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# SOME STATISTICAL PROPERTIES OF SIMPLE QUANTUM SYSTEMS STOCHASTIC IN THE CLASSICAL LIMIT

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## A b s t r a c t

The simple quantum models stochastic in the classical limit are studied numerically. It is shown that the correlation properties of the quantum and classical systems becomes quite different in a very short period of time $t_s$, and the dynamics of a quantum system, unlike the classical one, is stable reversible. In this case, the diffusive excitation of the quantum system occurs during the period $t^* \gg t_s$ at a diffusion rate close to the classical one, and the continuous component in the correlation spectrum is observed during the period $t_w \gg t^*$. It is shown that in the case of a quasiperiodic-in-time perturbation the time $t^*$ increases sharply.

## 1. Introduction

The interest to the dynamics of nonlinear quantum systems, (which are stochastic in the classical limit ($\hbar = 0$)) is currently growing (see, for example, /1-7/, /8,9/). A study of such systems is of large interest for an analysis of the statistical properties of quantum systems and, for example, for a study of the behaviour of molecules and atoms being in the field of a strong electromagnetic wave, in the stochasticity domain /10,11/, as well as for understanding the peculiarities of the intramolecular dynamics processes /17/. A theoretical study of such problems, even in the quasiclassic region, faces significant problems /2-7/, which are due to a local instability of classic trajectories, leading to an exponential fast spreading of the classical packet, and also due to an increase of quantum corrections with time. In view of this, to study the properties of stochastic quantum systems (SQS), by such systems the quantum systems stochastic in the classical limit are meant, the numerical experiments with a simple model of a quantum rotator in the field of periodical perturbation have been carried out. The main result /1/ is that the motion of a SQS is similar, under the definite conditions, to the stochastic motion of a classical system. So, for example, a diffusive growth of the rotator energy with time has been observed. But for large time the rate of diffusion has substantially reduced.

In the present work a number of numerical experiments with simple SQS models is described. These studies have resulted in that the correlations in the quantum rotator system, unlike the classical one (when the measure of the islands of instability is quite small) are not decaied exponentially with time, that confirm the theoretical result /12/ (section2). The dynamics of excitation of a quantum system with two degrees of freedom has been studied as well (section 3). The regime in which the leading degree of freedom, exciting to a definite level only (quantum limitation of diffusion /1/) affects the second degree of freedom, so that the exitation lasts diffusively much longer than for the leading degree of freedom (probably, unlimitedly). Section 4 analyses the excitation of a quantum rotator by the external perturbation, quasiperiodic in time (two or three non-commensurable frequencies). The numerical experiments indicate that this case differs qualitatively from the case of a periodic perturbation - for it is exempt from the quantum diffusive limitation and a growth of the energy is not practically time-limited.

## 2. Quantum correlations in the rotator model

Let us consider a rotator model in an external field with the Hamiltonian:

$$\hat{H} = -\frac{\hbar^2}{2J}\frac{\partial^2}{\partial\theta^2} + \tilde{k}\cos\,\delta_{\tilde{T}}(\tau) \qquad (2.1)$$

where $\tilde{k}$ is a parameter characterizing the magnitude of perturbation, $\delta_{\tilde{T}}(\tau)=\sum_{n=-\infty}^{\infty}\delta(\tau-nT)$ is the periodic delta-function (periodic kicks), $J$ is the moment of inertia of a rotator, $\theta$ is the angular variable. Below $J = 1$.

The revelant classic problem is described by the Hamiltonian:

$$H = \frac{p^2}{2} + \tilde{k}\cos\theta\,\delta_{\tilde{T}}(\tau) \qquad (2.2)$$

and, because of the periodic type of the delta-function, the motion of a rotator is convenient to describe by the mapping:

$$\bar{p} = p + \tilde{k}\sin\theta$$
$$\bar{\theta} = \theta + \tilde{T}\bar{p} \qquad (2.3)$$

where $\bar{p}, \bar{\theta}$ are the values of the variables after a kick.

The mapping (2.3) has been investigated in detail in /9/, where the value $\tilde{k}\tilde{T}\approx 1$ is shown to be the boundary of stability. At $\tilde{k}\tilde{T}<1$ the motion is stable and variation of the quantity $p$ is limited ($|\Delta p|\leq\sqrt{\tilde{k}/\tilde{T}}$). If $\tilde{k}\tilde{T}\gg 1$, already at $\tilde{k}\tilde{T} = 5$, the motion becomes stochastic. In this case, almost under any initial conditions, except for small (at $\tilde{k}\tilde{T}\gg 1$)



islands of stability, the close trajectories diverge exponentially: $d = d_0\exp(ht)$ where $d = \sqrt{(\tilde{T}\Delta p)^2+(\Delta\theta)^2}$, and $h\approx\ln(\frac{\tilde{k}\tilde{T}}{2})$ (at $\tilde{k}\tilde{T}>4$) is the KS-entropy /9,13/. Such a local instability of motion leads to that the phase $\theta$ becomes a random variable and the rotator energy grows by the diffusion law:

$$E = \frac{\langle p^2(t)\rangle}{2} \approx \frac{\tilde{k}^2}{4}t + E(0) \qquad (2.4)$$

and the momentum-distribution function has the Gaussian form:

$$f(p) = \frac{1}{\sqrt{\pi\tilde{k}^2 t}}\exp(-\frac{p^2}{\tilde{k}^2 t}) \qquad (2.5)$$

Here and below $t$ is the dimensionless time, measured by the number of kicks. The brackets < > imply the averaging over a large number of the trajectories corresponding to different initial data.

The motion of the quantum system (2.1) is also convenient to describe by the mapping for a wave function $\Psi$ in period $\tilde{T}$ /1/:

$$\Psi(\theta, t+1) = e^{-ik\cos\theta}\frac{1}{\sqrt{2\pi}}\sum_{n=-\infty}^{\infty}A_n(t)\exp(in\theta - i\frac{Tn^2}{2}) \qquad (2.6)$$

where $A_n(t) = \frac{1}{\sqrt{2\pi}}\int_0^{2\pi}\Psi(\theta,t)e^{-in\theta}d\theta$, $k = \frac{\tilde{k}}{\hbar}$, $T = \hbar\tilde{T}$.

It follows from (2.6) that one kick couples, with an exponential accuracy, $\approx 2k$ levels of the unperturbed system, that has been used in a numerical study of the model (2.1) /1/. The mapping (2.6) contains two independent parameters $k, T$ used below. We put $\hbar = 1$, and then, within the quasiclassic limit $k\to\infty$, $T\to 0$, $kT = const$. The performed numerical experiments /1/ have shown that at $kT > 1$, $k\gg 1$, in the quantum system (2.1) a diffusive growth of the rotator energy occurs with a rate close to the classical one during a certain period $t^*$, but at $t > t^*$ the rate of diffusion slows down and at $t\gg t^*$ an increase in energy practically stops /1,7/. The time $t^*$ increases with increasing the parameter $k$.

Additional numerical experiments /4/ have shown that the $t^*$ - dependence on $k$ may be approximated by the degree law $t^* = C k^{\alpha}$ (see Fig. 1). In this case, $t^*$ is taken as $t$, when the energy of the quantum system differs from the classical value by more than 25%. The root-mean-square values of parameters seem to be equal to: $\langle \lg C \rangle = -0.44$, $\langle \alpha \rangle = 1.5$. The theoretical dependence

$$t^* = C k^2 \tag{2.7}$$

obtained in /4,7/ turns out to be within the spread of experimental data with $\langle \lg C \rangle = -1.19$ (see Fig. 1). Hence, at $k \gg 1$ the quantum rotator energy grows, as in the classics, in a diffusive manner during a long period of time.

At the same time, according to the theoretical results /12/, in the quantum system the time correlations, exponentially decaied in the classic case at $kT \gg 1$, when the measure of the islands of stability is quite small, the time

$$t_s \sim \frac{\ln k}{\ln kT} \tag{2.8}$$

after, decrease not rapidly than the root of time ( $\tau^{-\frac{1}{2}}$ ). In this case, the quantum and classic correlations become completely different already at $t \gtrsim t_s$. But since the absolute magnitude of correlations proves to be small ( $\propto O(k^{-1})$ ), they affect the rotator energy only on the time $t \gtrsim t^* \gg t_s$.

For these predictions to be verified /12/, the quantum model (2.1) have participated in the numerical experiments in which the correlations

$$R_t(\tau) = \langle 0 | \cos\hat{\theta}_t \cos\hat{\theta}_{t+\tau} + \cos\hat{\theta}_{t+\tau} \cos\hat{\theta}_t | 0 \rangle \tag{2.9}$$

where $\cos\hat{\theta}_t = U_t^+ \cos\hat{\theta}\, U_t$ is the Heisenberg operator at the moment of time $t$, $U_t$ is the operator of evolution of the Hamiltonian (2.1), $\langle 0 | \ldots | 0 \rangle$ stands for the average over the initial state, have been calculated. In principle, one can consider the other correlations, for example, the correlations $\sin\hat{\theta}$. These correlations qualitatively behave in the same manner as the $\cos\hat{\theta}$ ones, exception is one specific feature ana-



lyzed below (see section 3).

The numerical algorithm of computing correlations was to define the wave functions $\psi_t = U_t|0\rangle$, $\varphi_{t+\tau} = U_\tau \cos\theta\, \psi_t$, $\psi_{t+\tau} = U_{t+\tau}|0\rangle$ by means of Eq. (2.6) by the method described in /1,4/ and then to calculate the average: $2\,\mathrm{Re}(\langle \psi_{t+\tau} | \cos\theta | \varphi_{t+\tau} \rangle) = R_t(\tau)$.

The results of numerical experiments are listed in Table 1. Also, the classical $R_{cl}$ and quantum correlations $R_q$ are compared at $t = 0, 0 \le \tau \le 7$ (see (2.9)) in the case of the initial classic state: $p = 0$, $0 \le \theta \le 2\pi$ $(R_{cl} = \frac{1}{\pi}\int_0^{2\pi} \cos\theta \cos\theta_\tau\, d\theta)$ and of the corresponding quantum one: $\psi(\theta, 0) = (2\pi)^{-\frac{1}{2}}$. It is seen from these data that at $kT = 5$, $5 + 2\pi$, when the measure of the islands of stability is negligibly small /9/, the classical correlations, at $\tau \le 7$ decay exponentially with time. The quantum correlations, in this case, are close to the classical ones only for $\tau \lesssim t_s \approx 3$, but at $\tau \gtrsim t_s$ they differ by a factor of a few times. The theoretical value of $t_s$ (2.8) seems also to be equal to several kicks, that is consistent with the results of numerical experiments. So, for example, for $k = 40$, $kT = 5$ the quantum rotator energy differs from its classical value by less than 25% during the period of time $t^* = 120 \gg t_s \approx 3$. In the case $kT = 2$, when the measures of the stable and stochastic components seems to be approximately the same, the classical correlations do not decay with time and the distinction between $R_{cl}$ and $R_q$ remains less than 20% for the period $\tau \approx 100 \gg t_s \approx 3$ ( $k = 40$). Thus, the characteristics not decreasing exponentially with time, for example, the rotator energy, correlations at $kT = 2$, seems close during the time $t^* \gg t_s$. Note also that in the stability region, $kT = 0.5$ ( $k = 20$) the distinction between the quantum and classical correlations is at a level of 0.1% for $\tau \approx 20$ (at $k = 5$, $\tau \approx 20$ at a level of 10%).

The typical behaviour of quantum correlations is shown in Figs. 2,3. It is seen that there are residual correlations not decreasing with time. The value of these correlations reduces as $k$ increases, but an explicit form of the dependence on $k$ fails to be found because of a sharp increase of the memory required and the computing time with the growth of $k$.

The magnitude of residual correlations may be evaluated as follows. Let $\tau \gg t^*$. Then the wave function $|\psi\rangle = e^{\pm i\theta} \cdot U^+_\tau e^{\pm i\theta} U_\tau |0\rangle$ contains approximately $\sqrt{k^2 t^*}$ harmonics $\theta$ (at $\tau \gg t^*$ an increase in energy practically stops). Since $\langle\psi|\psi\rangle = 1$, an average amplitude of harmonics $Q$ is determined from the condition $Q^2\sqrt{k^2t^*} \sim 1$. Then, from the relation $R(\tau) \sim \langle 0|\psi\rangle \sim Q$ and (2.7) we get the estimate

$$|R_t(\tau)| \sim (k^2t^*)^{-\frac{1}{4}} \sim k^{-1}, \quad t+\tau \gg t^* \qquad (2.10)$$

At $t_s \le t+\tau \ll t^*$ the number of harmonics in $|\psi\rangle$ will be of the order of $\sqrt{k^2\tau}$ and, hence, on this time interval the correlations slows down with the growth of $\tau$:

$$|R_t(\tau)| \sim (k^2\tau)^{-\frac{1}{4}}, \quad t_s \le t+\tau \ll t^* \qquad (2.11)$$

This slowing down seems to be very slow and the parameter $(t^*)^{1/4}$ not too large and for this reason the non-decreasing with-time residual correlations are observed in the numerical experiment practically immediately (see Figs. 2,3).

It is noteworth that, according to the estimates obtained, (2.10), (2.11) and to the results /12/, there is no exponential decay of quantum correlations in such systems wherein the measure of the islands of instabilities is strictly equal to zero (e.g., the system (2.1)) with the perturbation potential

$$V(\theta) = \begin{cases} -\dfrac{\theta^2}{2}, & 0 \le \theta \le \dfrac{\pi}{2} \\ \dfrac{(\theta-\pi)^2}{2} - \dfrac{\pi^2}{4}, & \dfrac{\pi}{2} \le \theta \le \pi \end{cases}$$

$$V(\theta) = V(-\theta), \quad V(\theta) = V(\theta+2\pi) \quad \text{at} \quad kT > 4.$$

The numerical experiments also show that in such a quantum system, unlike the classical one, there is no, exponential decay of correlations.

An interesting feature of quantum correlations of operators $\cos\hat\theta$ is that $R_t(\tau) > 0$ almost at any $\tau$. As a result of this, the frequency spectrum of correlations is sharply peaked at $\omega = 0$. The properties of the frequency spectrum are discussed in detail in section 3.



It should mention that, in the quantum model not only the exponential decay of correlations is absent but the KS - - entropy $h$ is zero /12/ (in the classical system $h \approx \approx \ln(\frac{kT}{2}) > 0$ at $kT > 4$ /9/). By virtue of this, in the quantum system there is no local instability of motion, which occurs in the classical model (2.2) at $kT > 1$. The presence of the local instability ( $h > 0$ ) leads to that the dynamics of a classical system turns out to be actually non-reversible. Indeed, although the equations of motion of the system with Hamiltonian (2.2) are reversible (the system's Hamiltonian is symmetric with respect to the replacement $\tau \to -\tau$ at the moments of time $lT + \frac{T}{2}$, $l$ is integer, and therefore at the substitution $p \to -p$ at the moments $lT + \frac{T}{2}$ the trajectory will move exactly in the inverse direction and returns to the initial point), as small perturbation $\varepsilon$ as possible will change the trajectory in the period $t_\varepsilon \sim \frac{|\ln\varepsilon|}{h}$ because of the local instability. In connection with this, in the numerical experiments where there are the errors of approximation at the level $\varepsilon \sim 10^{-12}$ (BESM-6) no reversibility in time is present (see Fig. 4). At the same time, the dynamics of a quantum system proves to be completely resersible (the accuracy of return is at a level of computer accuracy). Moreover, reversibility occurs even at a random variation of the phases of Fourier-components $A_n$ of a wave function $\psi$ within the interval $\Delta\psi$ at the moment of reversion (see Figs.5,6). Fig. 6 presents the distribution function $f(n) = |A_n|^2$ over the levels of an unperturbed quantum system in normalized coordinates $f_N = f(n)\sqrt{\pi k^2 t}$, $X = \frac{n^2}{k^2 t}$ (the classical distribution (2.5) in these coordinates has the simple form: $f_N = e^{-X}$). The sharp peak in Fig. 6 at $n = 0$ corresponds to the returning component ( $\psi(\theta, t=0) = (2\pi)^{-1/2}$), the fraction of the nonreturned component is $W_L \approx 7.9 \times 10^{-4}$ (phase variation takes place in the interval $\Delta\psi = 0.1$). The dependence of $W_L$ on $\Delta\psi$ is given in Table 2. It is interesting that $W_L \approx 0.079 \ll 1$ even at $\Delta\psi = 1$.

The total number of levels in the performed numerical experiments was $N$ = 2049 (-1024,1024). The initial conditions were different: excitation of the zero level only ($n_0$ = 0, the uniform distribution over $\theta$ ), the Gaussian distribution with the width $4 \le \Delta n \le$ 20. Just as in /1,4/, a no significant dependence of the motion on initial conditions was observed.

## 3. Two-dimensional model

The numerical experiments (see section 2) with the model (2.1) have shown that the statistical properties of a quantum system are much weaker than those of the classical one. At the same time, of interest is to study how such a system affect the other degree of freedom if a weak coupling is available. In the case when the coupling is weak, its influence on the first degree of freedom may be neglected and excitation of the second degree of freedom will be determined by the statistical properties of the motion with respect to the first degree of freedom.

As an example, let us consider a system with Hamiltonian:

$$H = \frac{\hat{p}_1^2}{2} + \omega \hat{p}_2 + (k\cos\theta_1 + \varepsilon\cos\theta_1\cos\theta_2)\,\delta_T(\tau) \qquad (3.1)$$

where $\hat{p}_1 = -i\frac{\partial}{\partial\theta_1}$, $\hat{p}_2 = -i\frac{\partial}{\partial\theta_2}$, $\hbar = 1$.

By solving the Schroedinger equation with Hamiltonian (3.1) one gets the mapping for a wave function in a period:

$$\psi(\theta_1,\theta_2,t+1) = \exp(-i(k\cos\theta_1 + \varepsilon\cos\theta_1\cos\theta_2))\cdot$$

$$\cdot\frac{1}{2\pi}\sum_{n_1,n_2=-\infty}^{\infty} A_{n_1 n_2}(t)\, e^{i(n_1\theta_1+n_2\theta_2)}\, e^{-i(\frac{T}{2}n_1^2+\omega T n_2)} \qquad (3.2)$$

which have been used in the numerical experiments. Let us consider the case when $\varepsilon \ll 1$ and for the second degree of freedom, only the ground level is initially excited ( $A_{n_1 n_2}(0) = A(n_1)\cdot\delta_{n_2,0}$ ). Then the number of excited levels is determined by the statistical properties of the system (2.1). Indeed, from the equations for Heisenberg operators we have:

$$\hat{p}_2(t) = \hat{p}_2(0) + \varepsilon\sum_{t_1=1}^{t-1}\cos\hat{\theta}_1(t_1)\,\sin(\hat{\theta}_2 - \omega T t_1) \qquad (3.3)$$

From (3.3) one can obtain the number of excited levels $n_2$ :

$$\langle n_2^2\rangle = \frac{\varepsilon^2}{2}\Big[\sum_{t_1=0}^{t}\Big(\sum_{\tau=0}^{t-t_1} R_{t_1}(\tau)\cos\omega T\tau - \frac{1}{2}R_{t_1}(0)\Big)\Big] \qquad (3.4)$$

Due to the exponential decay of correlations in the sums in (3.4), the main contribution, in the classical case, was given by the terms with $\tau = 0$, therefore the diffusive excitation occurs both over the first (2.4) and the second degree of freedom:

$$\langle n_2^2\rangle = \frac{\varepsilon^2}{4}t \qquad (3.5)$$

In the quantum system the presence of residual correlations (see Figs. 2,3) leads to a sharp restriction of energy growth over the first degree of freedom at $t > t^*$ (the influence of the second degree of freedom may be neglected, since $\varepsilon \ll 1$ ). Nevertheless, the question on their influence to the excitation of the second degree of freedom requires additional studies. If the frequency spectrum $\tilde{R}(\nu)$ of correlations $R(\tau)$ is purely discrete (this occurs when the quasienergy spectrum /14/ of the system (2.1) contains only discrete levels), then for the values of a parameter $\omega T$, conciding with discrete frequencies $R(\tau)$, will be quadratically increased, $\langle n_2^2\rangle \sim t^2$. If the spectrum $\tilde{R}(\nu)$ contains a continuous component (this may occur only if the spectrum of quasienergies turns out to be continuous), $\langle n_2^2\rangle$ grows diffusively with time ( $\langle n_2^2\rangle = D\frac{\varepsilon^2}{4}t$ ), the diffusion factor $D \sim \tilde{R}(\omega T)$.

The motion of the quantum system (3.1) has been studied numerically by means of formula (3.2). The parameter $\varepsilon$ has been chosen to be equal to $10^{-5}$ (variations of $\varepsilon$ within the interval $10^{-3}\div10^{-5}$ has remained the quantity $\langle n_2^2\rangle/\varepsilon^2$ unchangeable to an accuracy of up to 0.1%). The finite number of levels $-400 \le n_1 \le 400$, $-2 \le n_2 \le 2$ , has been used in the run, and, because of the smallness of $\varepsilon$, $<n_2^2>$ has been determined only by the probability $W_{n_2}$ of finding on the levels $n_2 = \pm 1$, because during the whole period of running $W_{n_2=\pm 2} < 10^{-14}$ and, in view of this, the influence of the levels with $n_2 = \pm 2$ could be neglected (these levels have been used to control the computing accuracy). The computing accuracy has been also controlled by conservation of the probability $W = \sum_{n_1,n_2=-\infty}^{\infty} |A_{n_1 n_2}|^2 = 1$. In all cases the error $\delta W$ for the total probability did not exceed $10^{-3}$ and the dynamics of excitation of the first degree of freedom (e.g., $<n_1^2>$) coincided with the case $\varepsilon = 0$ with an accuracy of up to 0.1%.

The numerical experiments have shown that excitation of the second degree of freedom depends sunstantially on a parameter $\omega T$. In this case, there are three different situations:

1. For the second degree of freedom, just as for the first one, the quantum limitation of the diffusion is observed (see Fig. 7); at $k = 5$, $T = 1$, this occurs for $\omega T = 1, 1.5, 1.87, 2.37, 2.42$.

2. For some values of $\omega T$, the resonance excitation of the second degree of freedom ($<n_2^2> \sim t^2$) is observed, that takes place for $\omega T = 0, 0.5, 1.27, 1.71$ if $k = 5$, $T = 1$ (see Figs. 8.9).

3. In some cases ($k = 5$, $T = 1$; $\omega T = 2.4, 2.5, 2.52$) the diffusive excitation has been observed. It is noteworth that for $\omega T = 2.5$ $<n_2^2>$ grows practically linearly with time up to $t = 2000$ (see Fig. 10), while the diffusion limitation over the first degree of freedom occurs in a few kicks ($t^* = 5$). For $\omega T = 2.4, 2.5$ a linear growth has been observed during the total time of computing ($t = 750$) with the average diffusion factor $D_q / D_{cl} \approx 0.7; 2.$ respectively ($D_{cl} = \varepsilon^2/4$).

When changing the initial conditions, substantial variations in the motion have not been observed (for example, the resonances have occured at the same values of a parameter $\omega T$), but when changing the parameters $k$, $T$ (even at kT = const), the dependence on $\omega$ T has become absolutely another (so, for example, at $k = 10$, $T = 0.5$, $\omega T = 1.27$ the limitation of the diffusion process was observed, instead of the resonance).



Expection was only the value $\omega T = 0$ at which $<n_2^2>$ grew quadratically with time for all used values of parameters k, T in the regions $k<1, k>1$ at $kT<1$ and $kT>1$. In the classical stability region $kT<1$, $k>1$ the dependence of $<n_2^2>$ on $t$ seemed to be close to the classical one, where the resonance excitation also occured because of the stability of classical motion. Hence, the resonant growth $<n_2^2>$ in the quantum system $kT>1, \omega T \neq 0$ should be interpreted as an indication to the presence of a stable quantum component. Of course, this problem should be examined in more detail.

Note also that if the Hamiltonian (3.1) contained $\sin\theta_1$, instead of $\cos\theta_1$, the excitation of the second degree of freedom should be determined by correlations $\sin\theta_1$. But since in this case, at $\omega = 0$ $<n_2^2> \sim <n_1^2>$, the quantum limitation of the diffusion should occur over the first and second degrees of freedom.

The availability of resonances for $\omega T \neq 0$ indicates the presence, in the spectrum of correlations $R(\tau)$, of a discrete component and, as a consequence (see above), of discrete levels in the spectrum of quasienergies ($\varepsilon_{n'} - \varepsilon_n = \omega$). Otherwise, a diffusive growth of $<n_2^2>$ at some values of $\omega$ T indicates the presence, in the spectrum of quasienergies, a continuous zone with the width $\Delta\varepsilon_2 \gtrsim 0.02$ ($k=5$, $T=1$). However, the finiteness of the computing time $t_c$ admits only the lines with $\Delta\varepsilon \gtrsim \frac{1}{T t_c}$ and therefore, strictly speaking, one can affirm that in the zone of quasienergies $\Delta\varepsilon_2$, the spectrum is either continuous or consists of close-lying discrete lines, the distance between which is $\Delta\varepsilon_L \lesssim 5\ 10^{-4}$. Summarizing, one can say that, besides two time scales of the motion of the quantum system (2.1) $t_s$ and $t^*$ (see section 2 and /7/) there is once more time scale $t_W$ on which some weak statistical properties are still conserved; so, for example, on these times the diffusive excitation of the second degree of freedom in (3.1) occurs. It is significant that the scale $t_W$ exceeds very much $t_W$ and $t_s$ ($t_W \gg t^* \gg t_s$), so at $k = 5$, $T = 1$ we have $t_s \approx 1$, $t^* \approx 5$, $t_W \gtrsim 2000$ (see Fig. 5). The question on the determination of a scale of $t_W$, whether it is finite or infinite, requires further examination. If $t_W = \infty$, the spectrum of quantum correlations and the spectrum of quasienergies will contain a continuous component. It is noteworth that the continuity of the spectrum of quasienergies does not yet mean the continuity of the spectrum of correlations. So, at $T = 4\pi$ (the case of a quantum resonance /1,15/) $\psi(\theta,t) = \exp(-ikt\cos\theta)\psi(\theta)$ and, according to (2.9), the spectrum of correlations consists of only one discrete line, while the spectrum of quasienergies, in this case, in continuous /15/.

## 4. A model with non-commensurable frequencies

Along with the case considered in the foregoing section, such a situation can occur when the motion over one degree of freedom during a certain period of time may be considered as a given and periodic one, and the motion over the other degree of freedom will be then determined by a field of external forces. For the system (3.1) such a situation takes place at $\omega T p_2 \gg \varepsilon T$, $\varepsilon T \ll 1$. In this case, one can suggest, in a first approximation, that the coordinate $\theta_2$ varies periodically with time: $\theta_2(\tau) = \theta_2(0) + \omega_0 \tau$ $(\omega_0 = \omega p_2(0))$, and the dynamics of the first degree of freedom is then described by the Hamiltonian (2.1) with time-variable $k$: $k(\tau) = k + \varepsilon\cos\omega_0\tau$. The experiments have shown that the dynamics of the quantum system with $k(\tau)$, periodically varying in time, differs substantially from the cases considered in /1,4,7/, when $k$ = const and $k \sim t^{\alpha}$ $(\alpha < \frac{1}{2})$. Therefore, a study of such a system is of interest and importance for understanding the SQS properties. In the classical system (2.2) with the variable $k(\tau)$, just as in the case with constant k, at $(k+\varepsilon)T \gtrsim 1$ the stochastization of motion occurs. The phase $\theta$ then varies randomly and the rotator energy grows diffusively with time:

$$E(t) = \frac{k_{ef}^2}{4} t + E(0) \tag{4.1}$$

with $k_{ef}^2 \approx k^2 + \varepsilon^2/2$. Thus, the dynamics of classical systems with the constant and variable $k$ have no principal distinctions. At the same time, the dynamics of excitation of a quantum rotator in these two cases is different.



For example, at constant $k$ the quantum limitation of diffusion occurs (see occurs (see /1/ and section 2). It leads to that at $t \gg t^*$ (see (2.7)) the rotator energy does not practically grow. The numerical experiments carried out with the model (2.1) at $k(\tau) = k + \varepsilon\cos\omega_0\tau$ have shown that in the case of non-commensurability of frequencies $\omega_0$ and $\Omega = \frac{2\pi}{T}$ and at $\varepsilon \gtrsim 1$ the quantum rotator energy grows diffusively with time, the diffusion factor being close to the classical one. So, at k = 0, $\varepsilon$ = 7, $\varepsilon T$ = 7, $\omega_0 T$ = 2 a diffusive growth of the energy continues during all the period of running t = 1000 (see Figs. 11,12), the distribution over levels being close to the Gaussian one (2.5) with $k = k_{ef.}$ In this case, for k = 7, T = 1, $\varepsilon$ = 0, the time of diffusive growth was only $t^* \approx 10$. If the frequencies were commensurable ($\omega_0 T = \frac{2\pi p}{q}$, $p, q$ are integer non-commensurable numbers), the quantum limitation of diffusion were observed, and the moment of time $t^*$, from which the diffusion slowed down, increased with increasing $q$. So, $t^* \approx 60$ at $\omega_0 T = \pi$; $t^* \approx 400$, $\omega_0 T \approx \frac{2\pi}{3}$; $t^* = 450$, $\omega_0 T = \frac{2\pi}{5}$; $t^* > 1000$, $\omega_0 T = \frac{5\pi}{13}$ for k = 0, $\varepsilon$ = 7, T = 1. Interesting is the regime of motion at $\omega_0 T = 0.1$. Because the time of phase shift $t_p = \frac{2\pi}{\omega_0 T} \approx 60 > t^* \approx 10$ the diffusion diceleration occurs during the period $\sim t^*$ and the growth of energy practically ceases, but in time $\sim t_p$ the phase variation becomes significant and the energy grows again. Thus, a step-like diffusive growth of energy with time occurs (Fig. 13).

A diffusive energy growth was also observed in the essentially quantum region at k = 0, $\varepsilon$ = 3.5, $\varepsilon T$ = 7, $\omega_0 T$ = 2 (see Fig.14). For small values, $\varepsilon \lesssim 4.5$, the quantum limitation of diffusion occured, the time $t^*$ was increased from $t^* \approx 1$ to $t^* > 2500$ when varying $\varepsilon$ from 1 to 4.65 (see Fig.15). Within the interval $1 \leq \varepsilon \leq 4.65$, the dependence of $t^*$ on $\varepsilon$ is close to the exponential one, but experimental potentialities do not allow to establish, what happens at $\varepsilon > 4.65$. Apparently, $t^*$ will continue to increase exponentially with further increasing $\varepsilon$ (some estimates for $t^*$ $(k_{ef.})$ are given at the end of this section). However, since $t^*$ increase sharply (by three orders of magnitude) with increasing $\varepsilon$ from 1 to 3.5, one can suggest that practically there is some $\varepsilon_{cr} \approx 3.5$ above which a non-limited excitation of the quantum rotator takes place. This value of $\varepsilon_{cr}$ is only $S_q = \frac{\varepsilon_{cr}}{\varepsilon_\ell} \approx 3.5$ times higher than the quantity $\varepsilon_\ell$ corresponding to the quantum border of stability /10/. So, for the parameters k = 0, $\varepsilon$ = 1, T = 5.6, $\omega_0$T = 2 the ratio of the quantum and classical diffusion factors at the time $t$ = 200 seems to be equal to $D_q / D_{cl} \approx$ $\approx 2.4 \times 10^{-4} \ll 1$.

At $k > \varepsilon \gtrsim 2$ **the rotator is also excited diffusively.** (see Figs.16,17)(with $D_q \approx D_{cl}$ and the Gaussian distribution over levels close to the classical)

Note that for k = 10, T = 0.5, $\varepsilon$ = 0, the time of quantum limitation of diffusion is $t^* \approx 25$, while if $\varepsilon$ = 2.5 ($\omega_0$T = 1) the energy growth continues during the entire period of computing, t = 1000 (Fig.16,17). The correlation function $R(\tau)$, determined by means of (2.9), is represented, for this case, in Fig. 18. The Fourier analysis has shown that $\tilde{R}(\nu)$, in contrast to the case $\varepsilon$ = 0, does not contain clearly observed peaks, that indicates the continuity of the spectrum of motion of the quantum system (with an accuracy of experimental resolution being equal to $\Delta\omega \sim 10^{-3}$).

At $\varepsilon \lesssim$ 2 and $k \gg 1$, the rate of diffusive excitation of rotator coincides, during some time, with the classical one and then slows down to a certain limiting value $\overline{D}_q$ ; in the following this quantity does not decrease during the entire period of running (t = 300). The dependence of the ratio $\overline{D}_q / D_{cl}$ on $\varepsilon$ is illustrated in Fig. 19. For comparison, this figure presents also the dependence of $\overline{D}_q / D_{cl}$ on $\varepsilon$ in the case when $k(t) = k + \varepsilon\,\xi(t)$, where $\xi(t)$ is randomly varied with time in the interval [-1,1]. Just as in the case of a periodical variation of $k(t)$ at $\varepsilon \gtrsim 2$, $D_q \approx D_{cl}$ and at $\varepsilon < 2$ there is a limiting coefficient $\overline{D}_q < D_{cl}$, which decreases with $\varepsilon$ . At the same values of $\varepsilon$ , k, T ($\varepsilon$<2) the diffusion rate $\overline{D}_q$ for the rotator model with randomly varying k(t) turns out to be larger than the value $\overline{D}_q$ in the model with periodically varying k(t) (see Fig. 19). However, the qualitative form of $\varepsilon$ - dependence seems to be the same.

Thus, there is the quantum border of stability $\varepsilon \sim 1$. At $\varepsilon \ll 1$ the diffusive excitation of a rotator is sharply decelerated, at $\varepsilon \gtrsim$ 2 both for the periodical (with noncommensurable frequencies) and random variation of k(t) the rotator energy grows diffusively during the entire period of computing (t = $10^3$) with the diffusion factor $D_q \approx D_{cl}$. One should mention that such a diffusive excitation, at $\varepsilon > 1$, occurs also in the essential quantum region T > 1 (for example, for k = 10, $\varepsilon$ = 2.5, T = 4.6, $\omega_0$T = 1). In the region of classical stability, $(k + \varepsilon)T \ll 1$, energy variation, just in the classical case, is limited.

In conclusion of this section, some estimations are given. Let us consider the case, when two frequencies are commensurable, i.e. $\omega_0 T = \frac{2\pi p}{q}$. Here the perturbations proves to be periodical with the period from $q$ kicks and the time $\tau^* = t^*/q$ will be determined by a distance between the discrete levels of the quasienergy (such estimation method of evaluating $t^*$ is used in /7/): $\tau^* = \frac{t^*}{q} \sim \frac{1}{\Delta}$. The quantity $\Delta$ is determined by the number of effectively excited levels of quasienergy $N_\psi$ , which may be estimated by the number of excited unperturbed levels $N_\psi \sim k_{ef}\sqrt{t^*}$. From these relations we obtain an estimate for $t^*$:

$$t^* \sim k_{ef}^2\, q^2 \tag{4.2}$$

Let now $\omega_0 T = \frac{2\pi p}{q} + \delta$ , where $\delta$ is a small deviation. In this case, the system moves approximately in the same manner as at $\omega_0 T = \frac{2\pi p}{q}$ during the time $t \lesssim t_p \sim \delta^{-1}$. If $t_p \sim \delta^{-1} \gtrsim$ $\gtrsim t^* \sim k_{ef}^2 q^2$, there occurs the quantum limitation of diffusion but at $2t_p > t > t_p$ a diffusive growth may begin again, similarly to the case with k = 0, $\varepsilon$ = 7, $\varepsilon T$ = 7, $\omega_0 T$ = 0.1 (Fig.13). Hence, the quantum limitation of diffusion will be observed at least during some time, if

$$\delta(q) \lesssim \frac{1}{q^2 k_{ef}^2} \tag{4.3}$$

The total measure of all the deviations grows with increasing $q$:

$$\sum_{q=1}^{q_{cr}} \sum_{p=1}^{q} \delta(q) \sim \frac{\ln q_{cr}}{k_{ef}^2} \sim 1$$

and becomes equal to 1 at $q_{cr} \sim e^{c k_{ef}^2}$. From this one can obtain an estimate for $t^*$ in the case of two noncommensurable frequencies:

$$t^* \sim k_{ef}^2 \, e^{2c k_{ef}^2} \tag{4.4}$$

where $c$ is a certain numerical constant. The estimates (4.2) and (4.4) coincide qualitatively with the available experimental data (see Fig. 15) but the more exact comparison fails because of a sharp increase of $t^*$ with increasing $k_{ef}$ and $q$. In the case when there are three non-commensurable frequencies (e.g., $k(\tau) = k + \varepsilon \cos\omega_0\tau \cos\omega_1\tau$) and $\omega_0 T = \frac{2\pi p_0}{q_0} + \delta_0$, $\omega_1 T = \frac{2\pi p_1}{q_1} + \delta_1$, the time $t^* \sim (q_0 q_1 k_{ef})^2$, if $\delta_0 = 0$, $\delta_1 = 0$. In order the diffusive limitation occurs, $t^* < \min(\delta_0^{-1}, \delta_1^{-1})$ is required. It follows that the measure of such frequencies seems to be small at large $k_{ef}$ ($\sum_{q_0,p_0,q_1,p_1=1}^{\infty} \delta_0\delta_1 \sim k_{ef}^{-4} \ll 1$). Therefore, in the case of three and more noncommensurable frequencies an unlimited diffusive excitation of a quantum rotator occurs almost for any $\omega_0, \omega_1$. So, at k = 0, $\varepsilon = 3.5$, $\varepsilon T = 7$, $\omega_0 T = 2$, $\omega_1 T = 2^{1/4}$, the time $t^* > 2000$.

## 5. Conclusive remarks

The performed studies show that the statistical properties of SQS, in comparison with the classical stochastic systems, are much weaker. So, the quantum systems are exempt from the exponential decay of correlations (section 2, /12/), which occurs in the classical systems when measure of the islands of stability is negligibly small. The total statistical properties of classical systems in the quantum case are conserved during a short period of time $t_s \propto \ln\frac{1}{\hbar}$ (2.8), only. At $t \gtrsim t_s$ the correlations in the quantum and classical systems become completely different (see Table 1, Figs. 2,3). The KS-entropy of the classical and quantum systems are strongly different, too: in a SQS the KS-entropy $h$ = 0 /12,16/, while in the corresponding classical system $h > 0$. Note that in a SQS $h$ = 0 not only when the spectrum of motion of a quantum system is discrete (this case was considered in /16/), but when the spectrum of motion is continuous, for example in the model (2.1) at $T = \frac{4\pi p}{q}$ (the quantum resonance /1/) the spectrum of quasienergies is continuous /15/, but $h$ = 0 /12/. The consequence of the zero KS-entropy is a stable reversibility of the quantum evolution (see section 2, Figs. 5,6) which is absent in classical stochastic systems (Fig. 4).



At the same time, the weaker statistical properties, for example diffusion, are conserved in a SQS on much larger times $t^* \propto \frac{1}{\hbar}$ ($t^* \gg t_s$). For the system (2.1) with one degree of freedom and a periodical external force, the time $t^*$ grows with increasing the quasiclassical parameter, according to (2.7). As the numerical experiments show (section 3), the continuous component in the spectrum of correlations and a diffusive excitation of the other degree of freedom on definite frequencies are observed during a larger time interval $t_w \gg t^* \gg t_s$. At k = 5 $t_w$ exceeds $t^*$ nearly by three orders of magnitude (section 3). The question what determines the third time scale and whether it is finite or infinite remain still open.

Numerical experiments with a one-dimensional model and a two-frequencies external force (section 4) have shown that in such a system the diffusive scale $t^*$ increases very sharply (probably, exponentially) as the quasiclassic parameter $k_{ef}$ increases (see Fig. 15 and (4.4)). One seems sufficient to exceed the quantum border of stability $\varepsilon_q$ by $S_q \approx 3.5$ times that $t^*$ would increase by three orders of magnitude. If there are three non-commensurable frequencies (or more) the diffusive scale proves to be, apparently, infinite (see the estimates of section 4). In this case, the unlimited diffusive excitation of a quantum rotator occurs, and the quantum correlations are decaid in a power manner (2,11), and hence the quantum system possesses the property of mixing.

Because the external force with non-commensurable frequencies, in a certain approximation, may be always represented as a system with a large number of degrees of freedom, then the unlimited diff[illegible]e growth of the energy with the diffusion factor close to the classical is possible already in the quantum system with two degrees of freedom (or more than two) and the external periodic force. This diffusive excitation takes place if the classic criterion of stochasticity if fulfilled and the quantum border of stability for a perturbation is exceeded /10/. As the numerical experiments show, the quantum border should be exceeded only by $S_q \approx 3.5$ times for a practically unlimited excitation.

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Table 1

| $\tau$ | $R_{cl}$ | $R_q/R_{cl}$ k = 5 | $R_q/R_{cl}$ k = 20 | $R_q/R_{cl}$ k = 40 | $R_q/R_{cl}$ k = 100 |
|---|---|---|---|---|---|
| 0 | I | I | I | I | I |
| | | | kT = 2 | | |
| I | 0.5767 | 0.9880 | 0.9993 | 0.9998 | 1.0000 |
| 2 | 0.4986 | 0.9651 | 0.9976 | 0.9994 | 0.9998 |
| 3 | 0.9614 | 0.9753 | 0.9982 | 0.9996 | 0.9999 |
| 4 | 0.6794 | I.1785 | I.0745 | I.0294 | I.0053 |
| 5 | 0.5688 | I.5397 | I.0742 | 0.9552 | 0.9754 |
| 6 | 0.6504 | 0.9371 | 0.9294 | I.1233 | I.0152 |
| 7 | 0.7648 | 0.8375 | I.0365 | 0.9678 | 0.9946 |
| | | | kT = 5 | | |
| I | -0.1310 | 0.8313 | 0.9908 | 0.9977 | I.0000 |
| 2 | 0.01229 | 14.9880 | 7.4768 | 2.7307 | 0.9723 |
| 3 | 0.3384 | 2.02554 | 0.8543 | I.0774 | 0.9069 |
| 4 | 0.08002 | 5.4849 | 2.7656 | I.4921 | I.3884 |
| 5 | 0.09999 | I.0701 | 2.3252 | I.5372 | 0.1946 |
| 6 | 0.09167 | 2.5472 | I.9941 | 2.4490 | 0.3505 |
| 7 | 0.00965 | 82.404 | -3.8520 | 12.615 | 11.703 |
| | | | kT = 5 + 2$\pi$ | | |
| I | -0.03770 | -0.6053 | 0.8963 | 0.9756 | 0.9960 |
| 2 | 0.08725 | -4.0183 | 0.04544 | I.0256 | 0.9012 |
| 3 | 0.1389 | I.7423 | I.4104 | I.1857 | 0.2191 |
| 4 | 0.01641 | -5.6684 | 0.4188 | 4.4308 | 0.2732 |
| 5 | 0.01945 | -9.3060 | -9.4807 | 4.9851 | 2.0925 |
| 6 | 0.02184 | 0.6062 | 2.1323 | -8.0998 | -3,0714 |
| 7 | 0.00752 | 33.524 | 8.7131 | 19.6676 | -I.1330 |



Table 2

k = 20, kT = 5, t = 30 (logs are base 10)

| | | | | | |
|---|---|---|---|---|---|
| $\log(\Delta\Psi)$ | 0.8 | 0.48 | 0 | -0.52 | -1.0 |
| $\log W_l$ | -0.02 | -0.26 | -1,1 | 2.13 | -3.1 |

## Figure Captions

Fig. 1. The dependence of the time of quantum limitation of diffusion $t^*$ on a parameter k. The points stand for experimental values, two straight lines correspond to a linear interpolation (slope $\alpha$ = 1,5) and to the theoretical formula (2.7) (slope $\alpha$ = 2); logs are base 10.

Fig. 2. The dependence of quantum correlations $R$ (see (2.9)) on $\tau$ for the system (2.1) at k = 5, kT = 5, t = 100, $\tau$ = 1024.

Fig. 3. The same as in Fig. 2 for k = 40, kT = 5, t = 0, $\tau$ = 128.

Fig. 4. The time dependence of the energy of a classic rotator (2.2) for the case of reversion of the motion at moment t = 150; the motion of the system is non-reversible (kT = 5).

Fig. 5. The time dependence of the quantum rotator energy (2.1) for the case of reversion of the motion and at a random variation of the phase of amplitudes $A_n$ in the interval $\Delta\varphi$ = 0.1 at the moment of time t = 150; the motion of the quantum system is entirely reversible (k = 20, kT = 5); the straight line corresponds to the classic diffusion (2.4), the vertical line corresponds to the moment of time reversion.

Fig. 6. The distribution function over the unperturbed levels of the system (2.1) in normalized coordinates (see section 2) at the moment of return t = 300, the straight line corresponds to the classic distribution (2.5), the broken line stands for the numerical result

Fig. 7. The dependence of $\langle n_2^2 \rangle$ on time for the system (3.1) with k = 5, T = 1, $\omega$ T = 1.0, $\varepsilon$ = $10^{-5}$, t = 400. The straight line corresponds to the classic diffusion (3.5).

Fig. 8. The same as in Fig. 7 for $\omega$ T = 0.

Fig. 9. The same as in Fig. 7 for $\omega$ T = 1.27.



Fig. 10. The same as in Fig. 7 for $\omega$ T = 2.5, t = 2000.

Fig. 11. The time dependence of the rotator energy for the system (2.1) with $k(\tau) = k + \varepsilon\cos\omega_0\tau$ at k = 0, $\varepsilon$ = 7, $\varepsilon T$ = 7, $\omega_0$T = 2. The straight line corresponds to the classic diffusion (4.1).

Fig. 12. The distribution function over the unperturbed levels of the system (2.1) with $k(\tau) = k + \varepsilon\cos\omega_0\tau$ in normalized coordinates (see section 2) ) for the values in Fig. 11 at the time moment t = 1000, the straight line "a" corresponds to the classic distribution (2.5) with $k = k_{ef}$, the straight line "b" denotes the linear interpolation, the broken line is the experimental result.

Fig. 13. The same as in Fig. 11 for k = 0, $\varepsilon$ = 7, $\varepsilon T$ = 7, $\omega_0$T = 0.1.

Fig. 14. The same as in Fig. 11 for k = 0, $\varepsilon$ = 3.5, $\varepsilon T$ = 7, $\omega_0$T = 2.

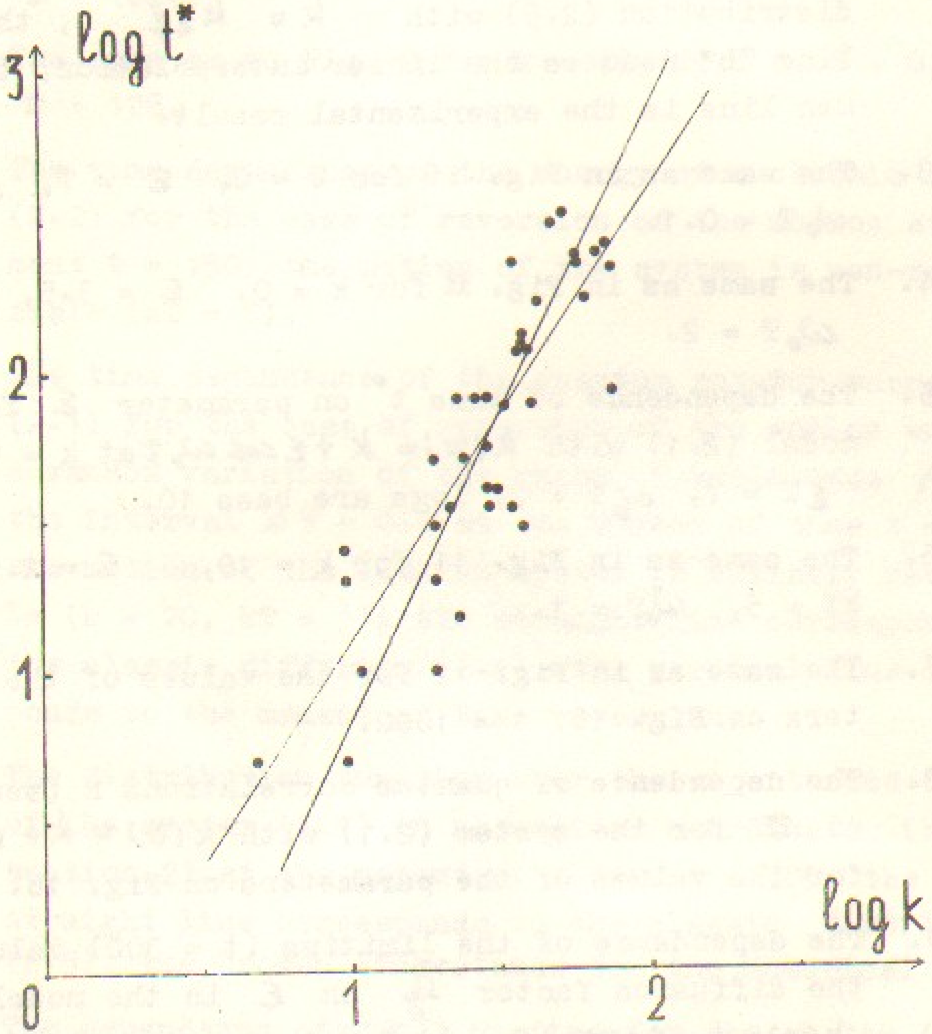
Fig. 15. The dependence of time $t^*$ on parameter $\varepsilon$ for the model (2.1) with $k(\tau) = k + \varepsilon\cos\omega_0\tau$ at k = 0, $\varepsilon$T = 7, $\omega_0$T = 2; logs are base 10.

Fig. 16. The same as in Fig. 11 for k = 10, $\varepsilon$ = 2.5, kT = 5, $\omega_0$T = 1.

Fig. 17. The same as in Fig. 12 for the values of the parameters on Fig. 16, t = 1000.

Fig. 18. The dependence of quantum correlations R (see (2.9)) on $\tau$ for the system (2.1) with $k(\tau) = k + \varepsilon\cos\omega_0 T$ for the values of the parameters on Fig. 16.

Fig. 19. The dependence of the limiting (t = 300) value of the diffusion factor $\overline{D}_q$ on $\varepsilon$ in the model with $k(\tau) = k + \varepsilon\cos\omega_0\tau$, $\omega_0$T = 1, 0 - k = 10, x - k = 20; and in the model with random perturbation: $k(t) = k + \varepsilon\xi(t)$, • - k = 10, + - k = 20; kT = 5.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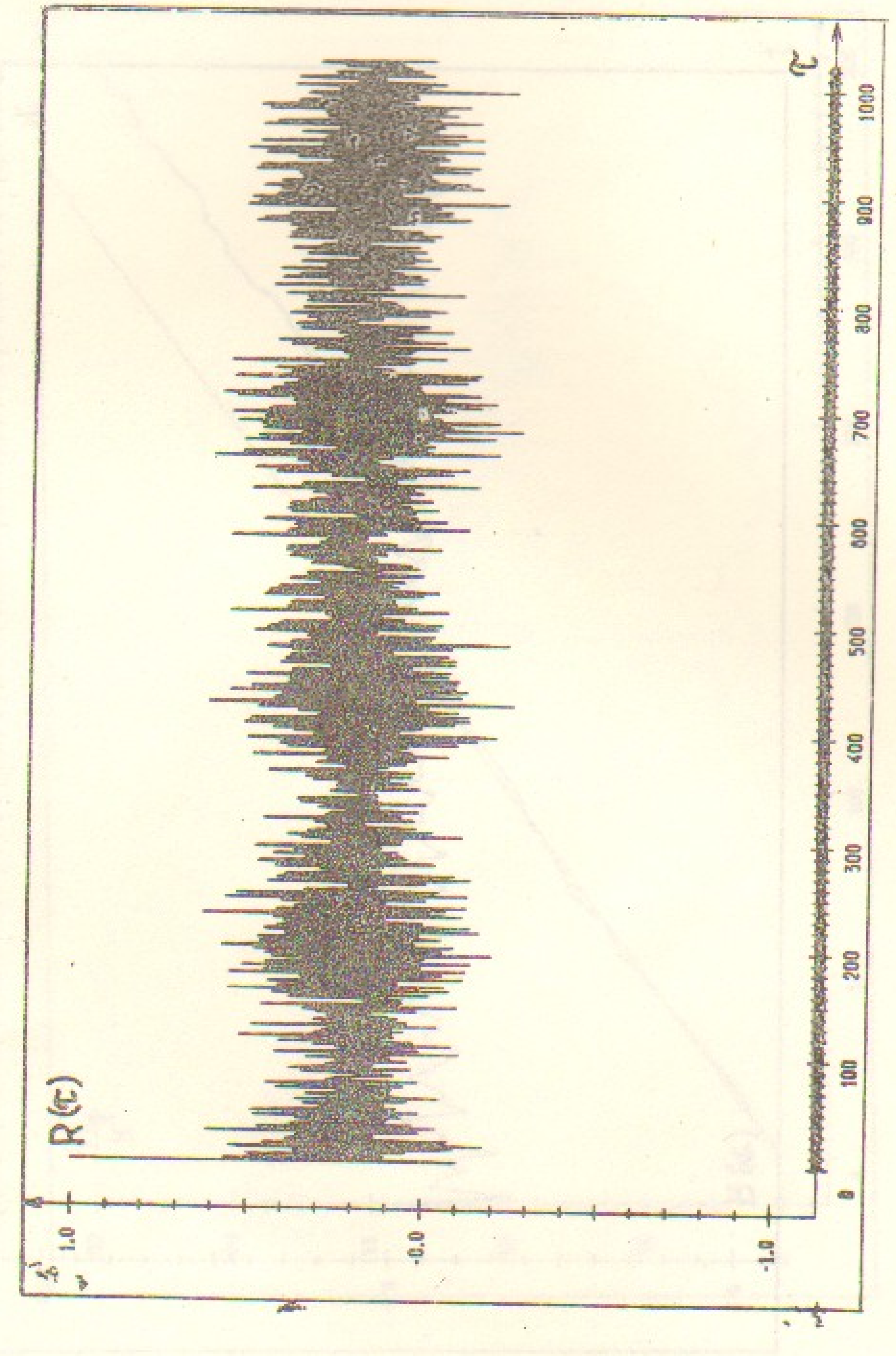
1
0
1
2
log k

Fi, 1.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-1.0

Fig. 2.

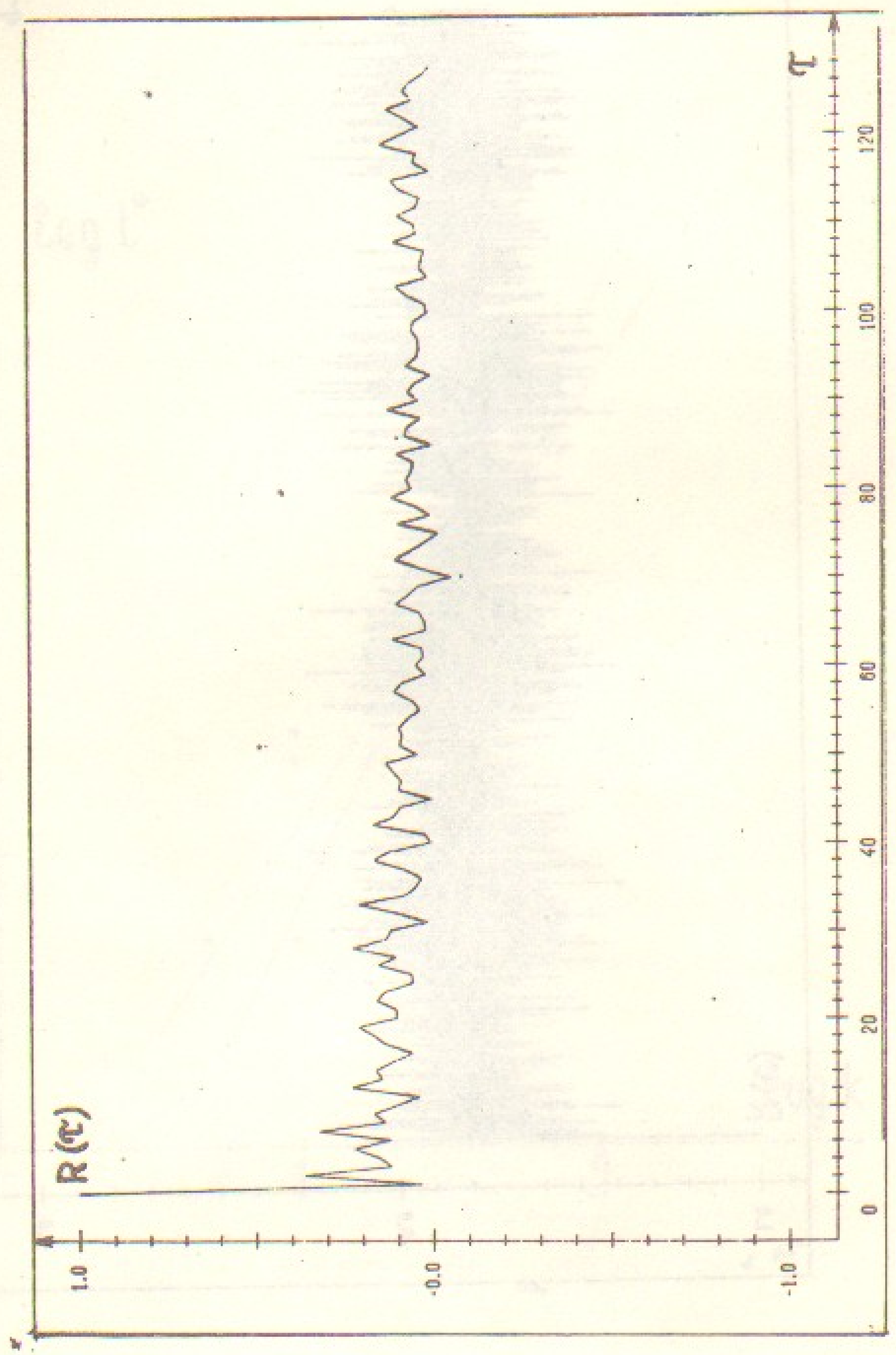
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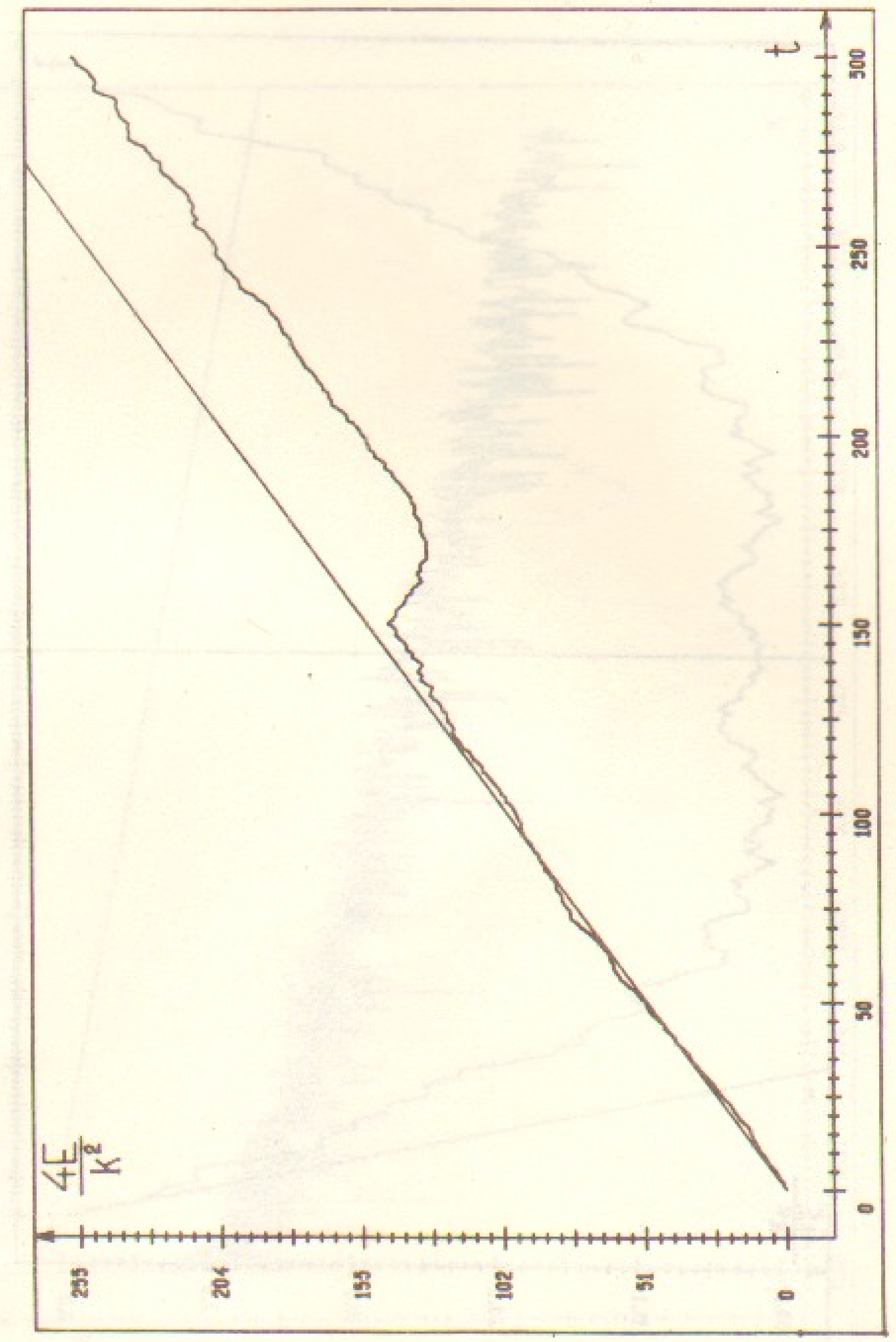
Fig. 3.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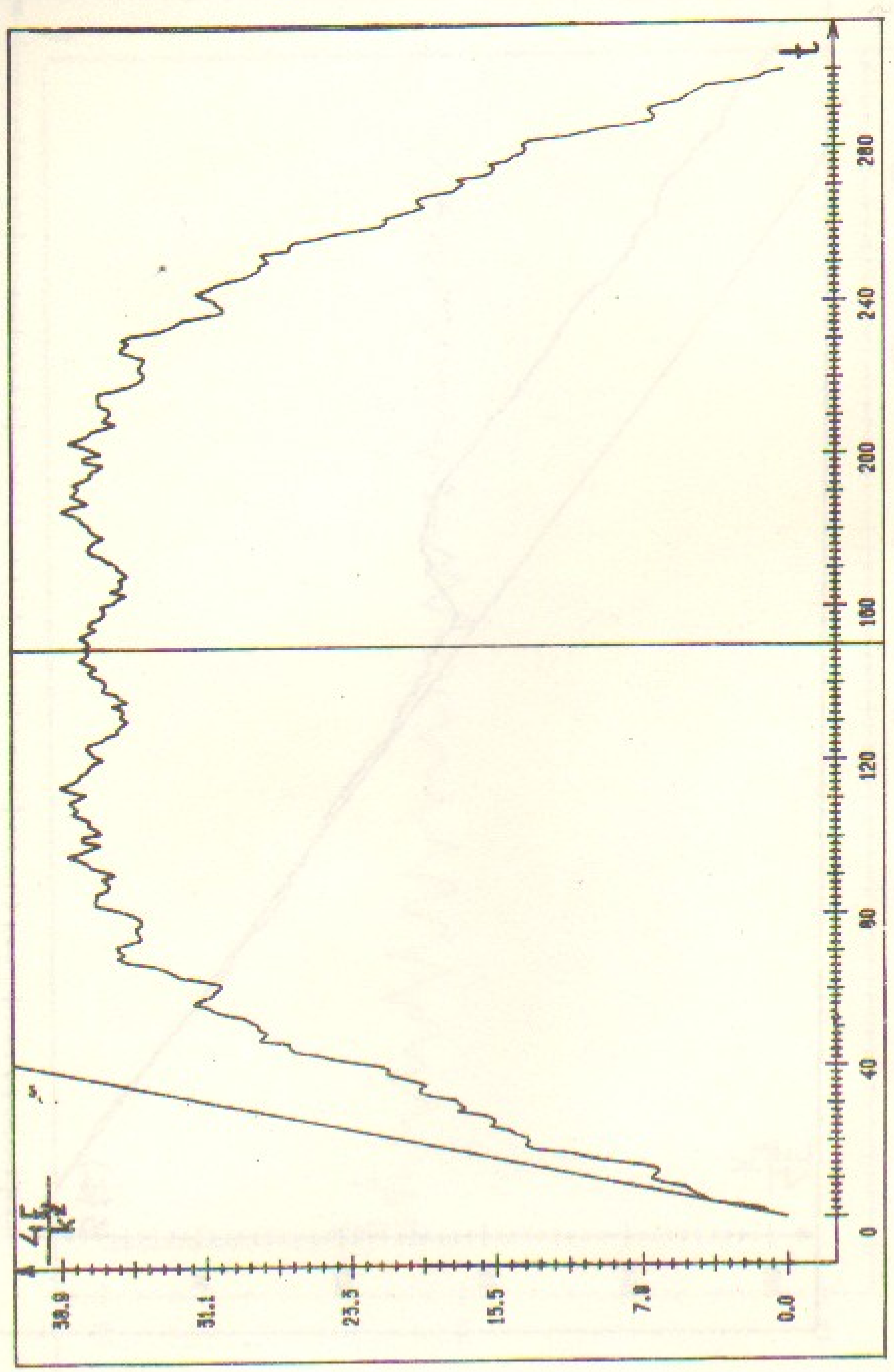
153
204
255
0
50
100
150
200
250
300

Fig. 4.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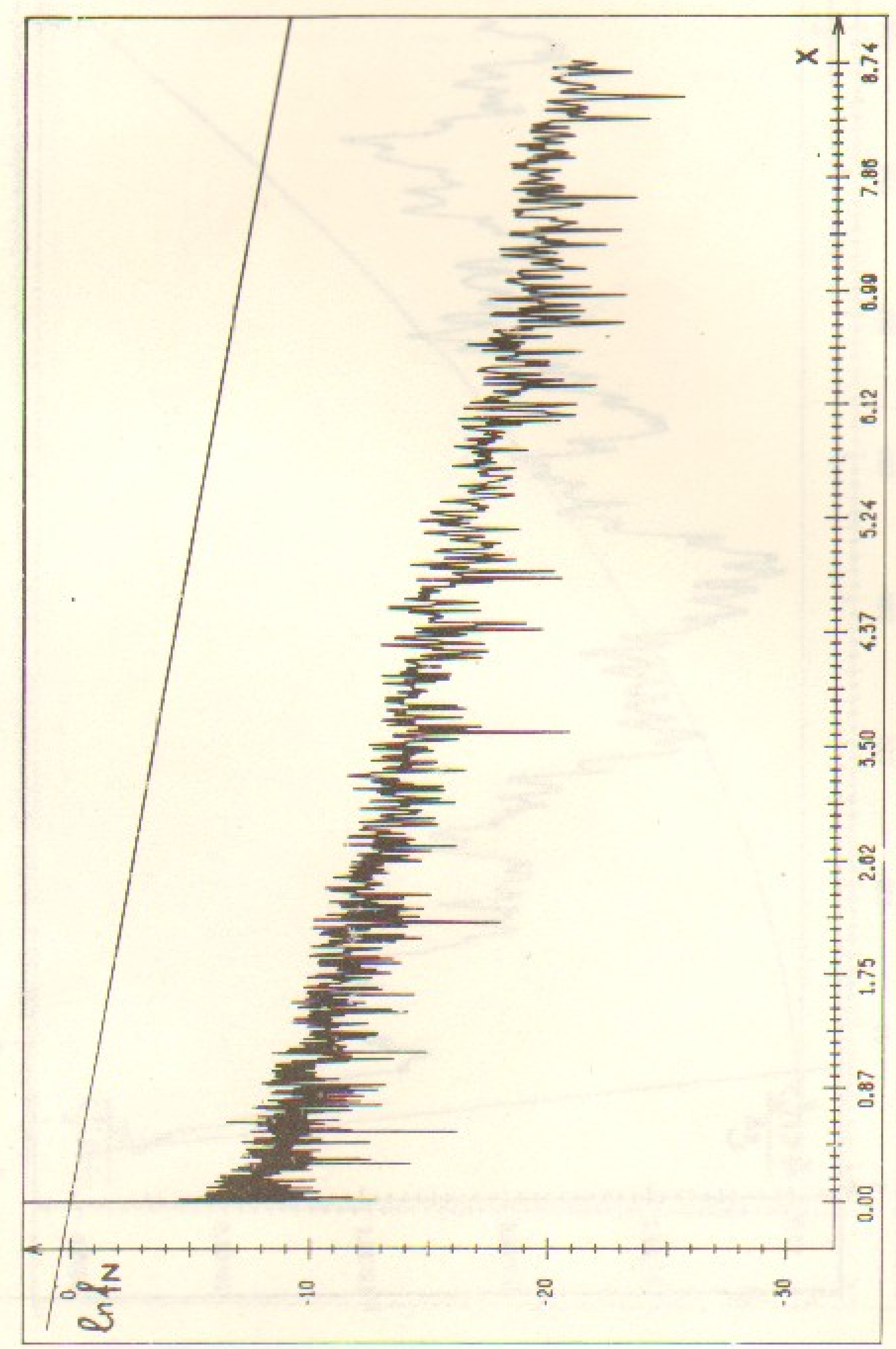
38.9

Fig. 5.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-10
-20
-30

Fig. 6.

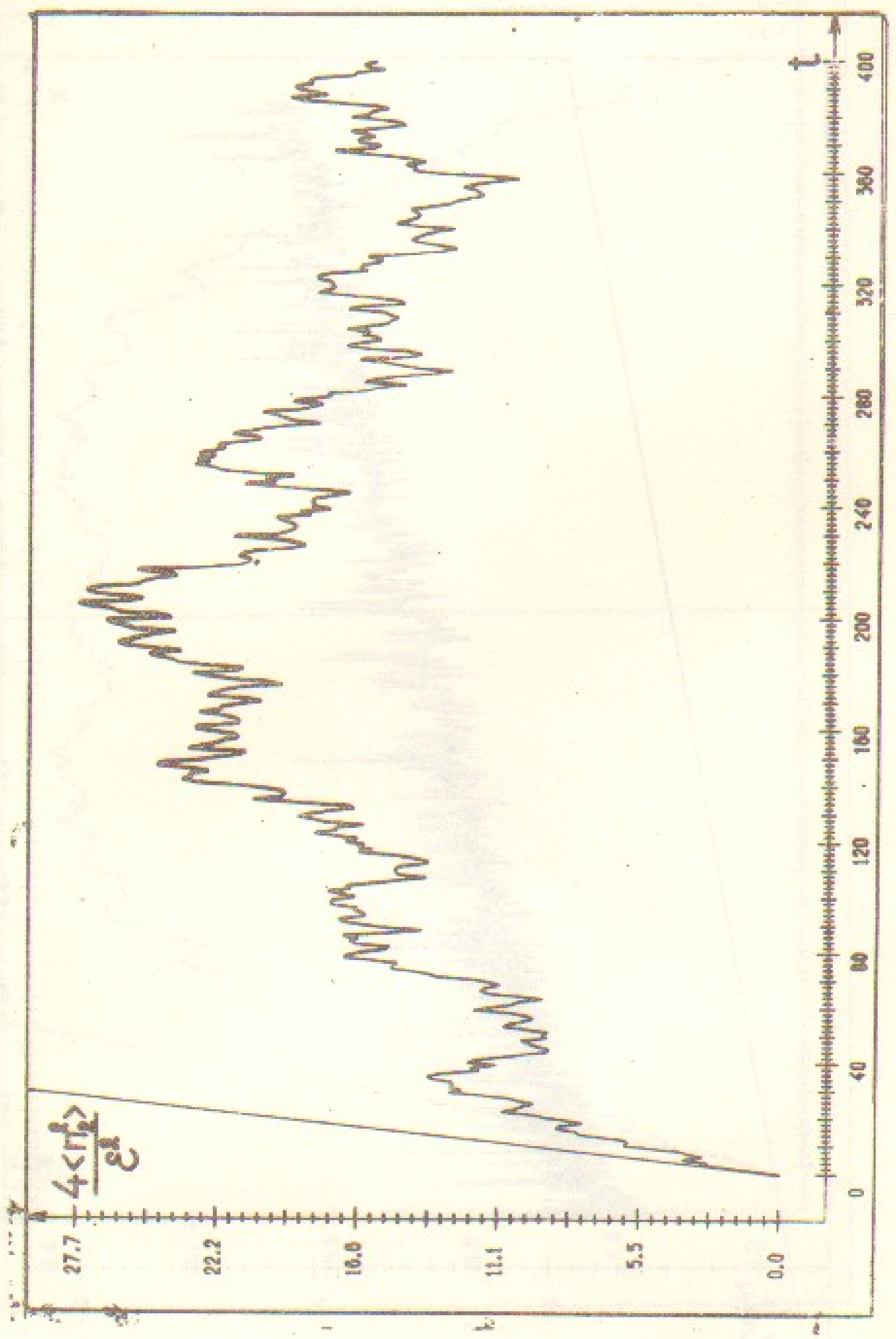
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Fig. 7.

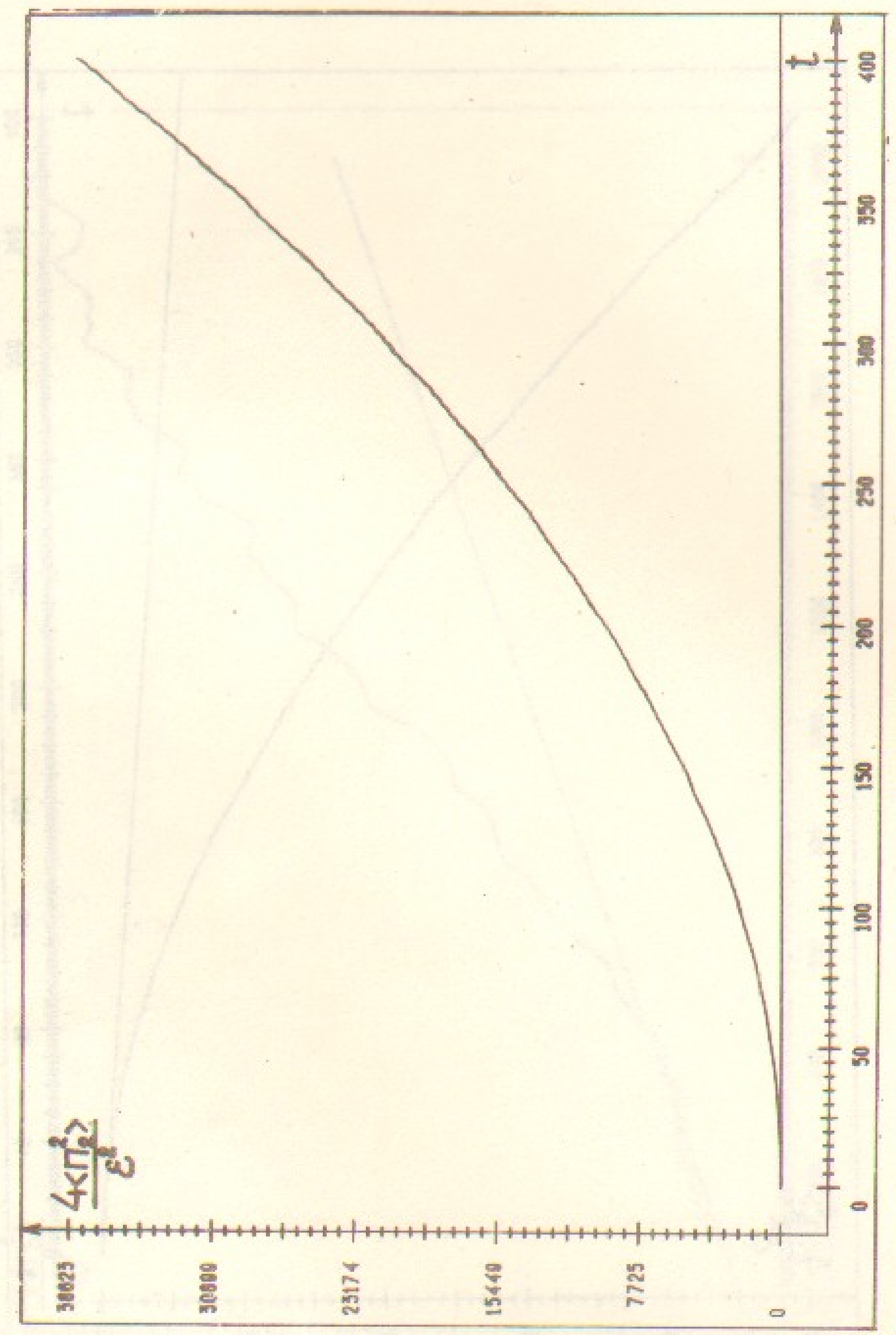
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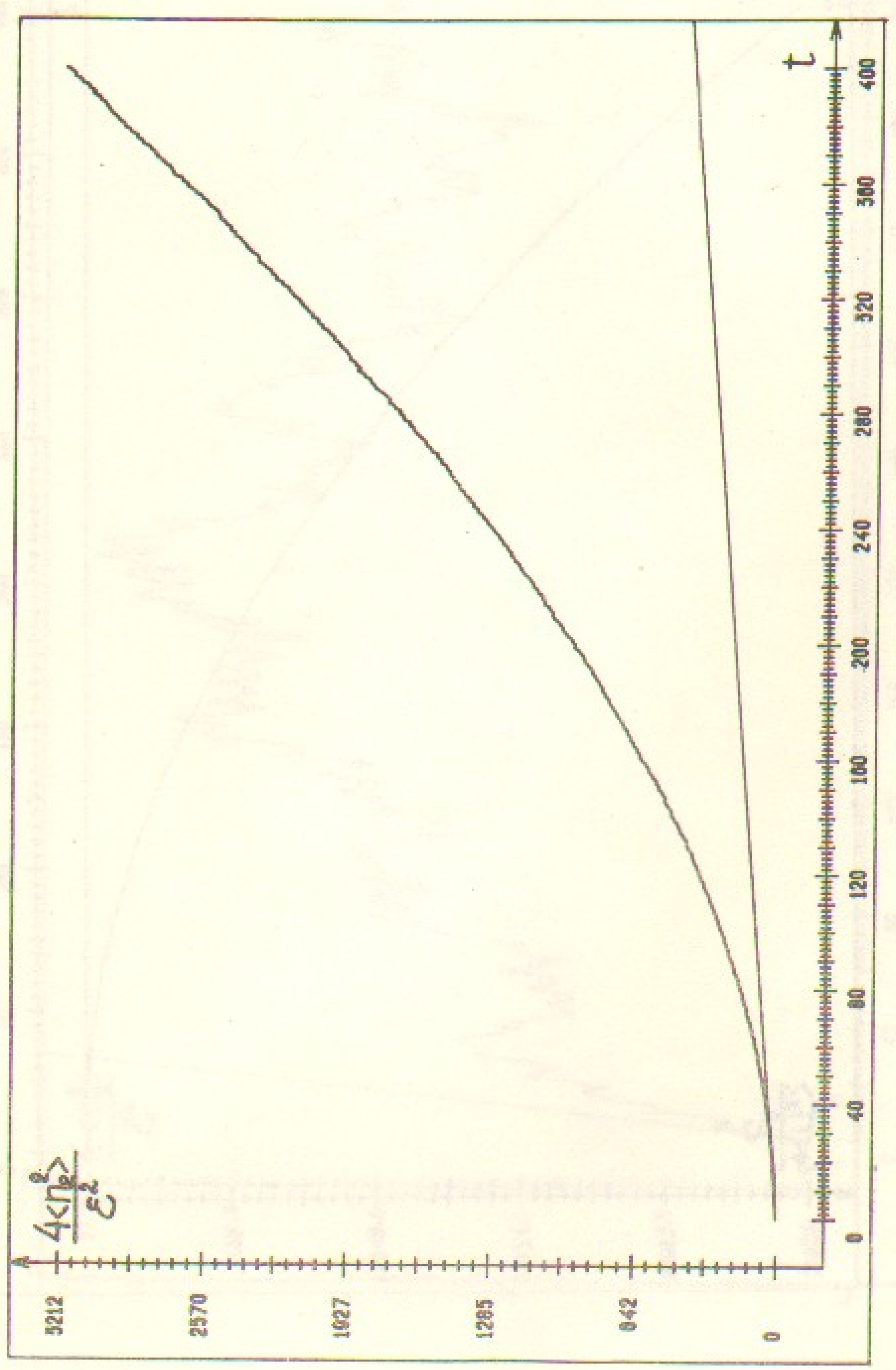
Fig. 8.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1927
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642
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t
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40
80
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160
200
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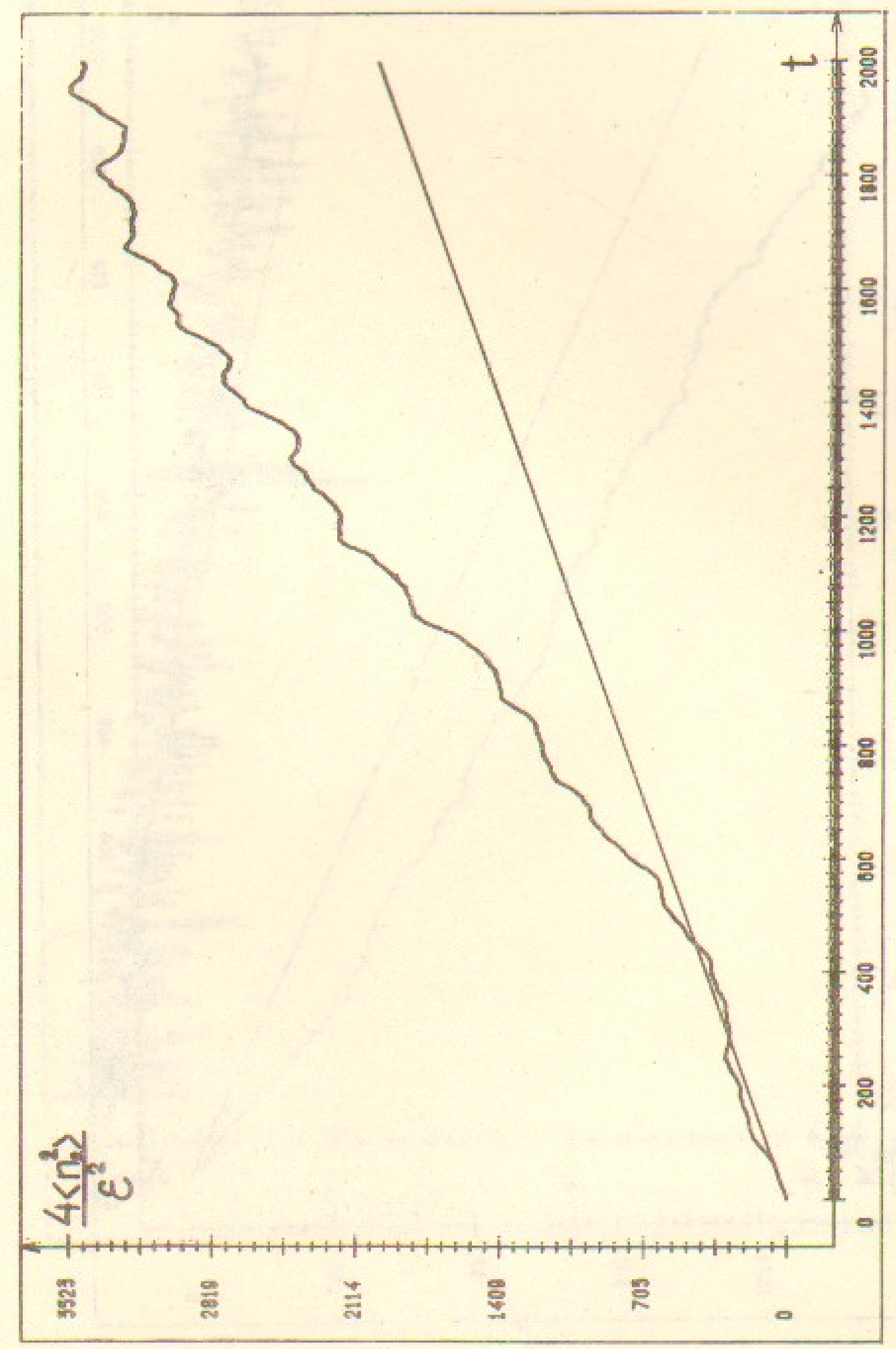
280
320
360
400

Fig. 9.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800
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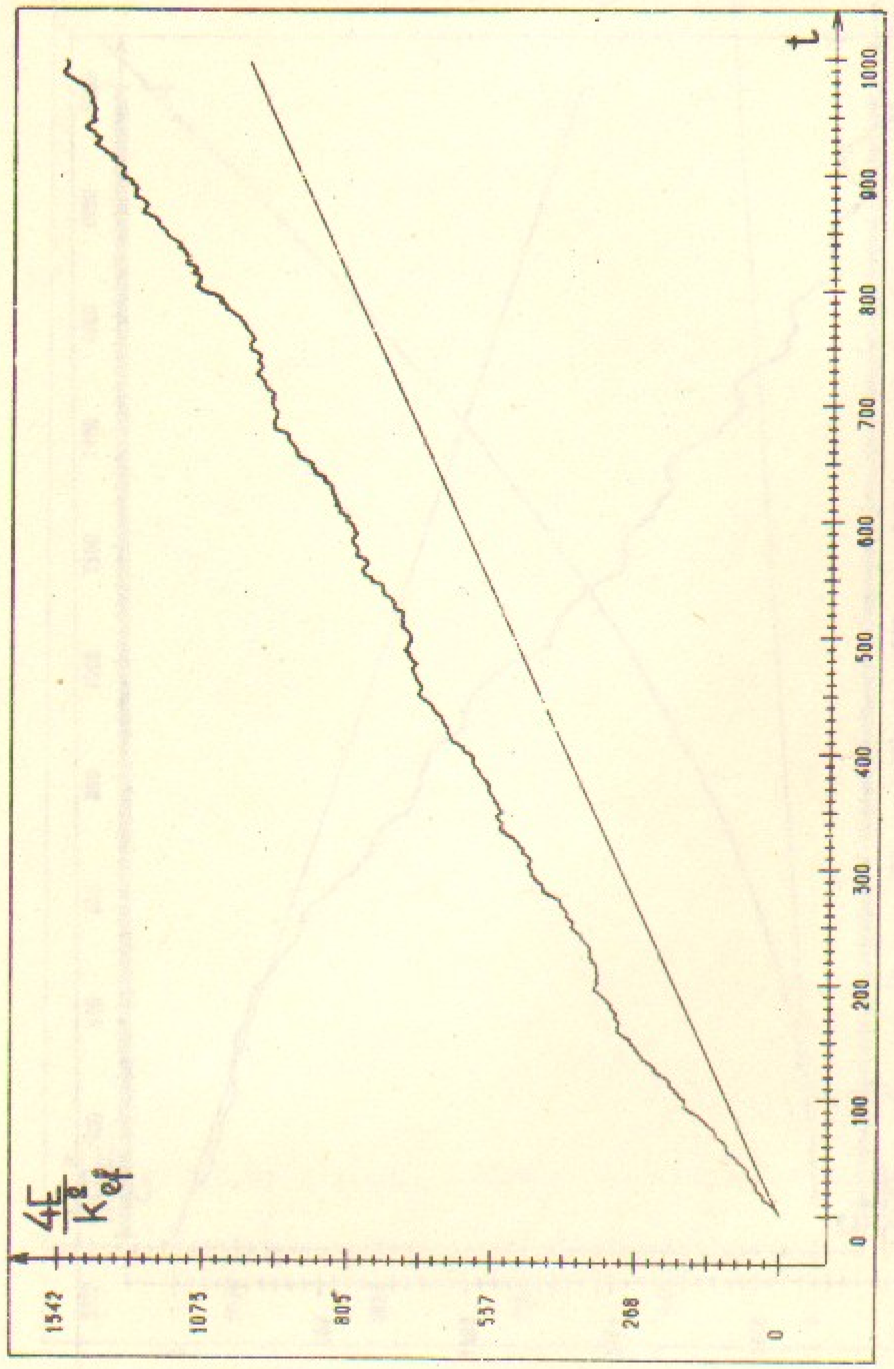
1200
1400
1600
1800
2000

Fig. 10.



$\frac{4E}{k_{ef}^2}$
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200
300
400
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600
700
800
900
1000
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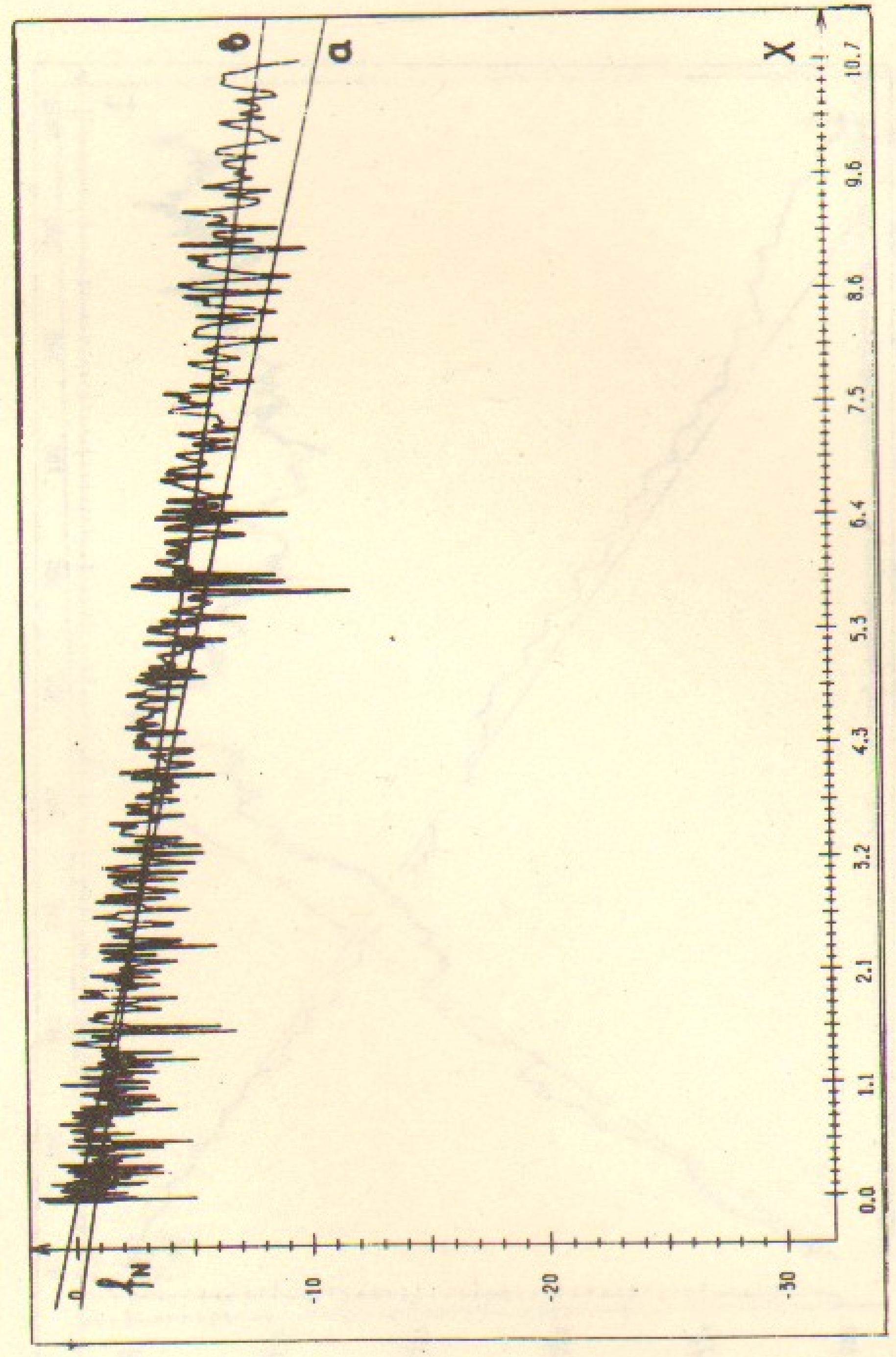
1073
805
537
268


Fig. 11.



X
a
б
$f_N$
0.0
1.1
2.1
3.2
4.3
5.3
6.4
7.5
8.6
9.6
10.7
0
-10
-20
-30


Fig. 12.

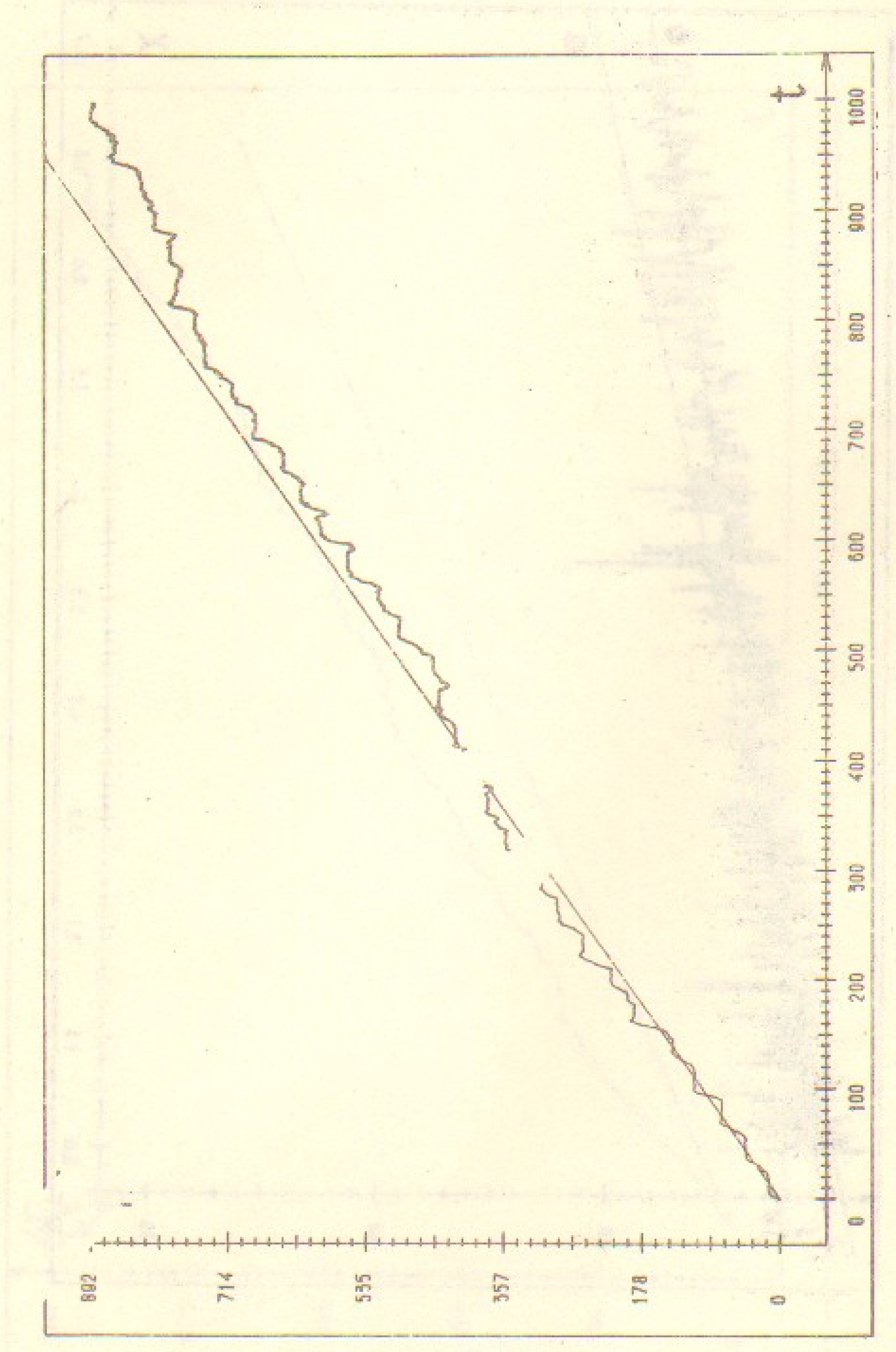

t
0
100
200
300
400
500
600
700
800
900
1000
0
178
357
535
714
892

Fig. 13.

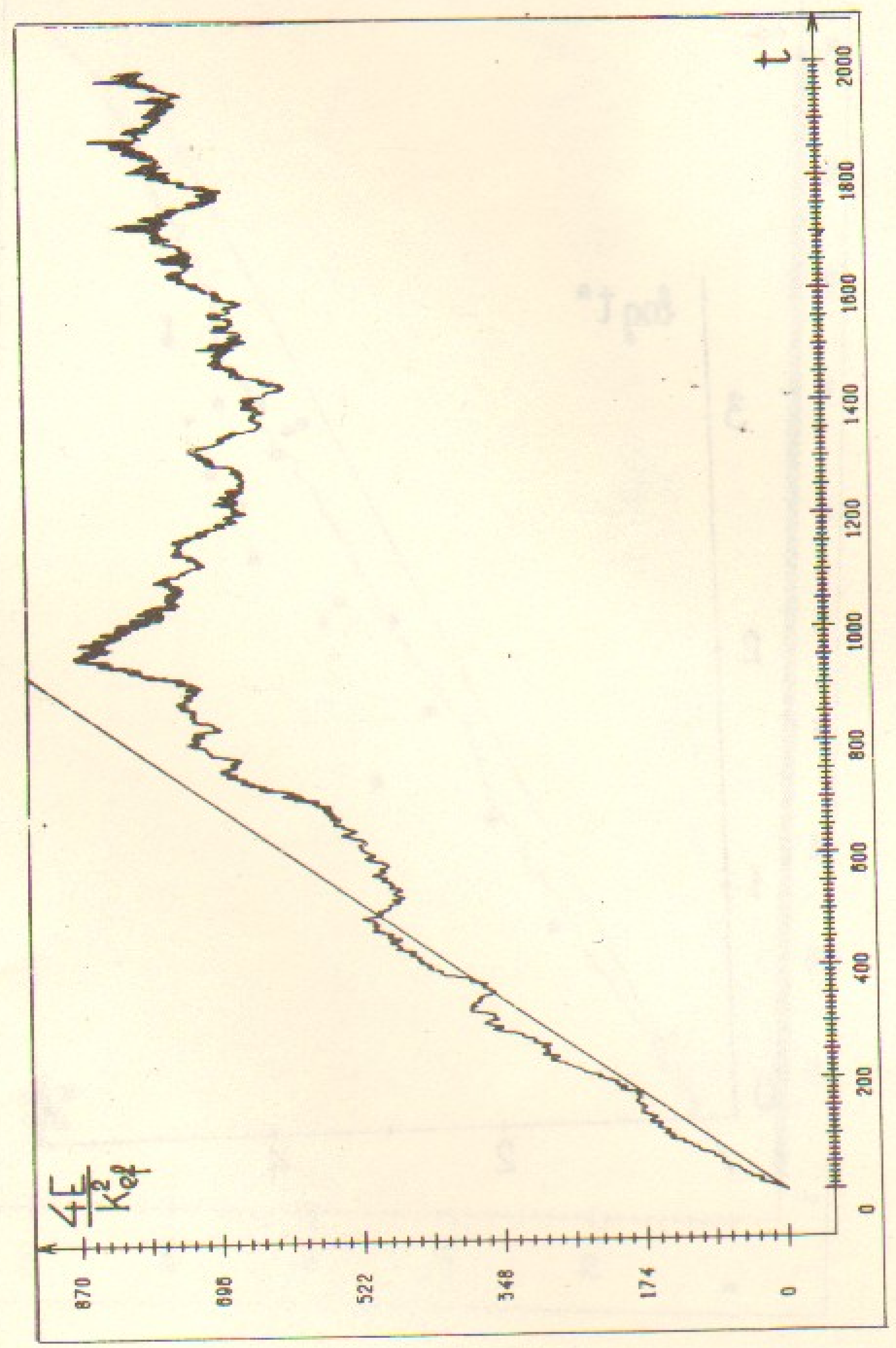

t
0
200
400
600
800
1000
1200
1400
1600
1800
2000
0
174
348
522
870

Fig. 14.

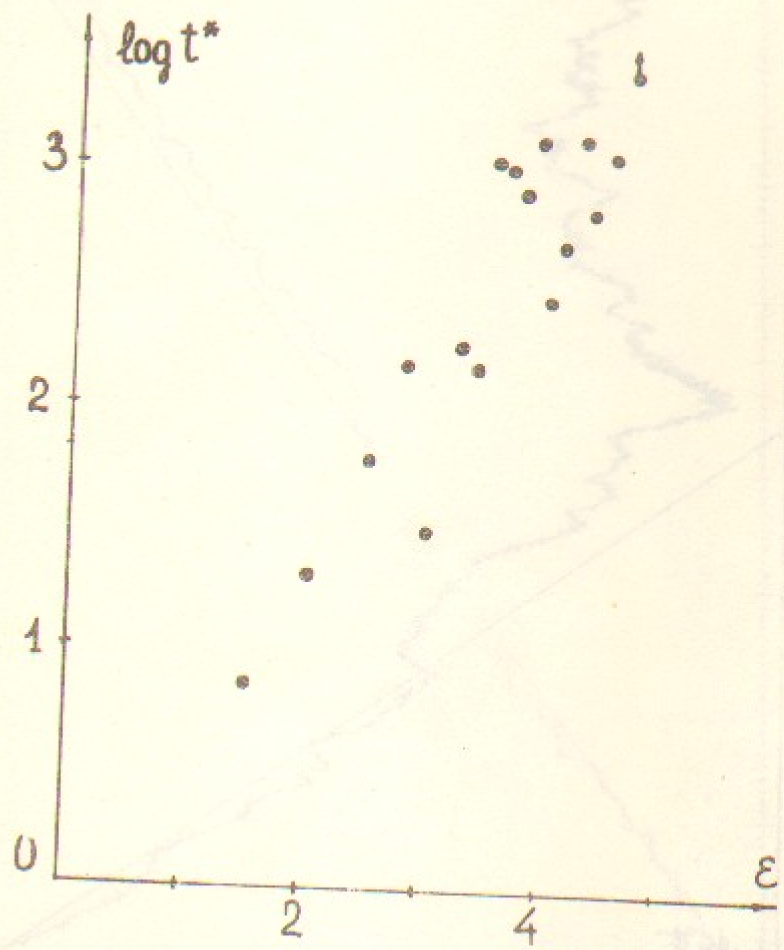

log t*
3
2
1
0
2
4
ε

Fig. 15.

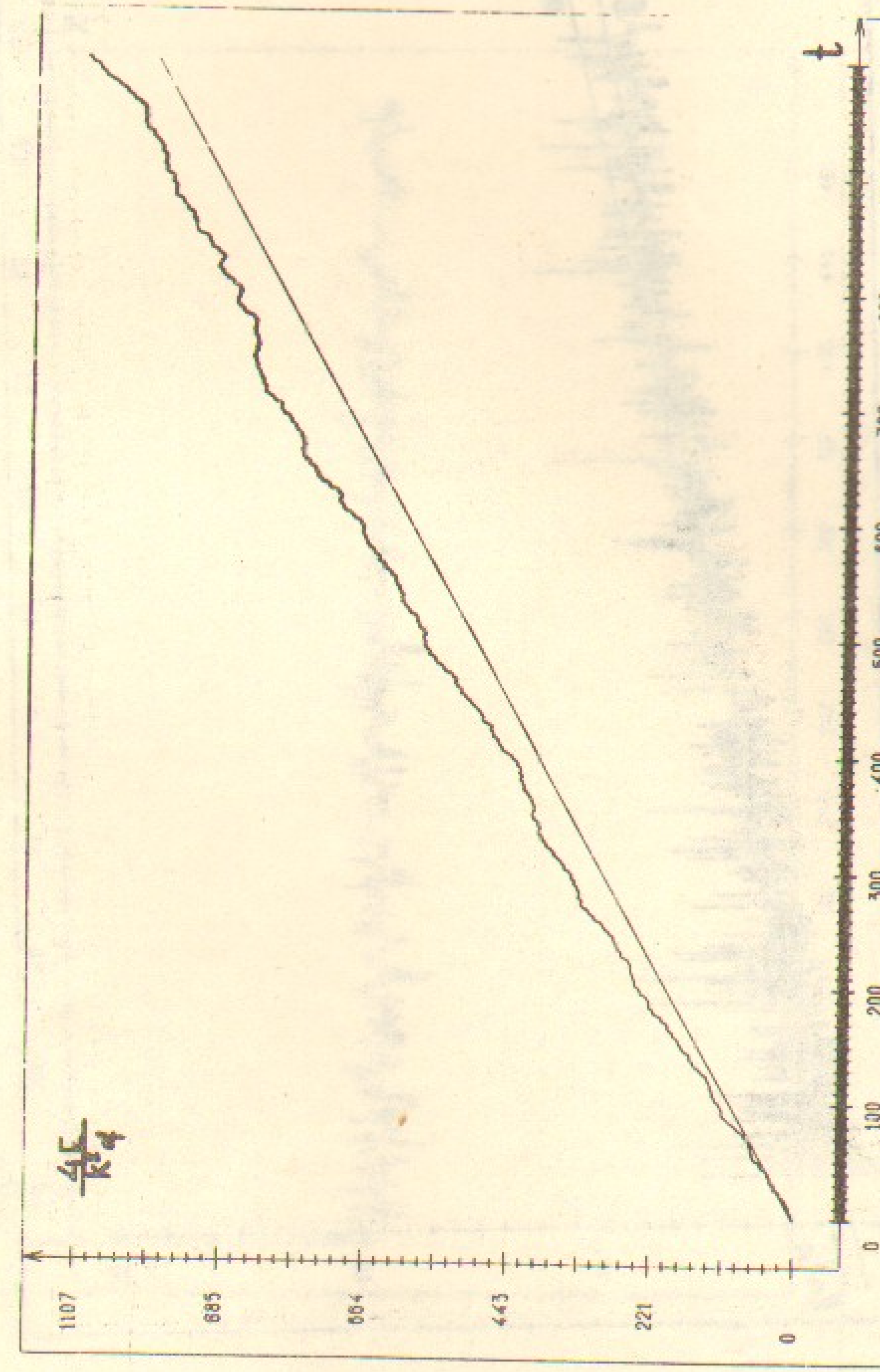

t
0
100
200
300
400
500
600
700
800
900
1000
1107
885
664
443
221
0

Fig. 16.

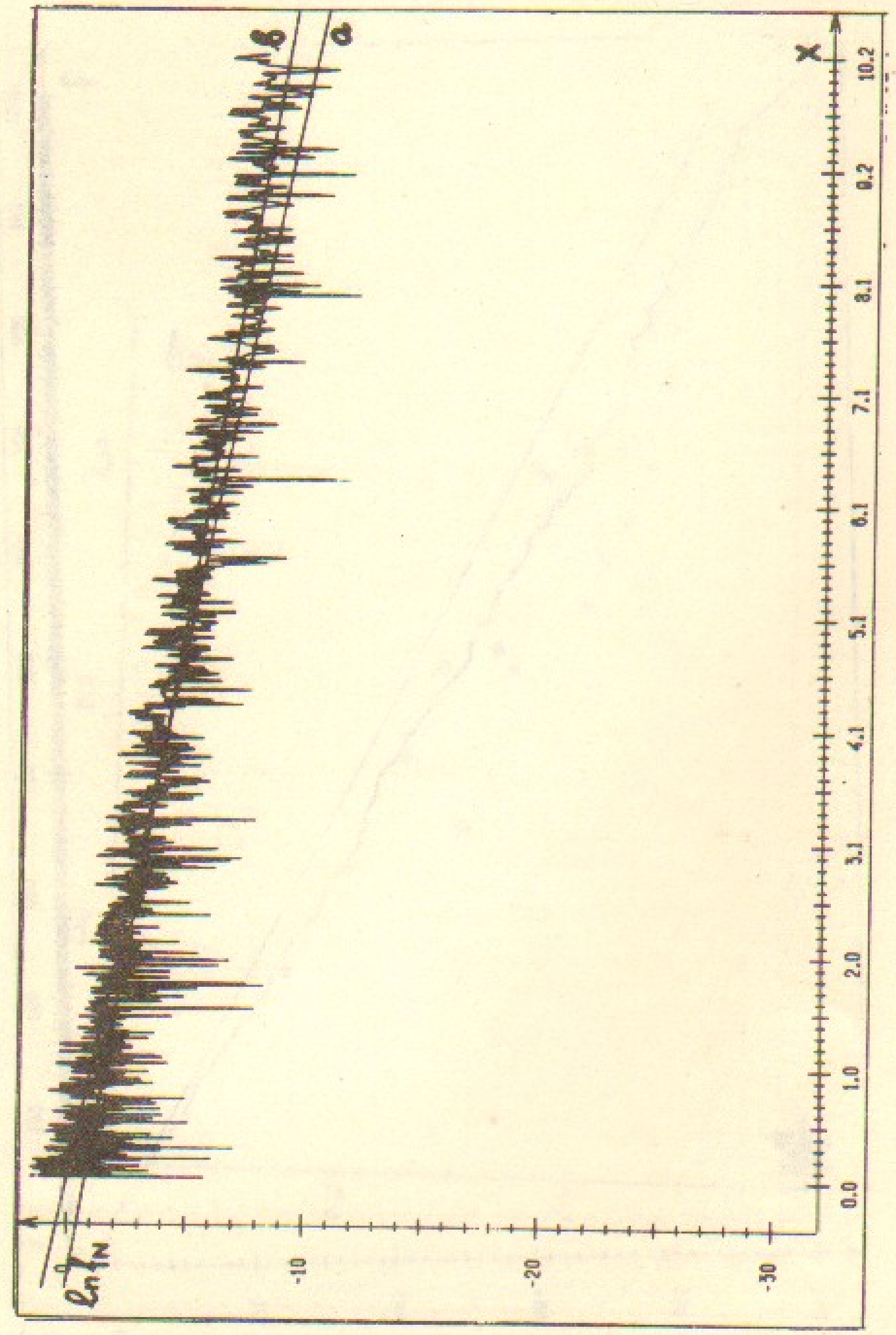

X
0.0
1.0
2.0
3.1
4.1
5.1
6.1
7.1
8.1
9.2
10.2
-10
-20
-30

Fig. 17.

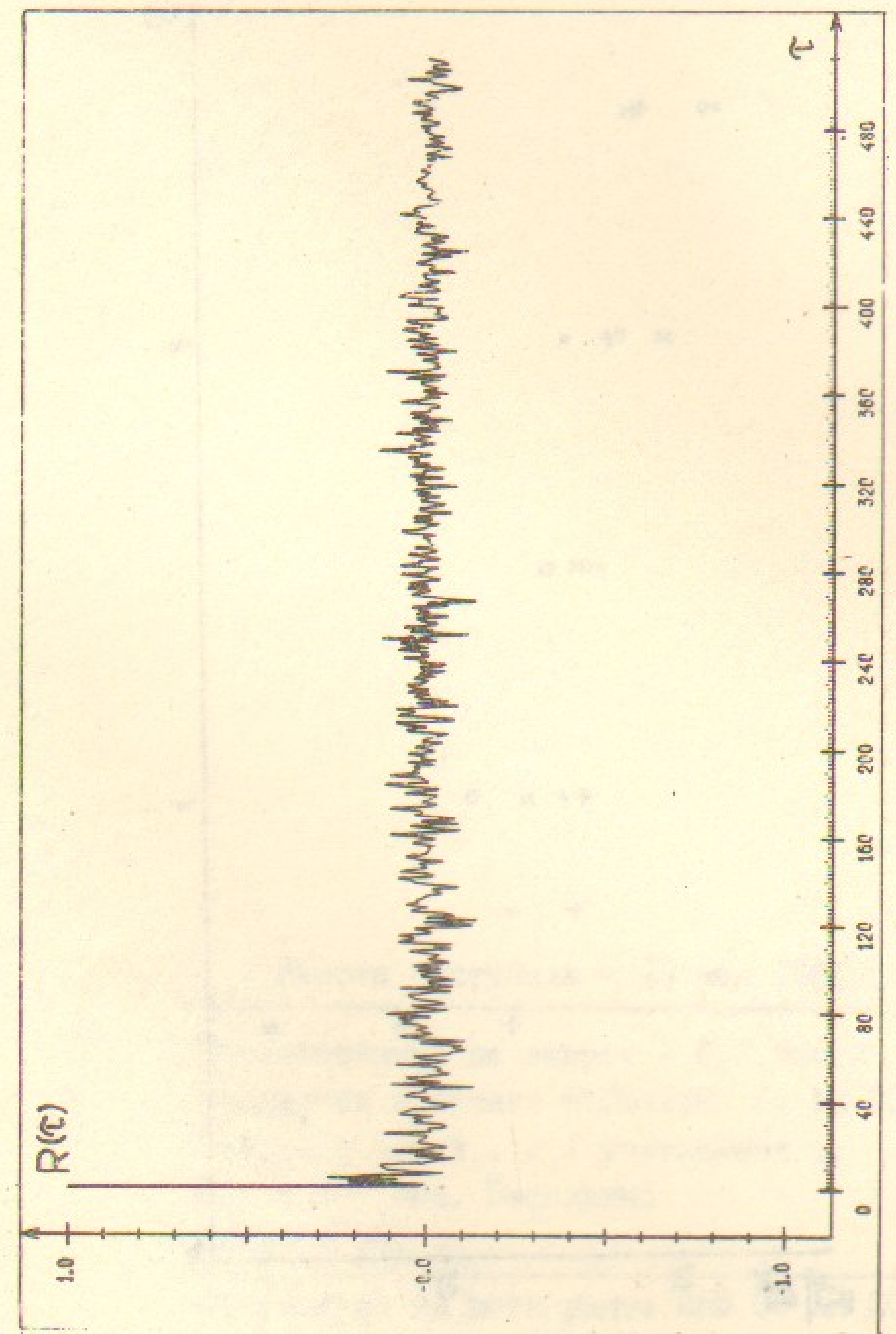

R(τ)
τ
0
40
80
120
160
200
240
280
320
360
400
440
480
1.0
0.0
-1.0

Fig. 18.

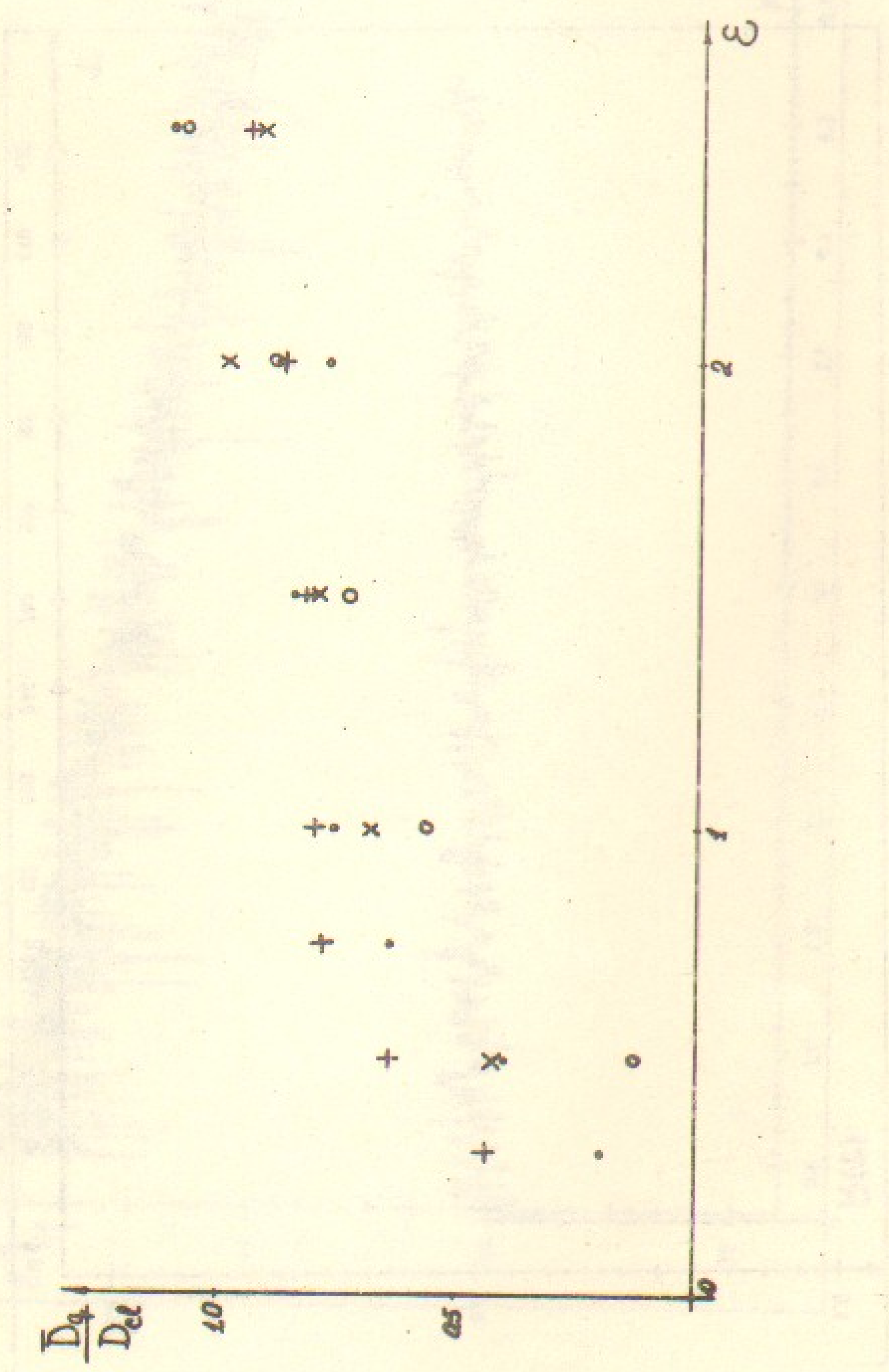


**Fig. 19.**

Работа поступила - 29 мая I98I г.

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