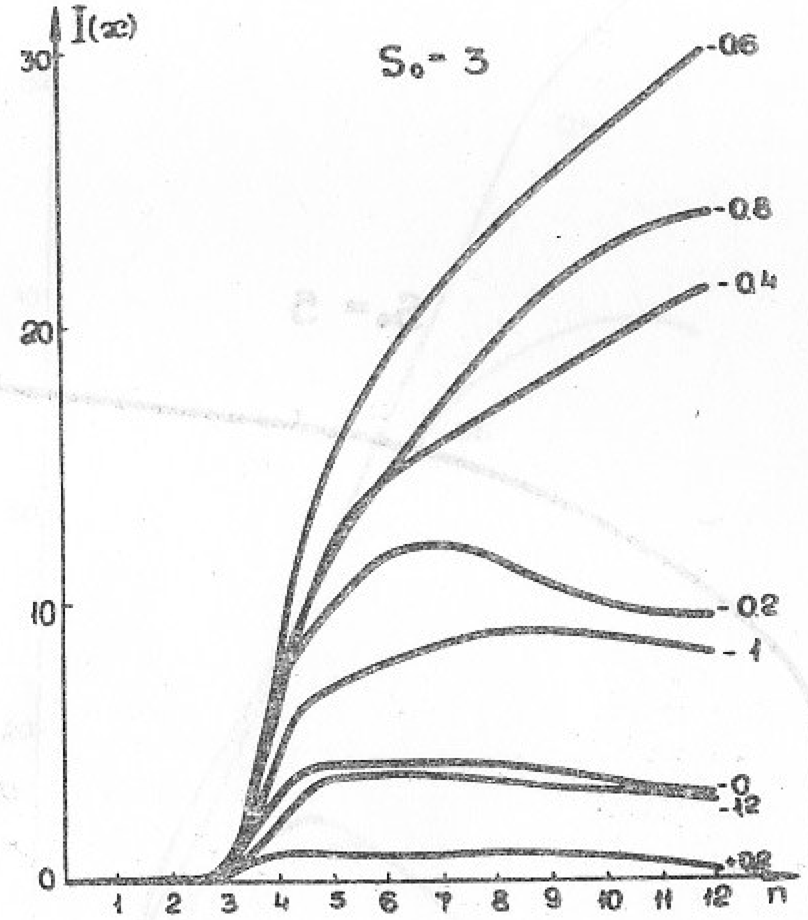

Fig.2. The dimensionless spectral density of the intensity $I(\varkappa)$ of the outgoing signal at a given harmonic as a function of the number of passages n at S_o = 3 and η = 0.05.

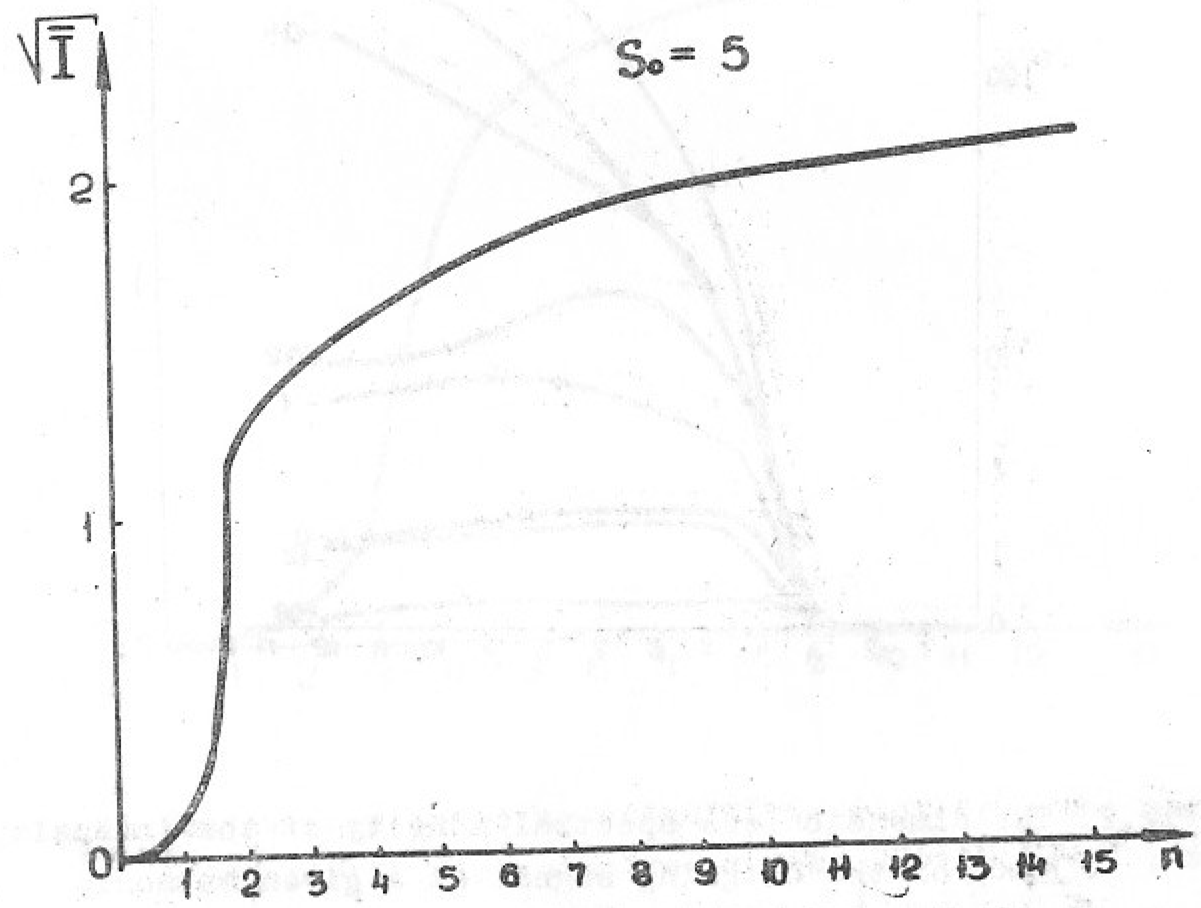

Fig.3. Similar to Fig.1 for S_o = 5.

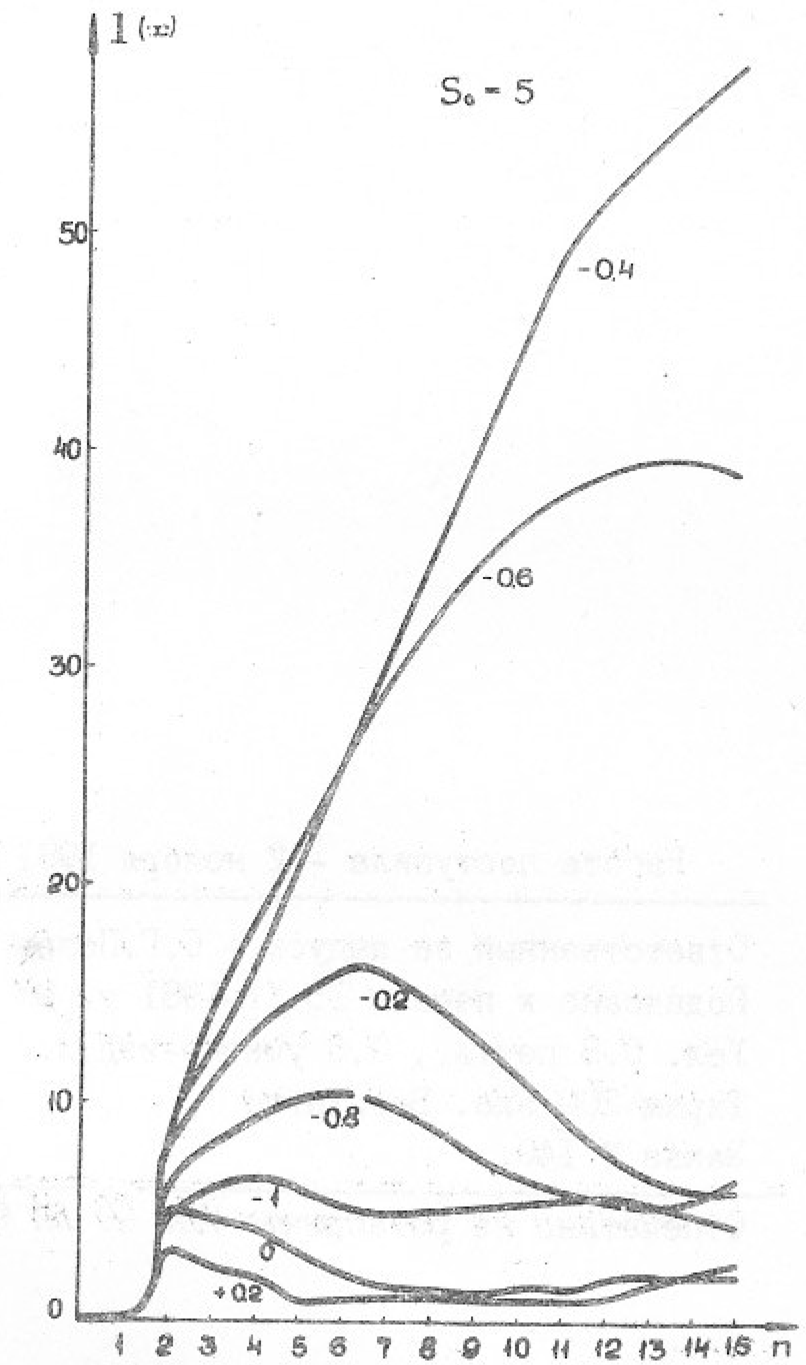

Fig.4. Similar to Fig.2 for S_o = 5.

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