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ON THE APPLICABILITY LIMITS OF PARTON AND HYDRODYNAMICAL DESCRIPTIONS IN HADRONIC COLLISIONS

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A b s t r a c t

The complementarity of parton and hydrodynamical descriptions is considered. The former is prescribed to the hard scattering of constituents of primary hadrons at the initial stage of hadron collision. The latter describes the subsequent multiple interactions between the comparatively soft constituents. The presented hydrodynamical picture agrees with data in many detailes. Simple considerations allow us to estimate the temperature of the phase transition from quark-gluon plasma to hadronic phase directly from the spectra of secondaries. The analysis of the available data (on the basis of simple model of hydrodynamical expansion) leads to the conclusion that up to highest ISR energies the space-time picture of the hadronic collision is rather similar to that of Landau model. Finally we dicuss the picture of hadronic collisions expected at very high energies.

1. INTRODUCTION

The existing multiparticle production models may be divided into two classes, depending on how "strong" the interaction is assumed to be. Historically the first class is mainly presented by the thermodynamical approach. It treats the interaction to be strong enough to "mix" the produced hadronic system up to the local thermal equilibrium (LTE) before the system "breaks up" into secondaries (see, e.g., reviews /1,2/ and references therein) In the most extreme (and earliest) Landau's hyprodynamical model /3/ the mixing is assumed to be so fast that the thermo- and hydrodynamical descriptions are applicable right from the beginning of the collision.

Another class includes the models based on the assumption that the effective "strong" coupling constant rapidly falls off with the increase of momentum transfer (at small distances). It should be mentioned here a variety of multiperipheral /4/, multireggeon /5/ and parton /6/ models. The latter being the most popular now have been considerably developed /7-9/ on the basis of the modern theory of strong interactions, the quantum chromodynamics (QCD). Keeping in mind mainly this parton model, we call such an approach the "parton description".

Now it becomes more and more clear that both the parton and thermodynamical descriptions are not incompartible but correspond to different stages of hadronic collision /10,11/.

A good agreement /1,2/ of thermodynamical predictions with the experimentally observed composition and $p_\perp$-distribution of softly produced hadrons ($p_\perp \lesssim 1$ GeV/c) ensure the LTE with the temperature $T_f \sim m_\pi$ at least during the system "break up" into secondaries.

Interpretation of hadronic spectra with higher $p_\perp$ as a "leakage" /12-14/ from the system at earlier stages of hydrodynamical expansion considerably extends the field of thermodynamical description.[1] This idea has been developed on the basis of Landau's space-time picture of hydrodynamical expansion and the lowest order QCD estimates of elementary processes in quark-gluon plasma. It provides rather good description of the yields of γ-quanta, dileptons, hadrons up to $m_\perp \equiv \sqrt{p_\perp^2 + M^2} \sim 4$ GeV/c^2 /10/. The fact that the agreement is not accidental in this region is supported at least by two observations /11/: (i) the coincidence of different estimates of the effective particle production temperature; (ii) the universality conditions /11/ are well confirmed by data. For higher $m_\perp$, where such "universality" is strongly violated, data agree with the predictions of the parton model obtained recently /7-9/ on the basis of QCD.

Thus the question on the compatibility of parton and thermodynamical descriptions appears to be quantitative rather than qualitative and it can be reduced to the division of hadronic collision into two successive stages /10,11/. Note, that such a division is quite adequate to the phenomenon. In fact, due to the "asymptotic freedom" of strong interactions the parton description seems to be the proper approach to describe the hard scattering of constituents, which proceeds at small space-time distances at the beginning of the hadronic collision. In contrast,

1) It should be noticed that the possibility to study earlier stages of the thermodynamical expansion has been first discussed in Ref.15, where the thermodynamical production of lepton pairs and γ-quanta has been considered long before the corresponding experimental data.

during the expansion the more and more soft processes take place, the effective strong coupling constant becomes larger, and the multiple interactions become more and more essential. They lead to the "mixing" and reaching of LTE and hydrodynamical expansion.

This paper is devoted to further developments of the thermodynamical picture and its detailed comparison with the current data. We also pay special attention to the predictions near the applicability limits and, if possible, compare them with those of the parton model. Schematically the thermodynamical picture consists of two rather independent parts: (i) the problem of space-time evolution of the (hydrodynamical) system and (ii) the problem of system decay into secondaries (formation of their spectra and composition). Accordingly, the paper can be divided into two parts. In the first one (sects. 2-4) we consider the formation mechanisms of the spectra of secondaries and also other phenomena rather independent of the details of space-time evolution. The space-time evolution and related phenomena will be considered in sects. 5-7, where we formulate some simple model which is the interpolation between the Landau model /3/ and Feynman parton picture /6/.

The main hadron production mechanism at the hydrodynamical stage appears to be the "leakage" of particles during the expansion /10-14/ and, of course, the final "break up" at the end of expansion, when the system cools down to the final temperature $T_f \sim m_\pi$ /3,16/. Although these mechanisms have been treated so far as the independent ones, it should be stressed that they correspond to the same decay process described by the same expression:

$$E\frac{dN}{d^3p} = \frac{g}{(2\pi)^3}\int \frac{d\sigma_\mu p^\mu}{\exp(\frac{p^\mu u_\mu}{T}) \pm 1} \qquad (1)$$

where g is the statistical weight of the produced particle, u_μ, T are the local 4-velocity and temperature respectively, which correspond to the element $d\sigma_\mu$ of space-time hypersurface, where the system turns into free particles. The space-time region of the hypersurface describes the usual "break up" at the end of expansion at $T=T_f$. The effective temperature of particles should be the same : $T_{eff} \approx T_f$, independent of their kinds and "transverse mass" $m_\perp$.

The time-like hypersurface describes the "leakage" ("evaporation") of particles at earlier stages of expansion with higher temperature $T > T_f$.[2] The effective production temperature of "evaporated" particles must grow with their $m_\perp$. One should expect that at $T \gtrsim T_c$, where T_c is the temperature of the phase transition of hadronic matter into the quark-gluon plasma, the "evaporation" of real hadrons is changed to the "evaporation" of quarks and gluons. The spectra of produced particles should be proportional to their statistical weights (or quark/gluon factors to fragment into the observed particle) and are determined by the same hypersurface σ_μ.

The analysis of data (sect.2) confirms the existence of all the mentioned above production mechanisms and allows the division of the region of thermodynamical description into three regions:

(i) the "soft" hadron region ($p_\perp \lesssim 0.8$ GeV/c), where the slopes of particle spectra correspond to the same production

2) Our comparison /11/ of the effective production temperature of hadrons with that of dileptons produced by the whole system volume shows that T does not differ from that averaged over the volume.

temperature $T_{eff} \approx 0.13$ GeV (the final "break up");

(ii) the "leakage" of hadrons ($p_\perp > 0.8$ GeV/c, $m_\perp \lesssim 2$GeV/c^2), where the ratio $\frac{\text{baryon}}{\text{meson}} \gtrsim 1$;

(iii) the "leakage" of quarks/gluons ($m_\perp \lesssim 2$ GeV/c^2), where the baryon production is strongly suppressed.

As a result, we have an estimate of the phase transition temperature $T_c \sim T_{eff}$ ($m_\perp \sim 2$ GeV) $\approx$ 0.25-0.28 GeV.

In section 3 we show that such a subdivision is in qualitative agreement with the experimentally observed energy dependence of hadronic spectra. The considerable growth of the energy dependence at $m_\perp \gtrsim 2$-3 GeV has found a natural explanation in a customary Landau model /3/ without any additional free parameter.

In our consideration, as well as in Ref.11 we neglect the transverse hydrodynamical expansion (THE). In section 4 we show that in contrast to the longitudinal expansion a considerable THE can appear only at the end of expansion. This means that the customary estimates of THE (for instance, numerical solution of Ref.17) are unreliable because they are sensitive to unknown conditions at the boundaries of the system. The data agrees with the absence of THE.

The following sections 5-7 are devoted to the detailed study of the space-time picture of hydrodynamical expansion, in particular, its initial conditions. We formulate the model with the free parameter which is the transition moment from the parton to hydrodynamical stage of hadronic collision. Note, that with the parton stage as the initial one we avoid a lot of difficulties /18/ which are intrinsic to the original Landau model /3/.

Let us briefly discuss the formation of the initial conditions of hydrodynamical expansion. As far as the parton systems of colliding hadron overlap, their coherency is destroyed /6,19/. After that we have the spatial ordering of partons, according to their longitudinal velocity: they fly out the collision point the farther the more rapid they are /13,20/. The partons being nearest in velocities appear nearest in space and their strong interactions provide their "mixing" and reaching of LTE.

This process as well as the following hydrodynamical expansion can be considered in a more convenient "accompanying" frame with the coordinates τ, χ defined by equations

$$th\,\chi = \frac{x}{t}, \qquad \tau = \frac{t}{ch\,\chi} \tag{2}$$

where x, t are the space-time coordinates, τ denotes the local proper time in a hadronic system, χ coincides practically with the local rapidity of collective motion. (The transition from one frame of reference to another is equivalent to common shift in χ.) The initial condition is completely characterized by $\tau_0(\chi)$ (the local relaxation time) and $T_0(\chi)$ (the corresponding local temperature).

For simplicity we assume that for any group of partons which are close in their rapidities their relaxation time and their temperature are rapidity-independent:

$$\begin{aligned} \tau_0(\chi) &= \tau_0 = const \\ T_0(\chi) &= T_0 = const \\ |\chi| &\lesssim Y_m \end{aligned} \tag{3}$$

It is equivalent to the hypothesis on existence of "Feynman plato" /6/, or "frame independence symmetry" /20/. As a consequence, the formation of the hydrodynamical system is not simultaneous in

laboratory coordinates x, t, and the total formation time increases with the energy of colliding hadrons in agreement with the customary (parton) picture.

The initial rapidity interval $[-Y_m, Y_m]$, the initial temperature T_0, and the characteristic relaxation time τ_0 depend on the unknown dynamics of relaxation processes and are free parameters of our model. We call the case of $Y_m \lesssim 1$ as the "stopping" of hadronic matter, otherwise we say about the "passing through".

It should be stressed, however, that we do not consider below the "passing through" of the rather energetic valence quarks, which are responsible for the formation of leading particles and their fragments. These particles appear already at rather low energies, starting from a few GeV. They do not participate in the hydrodynamical system and we exclude them from our consideration, introducing the inelasticity factor $k \approx 1/2$, the portion of energy going into the hydrodynamical system.

The object of our consideration is the hydrodynamical system formed by the softer and more interacting hadronic component (presumably, mostly gluons). At low energies this component can "stop", as it is assumed in the Landau model. With an increase of the energy of hadronic collision the "stopping" becomes unlikely and we have the "passing through" for gluon component too. The significant difference in gluonic and quark cross-sections /7/ give us a hope that such a transition occurs at higher energies ($\sqrt{s} \sim 10^2$ GeV /10/) than those at which the leading particles appear.

In section 5 we reduce the number of free parameters of our model, using the sum rules for the total energy and entropy. The latter is related with the total multiplicity of secondaries.

τ_0 remains the only essential parameter which is the relaxation time (to reach LTE). At $\tau_0 \to 0$ the model is transformed into Landau one /3/, at large $\tau_0 \sim \frac{1}{m_\pi}$ it is very close to the Feynman parton model /6/.

In section 6 we study the dependence of our predictions on τ_0 and assumptions about the equation of state of hadronic matter. Comparison with data shows that up to the maximum ISR energy the picture of hadronic collision is very similar to that expected in Landau model: at the beginning of hydrodynamical expansion the temperature of hadronic matter is rather high ($T_0 \sim 0.4$-0.5 GeV), and "stopping" takes place ($Y_m \lesssim 1$).

In section 7 we discuss the picture of hadronic collision expected at very high energies ($\sqrt{s} \sim 10^3$ GeV). We consider a model with the "passing through" and limited initial temperature. The initial temperature is assumed to be a constant $T_0 = 0.5$ GeV and the width of the initial rapidity distribution grows as $Y_m \sim \ln s$. We also discuss the predictions of different approaches in the high $p_\perp$ region.

2. HADRON PRODUCTION MECHANISMS

As we discussed in Introduction, the main thermodynamical mechanisms (forming the spectra of secondaries) are the final "break up" and "leakage" of particles (or quarks and gluons) during hydrodynamical expansion. Below we demonstrate some manifestations of these mechanisms. Let us remind their main features:

a) The final "break up" contributes to the "softest" region of hadronic spectra. The spectra should be thermal here with the same temperature $T \sim m_\pi$ independent of $m_\perp$ and of the kind of the observed particle.

b) The "leakage" forms the harder part of spectra. In this region the spectra should be universal as a function of $m_\perp$ /11/, all of the estimates of effective production temperature of the particles should coincide and grow with $m_\perp$. For $m_\perp$, where $T_{eff} < T_c$ (T_c is the critical temperature of the hadron gas $\leftrightarrow$ quark-gluon plasma transition), the "leakage of hadrons" should be dominant. In this case the particle yields are proportional to their statistical weights. At higher $m_\perp$ ($T_{eff}(m_\perp) > T_c$) the "leakage" of quarks and gluons become dominant and the particle composition is determined by the corresponding fragmentation functions.

The data /21-26/ plotted in Fig.1 and Fig.2 show that the slope of the soft part of spectra does really agree with the same value of effective temperature $T_{eff} = 0.13$ GeV.[3)] It is very interesting that the boundary of a region with the constant T_{eff} is rather well defined for π, $\bar{p}$, K and corresponds to the same value of $p_\perp \approx 0.8$ GeV/c. The following qualitative explanation is possible: the particles with the momentum less or order of that of nearest neighbours are well "mixed" or "closed up", while the particles with higher momentum fly away from the system.

Fig.3 shows that the composition of particles is in a reasonable agreement with the estimate of "break up" temperature $T_f \sim 0.13$ GeV. Some increase of production temperature for heavy particles can be

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3) The qualitative agreement between estimates of effective production temperature of the particles of different kinds has also been discussed earlier (see, for instance, Refs.13,27 and the references therein).

treated as a consequence of the transience of the system, namely, the cooling during expansion. Thus, the particles have time to be "mixed" kinematically (all of the slopes coincide) rather than in the composition (the heavier particles "remember" the earlier hotter stage of expansion). In this connection it seems to be very interesting to measure the spectra of ψ, χ, ψ' in the region $p_\perp \lesssim 1$ GeV/c with a higher accuracy. Although their yield can be mainly contributed by the hard production mechanism /33/, one should expect that their spectra have a region with the same slope corresponding to T_f = 0.13 GeV, that is a consequence of "thermolization" of slow particles. As the slopes of their spectra in the region $p_\perp$ = 1-3 GeV/c correspond to $T_{eff} \sim$ 0.2-0.25 GeV, the expected phenomenon should looks like a "break" in spectra, probably, at $E_\perp \equiv m_\perp - M \sim (1\ \mathrm{GeV})^2/2M$. The data /34-36/ do not contradict to this prediction, although their accuracy is not sufficient for definite conclusions.

In the region $p_\perp \gtrsim$ 0.8 GeV the slopes of spectra decrease (the effective temperature increases with $m_\perp$). Let us remind that in this region the spectra of hadrons should be proportional to the same " universal" function $f(m_\perp, y\approx 0, S)$ /11/, which characterizes the space-time picture of expansion. In order to exclude it, we have plotted the ratios $\frac{K^-}{\pi^-}$, $\frac{\bar{p}}{\pi^-}$ in Fig.4. Besides the "soft" region ($m_\perp \sim M$), where these ratios are changed rather rapidly[4)], there exist two regions with other behavior.

In the first of them ($p_\perp \gtrsim$ 0.8 GeV/c, $m_\perp \lesssim$ 2 GeV/c^2) the ratios depend on $m_\perp$ rather slightly: $\bar{p}/\pi^- \approx$ 1.2-1.3,

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4) For the "soft" region it is more natural to compare the spectra at the same $E_\perp \equiv m_\perp - M$ as that in Fig.2.

$K^-/\pi^- \approx$ 0.3-0.4. This is in qualitative agreement[5)] with the predictions of the thermodynamical picture: $\bar{p}/\pi^-$ = 2, K^-/π^- = 1.

In the second region ($m_\perp \gtrsim 2$ GeV/c^2) one observes a quite different behavior. While the ratio K^-/π^- remains almost the same, the ratio $\bar{p}/\pi^-$ rapidly falls. Such a behavior can be easily understood, assuming the dominance of "evaporation" of quarks and gluons. Firstly, one should expect the barion-to-meson ratio to be suppressed in fragmentation of quark (gluon) just from the combinatorial count /37/. Secondly, there exists the "kinematical" reason. When the barion is "evaporated" by the system, its barion number is compensated by the whole system. In contrast, when the system "evaporates" the quark (gluon), fragmenting into the observed barion, the barion number is compensated mainly inside the jet having considerably smaller phase space volume. Besides that, the production of "partner" shifts the particle in $m_\perp$ by the "partner" energy E_{part}, which leads to the additional suppression factor $\exp(-\frac{E_{part}}{T_{eff}})$.

In the framework of such an idea it is very easy to understand the ratio $K^-/\pi^- \sim$ 0.3-0.4. So far as the "partner" production leads to suppression, the most probable process is the fragmentation into meson having the "evaporated" quark as the valence one. Thus, u, $\bar{u}$, d, $\bar{d}$ fragment only into pions,

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5) The quantitative disagreement should be related with (i) different contributions to the observed spectra coming from decays of heavier particles and resonances, (ii) the dependence of the kinetic on the mass of "evaporated" particle. In addition, there exists an additional suppression for K, $\bar{p}$ production because of quantum number conservation.

and $s,\bar{s}$ fragment into kaons, whence $K^-/\pi^- = 3/8$[6)]. The contribution of gluon fragmentation remains unknown. One may expect that their fragmentation functions are "softer" than those for quarks, therefore this contribution should be small. Let us stress that this statement is true only in our thermodynamical picture when the number of "evaporated" quarks and gluons is assumed to be of the same order, proportional to their statistical weights. It is not the case in hard collision models /7-9/ when the contribution of gluon-gluon scattering is expected to be dominant at $p_\perp \lesssim$ 3-5 GeV/c.

Finally, let us suppose that the transition from hadron to quark-gluon "evaporation" reflects the phase transition of hadronic matter in our system. This allows us to estimate the phase transition temperature $T_c \sim T_{eff}(m_\perp \sim 2\ \mathrm{GeV/c^2}) \sim 0.25$-$0.28$ GeV. This value is in qualitative agreement with the theoretical expectations /38/.

To answer whether the considered phenomenon is really the phase transition or it is only some peculiarity of the (hard?) scattering mechanism of constituents, one should the following

i) The study of the dependence of T_c on the energy of initial hadrons. Since $T_{eff}(m_\perp)$ grows with the energy, the critical value of m should <u>decrease</u> corresponding to the constant T_c. The detailed measurements at Serpukhov and lowest FNAL energies are especially interesting because at these energies the initial (maximum) temperature of the collision T_0 is

6) This estimate does not take into account the excitation of resonances and should be treated as the qualitative one.

expected to be near T_c. At $T_0 < T_c$ the quark-gluon jets should be absent.

ii) The same measurements in high energy nucleus-nucleus collisions, where the applicability of our microscopical picture becomes better.

3. ENERGY DEPENDENCE OF HADRONIC SPECTRA

Different mechanisms of hadron production should also manifest themselves in the energy dependence of hadronic spectra. Let us define the "energy dependence index":

$$\alpha = \frac{d\ln\left(E\frac{dN}{d^3p}\right)}{d\ln s} \qquad (4)$$

and consider its behavior in different regions of $m_\perp$ of the produced particles.

In the "soft" region ($p_\perp \lesssim 0.8$ GeV/c) which is connected to the end of hydrodynamical expansion, the shape of the $m_\perp$-distribution is independent of s and is determined by the universal "break up" temperature $T_f \approx 0.13$ GeV. Hence, we have

$$E\frac{dN}{d^3p} \propto \left(\frac{dN}{dy}\right) \qquad (5)$$

where $\left(\frac{dN}{dy}\right)$ is the rapidity distribution of secondaries. Thus, in this region α $(\equiv \alpha^{(1)})$ should be independent of $m_\perp$ (or $p_\perp$). The data /21,22/ agree with the constancy of $\alpha^{(1)}$ within errors and correspond: $\alpha^{(1)}_{\pi^-} = 0.13$, $\alpha^{(1)}_{K^-} = 0.21$, $\alpha^{(1)}_{\bar{p}} = 0.25$. The stronger energy dependence of kaon and antiproton spectra ($\alpha^{(1)}_{K,\bar{p}} \simeq 2\alpha^{(1)}_{\pi}$) is a consequence of the strangeness and baryon number conservation /12,13/.

Another behavior is expected for the spectra of "evaporated" particles. Rather simple (but crude) estimate can be obtained with the assumption that hydrodynamical expansion is one-dimensional and scale invariant ("frame independent").[7] In this case the production of a particle with definite $m_\perp$ is conditioned by the effective temperature $T_{eff}(m_\perp)$ independent of s /11/. The characteristic longitudinal size $\ell_\parallel$ and time $\Delta t \sim \ell_\parallel$, while $T \sim T_{eff}$, are only parameters depending on s. The "soft" part of the spectra corresponding to the space-like hypersurface (eq.(1)) is proportional to $\ell_\parallel$. The spectra of "evaporated" particles correspond to the time-like hypersurface and are proportional to $\ell_\parallel \cdot \Delta t \sim \ell_\parallel^2$, whence

$$E\frac{dN}{d^3p} \sim \left(\frac{dN}{dy}\right)^2 \qquad (6)$$

Thus, in the region of "leakage" ("evaporation") the preceeding value $\alpha^{(1)}$ is changed to another constant $\alpha^{(2)} = 2\cdot\alpha^{(1)}$

This estimate does not take into account s-dependent influence of conservation laws (strangeness, baryon number) essential in $K, \bar{p}$ production. Assuming that s-dependent suppression of $K, \bar{p}$ (which represents itself the probability to have in the system a "partner" compensating the quantum number) is independent of $m_\perp$ of the observed particle, we obtain

$$\alpha^{(2)}_{\bar{p},K} = \alpha^{(1)}_{\bar{p},K} + \alpha^{(1)}_{\pi} \qquad (7)$$

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7) This approximation is rather fine even in the original Landau model /3/, where the ratio $x/t = f(x,t)$ is found to be a slowly varying function.

This result is also in a qualitative agreement[8] with the data (Fig.5). It demonstrates an evident increase of α for $p_\perp \gtrsim 1$ GeV/c.

It should be stressed that in the region of quark-gluon fragmentation the energy dependence of the pion and antiproton spectra should coincide, so far as the baryon number is compensated inside the jet and suppression of $\bar{p}$ becomes independent of s. This statement is confirmed by the data /22/. The more accurate measurements are required for a final conclusion.

At the beginning of hydrodynamical espansion the estimates (6) and (7) are not valid because the expansion is rather far from the scale-invariant one. The particles with $m_\perp > \tilde{m}_\perp$ (some characteristic value) are produced at the same $T_{eff} \approx T_0(s)$, the initial temperature of hydrodynamical expansion /14/

$$E\frac{dN}{d^3p} \sim \Delta t \cdot \Delta X_\parallel \cdot exp\left(-\frac{m_\perp}{T_0(s)}\right) \qquad (8)$$

Supposing $\Delta t \sim \Delta X_\parallel \sim s^{-\delta}$, $T_0 \sim s^{\eta}$, we have

$$\alpha^{(3)} = -2\delta + \eta\frac{m_\perp}{T_0} \qquad (9)$$

In Landau model /3/ $\delta = 1/2$, $\eta = c^2/(1+c^2)$. Fitting the energy dependence of the total multiplicity of secondaries /39/, we have $c^2 = 0.28$ in ISR region. The study of the effective production temperature shows that at $\sqrt{s} = 53$ GeV $T_0 \approx 0.5$ GeV /11/, whence $T_0(s) = 8.7\cdot 10^{-2}\, s^{0.22}$ and we have

$$\alpha^{(3)} = -1 + 0.22\frac{m_\perp}{T_0(s)} \qquad (10)$$

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8) The better agreement can hardly be reached because of the crudeness of the model and uncertainties of the experimental data.

The comparison with data /22, 40-43/ (Fig.6) shows rather good agreement without any free parameter ! This fact gives a hope that the hydrodynamical picture may be applied even up to $m_\perp \sim 3$-4 GeV/c^2.

4. TRANSVERSE HYDRODYNAMICAL EXPANSION

In our previous considerations (and also in Ref.11) we neglect the effects of <u>transverse</u> hydrodynamical expansion (THE). The existence of THE is not essential in the estimates of the <u>yields</u> of particles but its influence can be very noticeable in the $p_\perp$-distribution of produced particles /17/. As the yields of particles and the slopes of their spectra correspond almost to the same effective temperature, there exists a hope that the influence of THE is insignificant.

In this section we show that the considerable THE may only appear at the latest stage of expansion, when the system temperature is $T \sim T_f$. Thus, one can neglect the THE at earlier stages which give a main contribution to particle production at $p_\perp \gtrsim 1$ GeV. Moreover, the customary estimates /3,17/ are sensitive to the unknown boundary conditions between the matter and vacuum. For instance, one should take into account the vacuum pressure, its possible dependence on plasma density, etc. As a consequence, these estimates are unreliable.

It should also be noted that the longitudinal rapidity distribution is mainly formed at earlier stages of the expansion /3/, when the influence of such factors is negligibly small.

Since the velocity distribution of THE depends on the initial plasma distribution, we restrict ourselves only to a rather crude estimate of $\langle \eta_\perp \rangle$, the characteristic value of the average THE rapidity.

Let us suppose the system to be a cylinder with the axis in longitudinal direction and consider a disc which corresponds to the unit interval of longitudinal rapidity in the system central region. Assuming that the THE is nonrelativistic, we have (in the disc rest frame):

$$\frac{d}{dt}\langle \eta_\perp \rangle \sim \left(2\pi r_0 \frac{dx_\parallel}{dy} \cdot P\right) \Big/ \left(\pi r_0^2 \frac{dx_\parallel}{dy} \cdot \varepsilon\right) \qquad (11)$$

where $r_0 \sim \frac{1}{m_\pi}$ is the disc radius; P, ε are the pressure and density of hadronic matter. Assuming that the equation of state is $P = c^2 \varepsilon$ ($c^2 \approx 0.2$ /38,49/), we get

$$\langle \eta_\perp \rangle \sim \frac{2c^2}{r_0} \int_{t_0}^{t_f} dt \qquad (12)$$

where t_0, t_f are the initial and final time moments of hydrodynamical expansion. In the case of isoenthropic one-dimensional and scale-invariant expansion[9)] we have a simple relation between the proper time and temperature /20/ (see also Appendix of Ref.11):

$$t = \frac{a}{T^{1/c^2}}$$

(generally speaking, a depends on s of the initial hadrons). It is evident that earlier stages with the temperature $T > T_f$ correspond to $t \ll t_f$ and their contribution to eq.(12) is negligible (the main contribution comes from $T \sim T_f$).

The energy dependence of $\langle \eta_\perp \rangle$ can be estimated if the parameter a is related to the multiplicity of secondaries (in the central region):

9) See the note 7.

here $\frac{dS}{dy}$ is the plasma enthropy at the final "break up" of the system, the parameters A, b are explained in Appendix.

Using $\frac{dx_{\parallel}}{dy} \sim t$, one has

$$\frac{dN}{dy} = bA\cdot(1+C^2)\pi r_o^2 a$$

Whence,

$$\langle \eta_\perp \rangle \sim \frac{2c^2}{r_o(1+c^2)}\left[\pi r_o^2\, bA\cdot(1+C^2)\cdot T_f^{1/C^2}\right]^{-1}\cdot\left(\frac{dN}{dy}\right) \qquad (13)$$

The data /21,22/ show that $\frac{dN}{dy} \backsim s^{0.10-0.13}$, i.e. the estimate (13) leads to the stronger energy dependence than that obtained in Ref./17/: $\langle \eta_\perp \rangle \backsim s^{0.07-0.1}$ by the numerical solution of 3-dimensional equations of hydrodynamical expansion. This is the case because we do not take into account the fact that at $t \sim r_o$ the expansion becomes 3-dimensional. In 3-dimensional expansion the cooling is more rapid and t_f becomes a more slow function of S. Of course, this fact does not change the conclusion that the latest stage of expansion is the most important in formation of THE.

The numerical solution obtained in Ref.17 shows that at ISR energies $\langle \eta_\perp \rangle \approx 0.4\text{-}0.5$ and the dependence $\eta_\perp(r)$ can be approximated:

$$\eta_\perp(r) = const\cdot r\,, \quad r < r_o. \qquad (14)$$

The introduction of such THE into the model results in serious problems. First, in order to reach agreement with the pion spectra, one has to lower the "break up" temperature down to $T_f \sim 0.09\text{-}0.1$ GeV. This value is rather far from that obtained from the particle composition $T_f \sim 0.13\text{-}0.15$ GeV. Second, this THE distorts the nonrelativistic region of the $m_\perp$-distribution of heavy particles. For instance, in Fig.7 we show the antiproton spectrum calculated with the use of THE rapidity distribution (14) for $\langle \eta_\perp \rangle = 0.5$ and the final "break up" temperature $T_f = 0.095$ GeV which provides the best fit to the pion spectra /17/. The evident disagreement with data /21,22/ shows that such large THE is very unlikely.

In conclusion of this section let us note that the same objections can be raised for recent attempts /44/ to describe the large $p_\perp$ particle production by the THE.

5. A SIMPLE MODEL OF SPACE-TIME EVOLUTION OF HADRONIC SYSTEM

In the previous sections the phenomena have been considered which are quite insensitive to the detailes of the space-time evolution of the hydrodynamical system. To analize the phenomena sensitive to this evolution, we use a simplified hydrodynamical model, assuming that the expansion is one-dimensional[10] and isoenthropical. Although the accuracy of such a model is not very good, it reproduces the main features of the expansion process.

As it has been shown in Introduction, the initial conditions of hydrodynamical expansion can be characterized by the initial temperature T_o, the relaxation time τ_o (to reach LTE) and the width Y_m of the initial rapidity distribution. In order to reduce the number of free parameters, we use here the sum rules

10) As we have seen in Section 4, the effects of transverse hydrodynamical expansion are expected to be rather small.

for the total energy and enthropy of the system. Assuming the enthropy to be constant during the expansion, we relate it to the total multiplicity of secondaries (excluding the leading particles and their fragments):

$$N = bS\,, \quad N \approx \frac{3}{2}(N_{ch} - 2) \tag{15}$$

where $b \approx 0.23$ in the model of slightly imperfect hadronic gas (see Appendix), N_{ch} is the experimentally observed multiplicity of charged secondaries.

The energy sum rule has the form:

$$k E_{CM} = \int d\sigma_\mu T^{0\mu} \tag{16}$$

$E_{CM} \equiv \sqrt{s}$ is the energy of initial hadrons, $k \approx 1/2$ is the inelasticity factor (the fraction of energy transferred by the gluons and presumably going into the hydrodynamical system), $d\sigma_\mu$ is the element of hypersurface corresponding to $\tau(x,t) = \tau_0$, $T^{\mu\nu} = (\varepsilon + P)\,U^\mu U^\nu - P g^{\mu\nu}$ is the energy-momentum tensor and $U^\mu \equiv (ch\chi, sh\chi, 0, 0)$ is the local 4-velocity. Using eq. (3), we obtain

$$k E_{CM} = \pi r_0^2\, \tau_0\, \varepsilon(T_0) \cdot 2\, sh\, Y_m = V_0\, \varepsilon(T_0) \cdot \frac{sh\, Y_m}{Y_m} \tag{17}$$

$$V_0 = 2\pi r_0^2 \tau_0 \cdot Y_m$$

Analogously, for the total multiplicity of secondaries, we have

$$N = bS = \pi r_0^2\, \tau_0\, b\, S(T_0) \cdot 2 Y_m = b V_0 S(T_0) \tag{18}$$

where r_0 is the (transverse) radius of the system, $\varepsilon(T)$, $S(T)$ are the densities of energy and enthropy, connected together by the equation of state (see, e.g., /38/, Appendix).

Thus, our model has only one free parameter. In essence,

this parameter is τ_0 which is the characteristic time to reach LTE. If the hadronic interaction is strong so that $\tau_0 \ll \frac{2M}{m_\pi E_{CM}} \equiv \Delta_0$, the Lorentz contracted longitudinal size of the initial hadrons, the hydrodynamical description is valid from the very beginning of collision and Landau model /3/ predicts

$$Y_m \to 0\,, \quad V_0 = \frac{c^2}{1+c^2}\left(\frac{1}{m_\pi}\right)^3 \cdot \left(\frac{2M}{E_{CM}}\right),$$

$$N \sim E_{CM}^{(1-c^2)/(1+c^2)}\,, \quad T_0 \sim E_{CM}^{2c^2/(1+c^2)} \tag{19}$$

where c is the hydrodynamical velocity of sound in hadronic matter.

In the opposite limit $\tau_0 \sim 1/m_\pi$, $T_0 \sim m_\pi$ and from eqs. (17) and (18) we get:

$$Y_m \sim \ln \frac{E_{CM}}{m}\,,$$

$$N \sim \ln \frac{E_{CM}}{m}\,, \qquad m \sim 1\ GeV/c^2 \tag{20}$$

which are just the usual predictions of the parton and multiperipheral models.

In the intermediate case the model represents some interpolation which phenomenologically describes the parton final state interactions.

Below we study the following two problems: (i) how the predictions depend on the equation of state and (ii) how they depend on the "passing through" of the gluon component.

1. The equation of state. Let us consider three cases. The first two ("a" and "b") assume the following power behavior

$$\varepsilon = A \cdot T^{(1+c^2)/c^2} + B$$
$$S = A \cdot T^{1/c^2} \cdot (1+c^2) \quad (21)$$

with c^2 equal to 0.2 and 1/3 respectrively. The parameters A and B have been evaluated from the normalization point $T = m_\pi$, where the estimates for slightly imperfect hadron gas seem to be reliable (see Appendix). The last variant "c" corresponds to the more complicated equation of state with the "phase transition". For the temperatures less than the critical value T_c it is described by the imperfect gas of hadrons with $c^2 = 0.2$. For $T > T_c$ it corresponds to the quark-gluon plasma with $c^2 \approx 1/3$ /38/. Phenomenologically the first part describes the growth of the effective number of excited degrees of freedom, while the second one corresponds to their asymptotic number in QCD proportional to the total number of different quarks and gluons. In our model we have chosen T_c to be equal to 0.3 GeV.

All of the three variants ("a", "b", and "c") correspond to the "stopping" of hadronic matter ($Y_m = 0.5$). The remaining parameters are determined by eqs. (17) and (18) from the data /39/ on the multiplicity of secondaries $N_{ch}(E_{CM})$. The calculated values are given in Table 1.[11)]

11) Let us note that the model "c" seems to be more preferable than "a", because in the model "c" we have approximately $V_0 \sim 1/E_{CM}$, which can be easily understood in the Landau model. At the same time, in the model "a" the problem raises that V_0 decreases too rapidly.

2. The "passing through". The model "a"-"c" assumes the "quick mixing" and reading of the LTE, i.e. the "early" transition from the hydrodynamical stage of the partonic to hadron-hadron collision. For comparison, we consider also the opposite case of the "late" transition (the model "d") when the initial temperature is close to the final "break up" temperature $T_0 \sim T_f \approx m_\pi$. In this case the stage of hydrodynamical expansion and "cooling" is inessential, all of the particles are produced at the same temperature $T \sim T_f$.[12)] Predictions of such a model coincide with those of the multiperipheral cluster models in which the higher initial temperatures are not considered either. In the central region this model predicts a "plato" like the parton model /6/.

Since the particular choice of T_0 and c^2 is not essential in our model "d", we take only one set of them, $T_0 = 0.19$ GeV, $c^2 = 0.2$. The rest of parameters defined by eqs. (17) and (18) from the data /39/ is also given in Table 1.

The equations of hydrodynamical expansion have been solved numerically in a one-dimensional approximation, neglecting the viscosity effects.

So far as the spectra of hadrons produced at the "break up" are strongly restricted by the normalization on the total energies and multiplicity of secondaries, in small $p_\perp$-region the predictions of all the models "a"-"d" are practically the same and are in reasonable agreement with data /22,46/ (Fig.8). In order to make a choice between these models, in the next section we discuss the phenomena depending on the earlier stages of hydrodynamical expansion.

12) A similar model has been considered in Ref./45/.

6. COMPARISON WITH DATA

1. Dileptons and hadrons with $m_\perp = 1\div 4$ GeV/c^2. The most direct way to study the space-time picture of hydrodynamical expansion seems to be the analysis of a "universal" function $f(m_\perp, y, s)$ /11/ which is some integral characteristic of this picture:

$$f(m_\perp, y, s) = \sigma_{in} \int dt\, d^3x\, \exp\left(-\frac{m_\perp ch(y-\tilde{y}(x,t))}{\tilde{T}(x,t)}\right) \qquad (21)$$

where $\tilde{y}(x,t)$ and $\tilde{T}(x,t)$ are the local rapidity and temperature in the hydrodynamical system, σ_{in} is the inelastic cross-section for initial hadrons. This function is directly connected with the inclusive spectra of dileptons:

$$\frac{d\sigma}{\pi dM^2\, dm_\perp^2\, dy} = \frac{\alpha^2}{24\pi^4} R(M^2) \cdot f(m_\perp, y, s), \qquad (22)$$

where $\alpha = 1/137$ is the fine structure constant, and

$$R(M^2) \equiv \frac{\sigma_{e^+e^- \to hadrons}}{\sigma_{e^+e^- \to \mu^+\mu^-}}.$$

For hadrons, we have

$$\left.\frac{d\sigma}{\pi dm_\perp^2 dy}\right|_{y=0} = \frac{g_h P_\perp}{2r_0(2\pi)^3} \cdot f(m_\perp, y, s) \qquad (23)$$

As it has been shown in Ref./11/ and in section 2, the main contribution to hadron spectra at $m_\perp \sim 2$-4 GeV/c^2 comes from the "evaporation" of quarks and gluons rather than common hadrons. Hence, g_h is proportional to the statistical weights of quarks and gluons, as well as to their fragmentation functions into the observed particle. Taking the fragmentation functions from Ref. /8/, we can numerically parametrize

$$g_\pi = 97.5\, n^{-2.82} + 7 \cdot 10^{-3}(n-6), \qquad (24)$$

where

$$n \equiv -m_\perp \frac{\partial}{\partial m_\perp} \ln(f(m_\perp, y, s)).$$

The invariant mass distribution for dilepton "continuum" is related to $f(m_\perp, y, s)$ in a more complicated way:

$$\frac{d\sigma}{dM dy} = \frac{\alpha^2 M R(M^2)}{12\pi^3} \int_M^\infty dm_\perp^2\, f(m_\perp, y, s) \qquad (25)$$

Since the extraction of $f(m_\perp, y, s)$ from existing data is a rather difficult procedure, we restrict ourselves to the comparison with data of the model predictions for $\left.\frac{d\sigma}{dM dy}\right|_{y=0}$.

Our comparison of the "experimental" $f(m_\perp, y, s)$ calculated by eqs.(22) and (23) from the data /22, 30, 40, 41/ (Fig.9) and the data for $\left.\frac{d\sigma}{dM dy}\right|_{y=0}$ /30,35/ (Fig.10) with the predictions of the models "a"-"d" shows that:

a) The best agreement with the data is provided by the model "a" corresponding to $C^2 = 0.2$, and also by the model "c" with the "phase transition" equation of state.

b) The predictions of the model "b" ($C^2 = 1/3$) are too high.

c) The "passing through" leads to too low predictions("d").

Thus, the experimental data can be well described by the models "a" and "c" with the space-time evolution which is very close to that of Landau model /3/ (the "stopping" of hadronic matter). It should be however noticed that one can introduce some "passing through" (incomplete "stopping") with the simultaneous increase in C^2 (or decrease in the "phase transition" temperature T_c in the model "c").

In Fig.11 we plot the different contributions to the pion spectra expected from

1) the final "break up" into secondaries at T_f = 0.13 GeV,

2) the "leakage" of hadrons (quarks and gluons) during the hydrodynamical expansion (the model "a"),

3) the hard collisions of the constituents of initial hadrons /8/ proceeding at the parton stage of hadronic collision.

The whole picture shows a nice agreement with the data, each of contributions rather well reproduces the $p_\perp$-dependence in the region of its application. Fig.6 demonstrates that the models "a" and "c" reproduce also the energy dependence of (pion) spectra. The more accurate data are needed at highest energies of ISR, where the experimental uncertainties are so far too large.

2. The $p_\perp$-distribution of dileptons. The detailed study of a shape of this distribution can provide an additional important test of the models. In the region of thermodynamical description this distribution is immediately connected with the mentioned above function $f(m_\perp, y, s)$, but outside this region this relation is strongly violated /11/. Therefore, a study of this distribution can clear up the limits of the thermodynamical description.

The model predictions for $\langle p_\perp \rangle$ and $C_2 \equiv \frac{\langle p_\perp^2 \rangle}{\langle p_\perp \rangle}$ are plotted in Fig.12. The data available /30/ on $\langle p_\perp \rangle$ disagree with any model, except "d" (the "passing through"). In contrast, the estimates of C_2 obtained from parametrization of the same data confirm strongly the models "a" and "c" which are the best ones in the above considerations. Such a disagreement can be ascribed to the absence of LTE or large contributions of hard processes [13], as well

13) The data /30/ correspond to the dileptons produced mainly outside the central region ($X_F \equiv \frac{2p_\parallel}{\sqrt{s}} \gtrsim 0.15$).

as to the large experimental uncertainties at small $p_\perp$. In any case, the new measurements of the dilepton $p_\perp$-distribution are required.

Finally, let us note that the difference between the models "a" and "c" appears to be more essential here than that in the inclusive spectra (Figs.9,10). At $M < 2$ GeV/c^2 the model "c" (dashed line) is close to "a", while at $M > 3$ GeV/c^2 its predictions coincide with those of "b".

7. HADRON-HADRON COLLISIONS AT VERY HIGH ENERGIES

As it has already been mentioned in Introduction, at high enough energies the asymptotic freedom makes the "stopping" impossible even for the gluon component. At $\sqrt{s} > \sqrt{s^*}$ (some critical energy of colliding hadrons) one has some initial rapidity distribution of hadronic matter at the beginning of hydrodynamical expansion. With an increase in s, this distribution should become more and more wide, but the growth of the initial temperature becomes slower or disappears at all. So, we come to a new regime where the parton stage plays the main role, while the hydrodynamical one gives the comparatively small corrections. Here it is expected that the power dependence of the total multiplicity on the energy (intrinsic to the Landau model) is changed to the logarithmical one, the scaling is reached in the central "plateau", etc.

The estimates in Ref./10/ show that such a transition is expected at $\sqrt{s^*} \sim 10^2$ GeV, which corresponds to the limiting value of the initial temperature $T_0 \simeq 0.5$-0.7 GeV. We consider below the model "e", having the equation of state "c" (with the "phase transition") and fixed initial temperature T_0 = 0.5 GeV. The remaining parameters (see sect.5) have been determined by eqs. (17) and (18) and also by the logarithmical extrapolation of multiplicity data /47/. In Fig.13 we compare the rapidity distribution in the "soft" $p_\perp$ region ($p_\perp$ = 0.4 GeV/c) with that predicted by the Landau model. In the latter we have used a "more realistic" (at high temperatures) equation of state "c" with the "phase transition". The main distinction in spectra is connected with the different expected total multiplicities of secondaries. In Landau model the power extrapolation $N \infty s^{1/4}$ leads to $N \approx 60$, while in the model "e" the logarithmical one gives $N \approx 30$ (at $\sqrt{s} \sim 10^3$ GeV).

The more interesting picture is expected in transverse momentum distribution (Fig.14). The existence of the limiting value T_0 leads to the independence of the shape and normalization of the thermodynamical contributions on the energy of initial hadrons. If the hard scattering contributions will continue to grow, as it is predicted by Ref./8/, they would be dominating even in the region of $p_\perp = 1 \div 3$ GeV, which at present energies is dominated by the "leakage" of quarks and gluons from the thermodynamical system. Since in hard scattering models the main contribution is here the gluon-gluon scattering, the growth of hadronic spectra should be more rapid than that of dilepton spectra and the "universality" /11/ of hadron and dilepton spectra should strongly violate.

Let us finally note that some increase in spectra at $m_\perp \gtrsim 2$ GeV is also predicted by the (one-dimensional[14]) Landau model,

14) The transverse hydrodynamical expansion is neglected.

but it is too slow in comparison with the mentioned above contribution of the hard scattering processes.

8. SUMMARY

In the present study we discuss in dètail the place of the thermodynamical description (TD) of high energy hadron-hadron collisions. In Introduction it has been shown that TD is natural for comparatively soft processes proceeding at late stages of hadronic collision, while the parton approach describes the hard scattering of constituents of the initial hadrons taking place at the beginning of the collision. The time moment when the collision comes from the parton stage to the hydrodynamical one (the moment of reaching of LTE) is governed by the rate of relaxation processes.

The main applications of TD are: (i) the description of a space-time picture of the system evolution (expansion) from the moment of reaching of LTE up to the system "break up" into secondaries, (ii) the description of the system decay into the observed particles.

In the first part of our study (sects.2-4) we have consider the topics unrelated to the particular picture of system expansion. In sect.2 we have studied the particles ratios and slopes of their spectra with the conclusion that the region of the thermodynamical description may be divided into three regions: (1) the "soft" hadron region ($p_\perp \lesssim$ 0.8 GeV/c), where the main contribution comes from the final "break up" at the end of hydrodynamical expansion, (2) the region ($p_\perp \gtrsim$ 0.8 GeV/c, $m_\perp \lesssim 2$ GeV/c^2), where the "leakage" ("evaporation") of hadrons dominates, (3) the region ($m_\perp > 2$ GeV/c^2), where the main contribution comes from the "leakage" of quarks and gluons. Assuming that the "leakage" of hadrons is changed by the "leakage" of quarks and gluons because of the phase transition in the system, we have estimated the critical temperature to be $T_c \sim T_{eff}(m_\perp \sim 2$GeV) $\approx$ 0.25-0.28 GeV. The study of the energy dependence of hadronic spectra (sect.3) has provided additional arguments in favor of our picture.

The estimates of the transverse hydrodynamical expansion (THE) have shown (sect.4) that the main contribution to THE comes from the latest stages of expansion, when the various unknown effects (e.g., the boundary conditions between the plasma and vacuum, etc.) become important. We have found that at the earlier stages of expansion the THE is negligible and at the latest stages the estimates of THE are unreliable. The analysis of the "soft" region of spectra of heavy secondaries (e.g., antiprotons) has shown that the THE is suppressed (or practically absent). Therefore, in the next sections (5-7) we have used the one-dimensional approximation for the longitudinal expansion.

In sect.5 we have formulated a simple model for the space-time picture of the (longitudinal) hydrodynamical expansion. The main parameters of the model are the characteristic relaxation time (to reach LTE) τ_0 and the equation of state of hadronic matter. For small τ_0 the model is close to the Landau one; in the opposite case, it corresponds to rather free "passing through" of hadronic matter without "heating" and is close to the Feynman parton picture. The comparison with data (sect.6) has shown that for the present energies (FNAL, ISR) the best agreement can be reached for rather small

τ_0 and therefore the initial conditions of hydrodynamical expansion are close to those in Landau model. The predicted change in the energy dependence of hadronic spectra at $m_\perp \sim 2$-3 GeV/c^2 agrees rather well with the data available.

In sect.7 we have considered some extrapolation of the thermodynamical picture to the superhigh energies of colliding hadrons. As far as the strong interactions are "asymptotically free", we expect that "passing through" leads to the scaling of the thermodynamical contributions to the spectra of secondaries. Comparison with the predictions of the hard scattering model has shown that the latter become dominating even at $p_\perp \sim$ 1-3 GeV/c, where at present energies we find the "evaporation" mechanism.

9. CONCLUSIONS

The main attractive features of the formulated thermodynamical picture are its comparative simplicity and clarity and the possibility of unified explanation of the available data. The detailed studies of the dilepton and hadron production in the applicability region of our picture ($m_\perp \lesssim 3$-4 GeV/c^2) allow us to obtain the information on the thermodynamics of hadronic matter, including its phase transition from quarks to hadrons and related non-perturbative effects.

At the same time, the picture considered above is mainly qualitative and approximate and requires the further theoretical studies (e.g., the account for the kinetic during the expansion and system decay into the observed particles, the impertfect LTE and so on), as well as the new, more detailed and accurate measurements of the spectra of produced particles and resonances, their energy dependence, etc.

It would appear to be very interesting to measure the same spectra in high energy nucleus-nucleus collisions. So far as the applicability of the thermodynamical approach becomes considerably better, such collisions could provide a nice possibility to study the properties of quark-gluon matter.

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At the low enough temperatures $T \sim m_\pi$ the thermodynamics of slightly imperfect hadron gas can be described in the first order of virial expansion by inclusion in the statistical sum besides the stable particles their bound states, the resonances /48,49/. In this approximation the energy and enthropy density ε, S can be calculated as follows:

$$\varepsilon = \sum_a T^4 \left(\frac{g_a}{2\pi^2}\right) \Phi\left(\frac{m_a}{T}\right) \tag{A.1}$$
$$S = \sum_a T^3 \left(\frac{g_a}{2\pi^2}\right) G\left(\frac{m_a}{T}\right)$$

where g_a is the statistical weight of a particle (or resonance) of the kind "a", m_a is its mass. The summation is done over all of the strongly interacting particles (resonances). The functions $\Phi(z)$, $G(z)$ are defined in Ref./3/. The estimates of the velocity of sound c obtained in Ref./49/, show that in the temperature interval T = 0.14-0.7 GeV the result c^2 = 0.2 very little depends on the detailes of calculations. Although at $T \sim m_\pi$ these estimates seem to be reasonable, their reliability at higher temperatures is rather questionable. We consider the power parametrization of the equation of state, being in essence an extrapolation into the high temperature region:

$$\varepsilon = A T^{(1+c^2)/c^2} + B \tag{A.2}$$
$$S = A\cdot(1+c^2)\, T^{1/c^2}$$

In the model "a", according to /49/, we suppose $C^2 = 0.2$, in the model "b" we take $C^2 = 1/3$. The parameters A and B are fixed by the normalization of eq.(A.2) at $T = m_\pi$. At lower temperatures the system "breaks up" into secondaries, at higher ones the estimates became unreliable. Taking into account $a = \pi, K, \eta, \rho, \omega, \eta', K^*$, we obtain $\mathcal{E}(m_\pi) = 7.7\cdot10^{-4}$ GeV, $S(m_\pi) = 6.5\cdot10^{-3}$ GeV^3. So, in the models "a" and "b" we have $A = 10^2$, $B = 1.7\cdot10^{-5}$ GeV^4 and $A = 1.8$, $B = 8.6\cdot10^{-5}$ GeV^4, respectively.

As has been pointed out in Ref./38/, the phase transition from the hadron to quark-gluon phase is expected at $T_c \sim 0.3$-0.7 GeV. Our considerations in sect.2 show that $T_c \sim 0.25$-0.28 GeV. Up to now, in this region the behavior of the equation of state remains unknown. We consider a simple "composite" model "c". For $T < T_c$ it coincides with "a", for $T > T_c$ it parametrizes the QCD estimates /38/ by the power form (A.2). Taking $T_c = 0.3$ GeV and assuming $S, \mathcal{E}$ to be continuous[15)], we have $c^2 \approx 0.3$, $A = 12$ GeV$^{1/3}$, $B = 5.5\cdot10^{-3}$ GeV^4.

Finally, we estimate the proportionality coefficient b between the enthropy of decaying plasma and the density of produced particles (π and K -mesons) at $T \sim m_\pi$. Doing in analogy to eq.(A.1), we obtain $n_{K+\pi}(T \sim m_\pi) = 1.2\cdot10^{-3}$ GeV^3, whence $b \simeq \frac{n}{s}$ 0.23.

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15) Strickly speaking, this is valid only for the second type of the phase transition.

References

1. E.L.Feinberg. Phys.Rep. 5C,237,1972.
2. E.V.Shuryak. Proc. of 18-th Intern. Conf. on High Energy Physics,Tbilisi,July 1976,p.A3-1.
   I.M.Dremin,ibid,p.A3-31.
   E.V.Shuryak. Quantum Chromodynamics and the Theory of Superdense Matter,1979,Phys. Reports, in press.
3. L.D.Landau. Izv. Akad. Nauk SSSR,ser.fiz.17,51,1953.
   S.Z.Belen'ky,L.D.Landau. Uspekhi Fiz. Nauk,56,309,1955.
4. D.Amati,A.Stanghellini,S.Fubini. Nuovo Cim.,26,896,1962.
5. K.A.Ter-Martirosyan. Nucl.Phys.,68,591,1965.
6. R.P.Feynman. Photon-Hadron Interactions,Benjamin Inc., N.Y.,1972.
7. B.L.Combridge,J.Kripfganz,J.Ranft. Phys.Lett.70B,234,1977.
   R.Cutler,D.Sivers. Preprint ANL-HEP-PR-77-40.
8. J.F.Owens,E.Reya,M.Glück. Phys.Rev.D18,1501,1978.
9. R.D.Field.Phys.Rev.Lett.,40,997,1978.
   A.P.Contogouris,R.Gaskell,S.Papadopoulos.Phys.Rev.D17, 2314,1978.
   J.F.Owens,J.D.Kimmel. Phys.Rev.D18,3313,1978.
   R.P.Feynman,R.D.Field,G.C.Fox. Phys.Rev.D18,3320,1978.
10. E.V.Shuryak. Phys.Lett.78B,150,1978;Yadern.Fiz.28,796,1978.
11. O.V.Zhirov. Yadern.Fiz.30,1102,1979;Novosibirsk,preprint IYaF-79-1,1979.
12. I.N.Sisakyan,E.L.Feinberg,D.S.Chernavsky,ZhETF,52,545,1967.
13. E.V.Shuryak. Yadern.Fiz.20,549,1974.
14. M.I.Gorenstein,V.P.Shelest,G.M.Zinovjev.Phys.Lett.60B,283,197
15. E.L.Feinberg.Izv.Akad.Nauk SSSR,ser.fiz.,26,622,1962.
16. I.Ya.Pomeranchuk. Dokl.Akad.Nauk SSSR,78,884,1951.
17. B.Andersson,G.Jarlskog,G.Damgaard.Nucl.Phys.B112,413,1976.

18. D.I.Blokhintzev. ZhETF,32,350,1957.
19 J.Novakowsky,F.Cooper. Phys.Rev.,D9,771,1974.
M.J.Morawcsic,M.Teper. Phys.Rev.,D16,1593,1977.
19. S.A.Voloshin,Yu.P.Nikitin. Yadern.Fiz.,27,1703,1978.
20. C.B.Chiu,E.C.G.Sudarshan,Kuo-hsiang Wang,Phys.Rev.,D12,902,1975
21. K.Guettler et al. Phys.Lett.,64B,111,1976;Nucl.Phys.B116,77,1976
22. B.Alper et al. Nucl.Phys.,B100,237,1975.
23. A.Suzuki et al. Lett.Nuovo Cim.,24,449,1979.
24. H.Kichimi et al. Lett.Nuovo Cim.,24,129,1979.
25. H.Kichimi et al. Phys.Lett.,72B,411,1978.
26. F.W.Busser et al. Phys.Lett.,61B,309,1976.
27. J.Bartke et al. Nucl.Phys.B120,14,1977.
28. A.Sheng et al. Phys.Rev.,D11,1733,1975.
29. K.J.Anderson et al. Phys.Rev.Lett.,37,799,1976.
30. J.G.Branson et al. Phys.Rev.Lett.,38,1331,1334,1977.
31. R.Singer et al. Phys.Lett.,60B,385,1976.
32. A.Böhm et al. Phys.Rev.Lett.,41,1761,1978.
33. V.G.Kartavelishvili,A.K.Likhoded,S.P.Slabospitzky.Yadern.Fiz., 28,1315,1978.
34. H.D.Snyder et al. Phys.Rev.Lett.,36,1415,1976.
35. A.G.Clark et al. Nucl.Phys.,B142,29,1978.
36. C.Kourkomelis et al. Phys.Lett.,81B,405,1979.
37. V.V.Anisovich. Preprint LIYaF-351,1977.
38. E.V.Shuryak. ZhETF,74,408,1978.
39. W.Thomé et al. Nucl.Phys.B129,365,1977.
40. P.Darriulat et al. Nucl.Phys.B110,365,1976.
41. K.Eggert et al. Nucl.Phys.B98,93,1975.
42. D.Antreasyan et al. Phys.Rev.Lett.,38,112,1977.
43. F.W.Büsser et al. Nucl.Phys.,B106,1,1976.
44. P.Carruthers,M.Duong-Van.,Phys.Rev.Lett.,21,133,1973.
Y.Hama,Nuovo Cim.,46A,569,1978;Lett.Nuovo Cim.,24,530,1979.

45. V.Cherny,P.Lichard,J.Pišut. Phys.Lett.,70B,61,1977.
46. P.Capiluppi et al. Nucl.Phys.,B79,189,1974.
47. M.Antinucci et al. Lett.Nuovo Cim.,6,121,1973.
48. S.Z.Belen'ky et al. Uspekhi Fiz.Nauk,62,1,1956.
49. O.V.Zhirov,E.V.Shuryak. Yadern.Fiz.,21,861,1975.

Table 1.

model	$\sqrt{s}$, Gev	T_o, Gev	Y_m	V_o, Gev^{-3}
a	23	0.32	0.5	97
	53	0.48	0.5	21
b	23	0.36	0.5	365
	53	0.53	0.5	178
c	53	0.52	0.5	46
d	23	0.19	1.9	390
	53	0.19	2.6	430
e	1000	0.5	4.4	80

FIGURE CAPTIONS

Fig.1 The dependence of invariant cross sections $E\frac{d\sigma}{d^3p}$ on the "transverse mass" $m_\perp$. The solid lines show the slope corresponding to $T_{eff} = 0.13$ GeV. The arrows correspond to $p_\perp = 0.8$ GeV/c. Data are taken from: • - [21]; ∗,+,□ - [22].

Fig.2 The dependence of particle spectra on the "transverse energy" $E_\perp \equiv m_\perp - M$. The solid lines show the slope corresponding to $T_{eff} = 0.13$ GeV, the arrows show $p_\perp = 0.8$ GeV/c. The data are taken from ○,● - [23]; • - [21]; ▼, ▲, △ - [24]; x - [25]; □ - [22]; ■ - [26].

Fig.3 The ratios of particle spectra divided by the particle statistical weight to the spectra of pions of the same sigh. The data are collected from: K - [21,25,28], ρ^0 - [23,29-31], $K^*(890)$ - [24], $\bar{p}$ - [21], φ - [29,30], Λ^0 - [25,26,28], $f(1270)$ - [23,32], $K^*(1420)$ - [24,32], $g(1630)$ - [32].

Fig.4 The particle ratios $\bar{p}/\pi$, K^-/π^- versus $m_\perp$. The data are taken from: • - [21], □, x - [22]. The arrows correspond to $p_\perp = 0.8$ GeV/c.

Fig.5 The energy dependence of hadronic spectra. The solid lines represent the data [21], the point are taken from [22]. The dashed lines are the data [22] averaged over regions ($p_\perp \lesssim 0.8$ GeV/c) and ($p_\perp \gtrsim 0.8$ GeV, $m_\perp \lesssim$ 2 GeV/c^2). The arrows correspond to $p_\perp = 0.8$ GeV/c.

Fig.6 The energy dependence of pionic spectra. The data are taken from: △ - [22] ; □ - [40] ; ○ - [41] ; ∗ - [42] ; ● - (π^0), ▼ - ($\pi^+\pi^-$) [43] . The dotted lines show the predictions of Landau model (eq.(8)). The solid lines correspond to the model "a" and "b" described in sects. 5 and 6.

Fig.7 The antiproton spectra $E\frac{d\sigma}{d^3p}$ [21,22] . The solid line corresponds to the absence of THE, the dashed line is calculated for the THE predicted in Ref./17/.

Fig.8 The inclusive spectra of the "soft" pions ($p_\perp = 0.4$ GeV/c). The predictions of the models "a", "b", and "d" are compared with data [22,46].

Fig.9 The "universal" function $f(m_\perp, y, s)$ predicted by the models "a"-"d" in comparison with that extracted from the data ▽ - π^- [22] , ● - π^0 [40] , ○ - π^0 [41] , x - $\mu^+\mu^-$ [30].

Fig.10 The dilepton spectra $\left.\frac{d\sigma}{dM\,dy}\right|_{y=0}$ predicted by the models "a"-"d" in comparison with data [30,35] .

Fig.11 The contributions to the pion spectrum expected from: (i) the system "break up" at the end of hydrodynamical expansion, (ii) the "leakage" of hadrons (quarks and gluons) during the expansion (the model "a"), (iii) the hard scattering of the initial hadrons. The data are: ▽ - π^- [22] , ● - π^0 [40] , ○ - π^0 [41] .

Fig.12 The dependence of $\langle p_\perp \rangle$ and $C_2 \equiv \frac{\langle p_\perp^2 \rangle}{\langle p_\perp \rangle}$ on the dilepton invariant mass. The data are taken from [30]. The dashed curve corresponds to the model "c".

Fig.13 The spectra of "soft" pions ($p_\perp = 0.4$ GeV/c) expected at very high energies ($\sqrt{s} = 10^3$ GeV) of the initial hadrons. The curves 1 and 2 correspond to the predictions of Landau model and the model "c" described in text. For the comparison we plot 3 - the parametrization of ISR data [22,46] at $\sqrt{s} = 53$ GeV.

Fig.14 The pion spectra $E\frac{d\sigma}{d^3p}$ versus $p_\perp$ predicted by: 1 - the model "c", 2 - Landau model, 3 - the hard collision model (Ref.8).

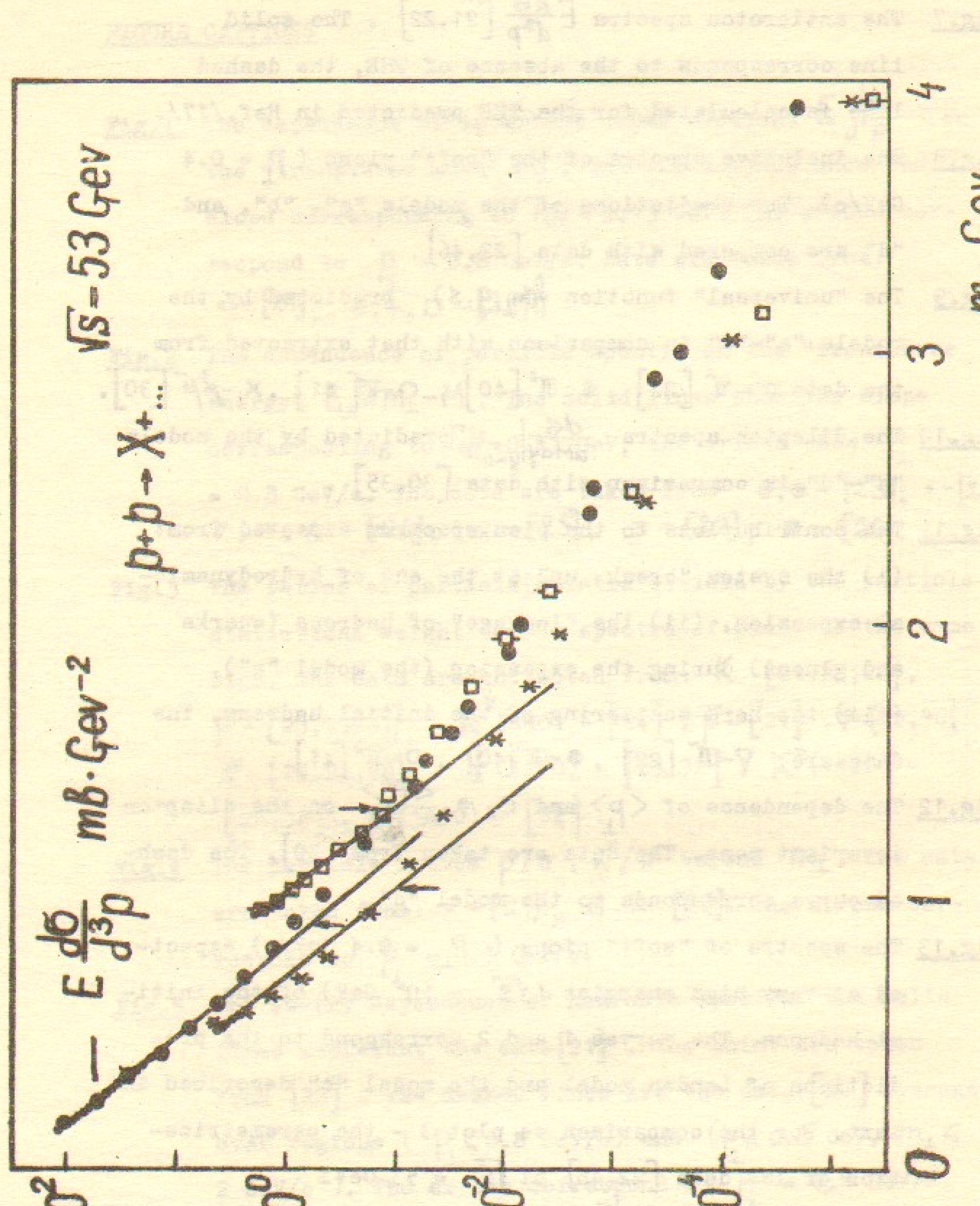

E dσ/d³p mb·Gev⁻²
p+p → X+...
√s = 53 Gev
m⊥ Gev
10²
10⁰
10⁻²
10⁻⁴
0
1
2
3
4

Fig.1

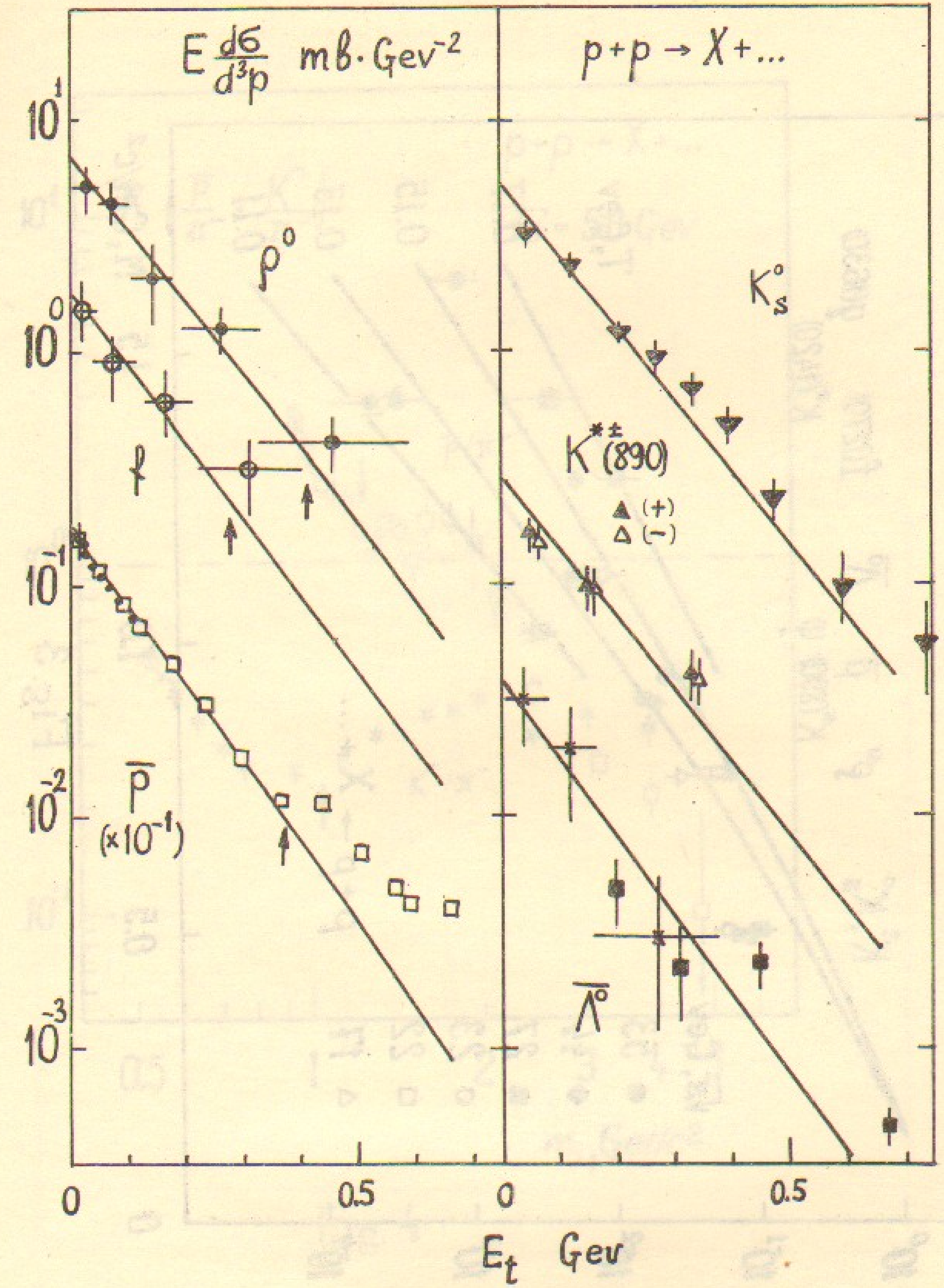
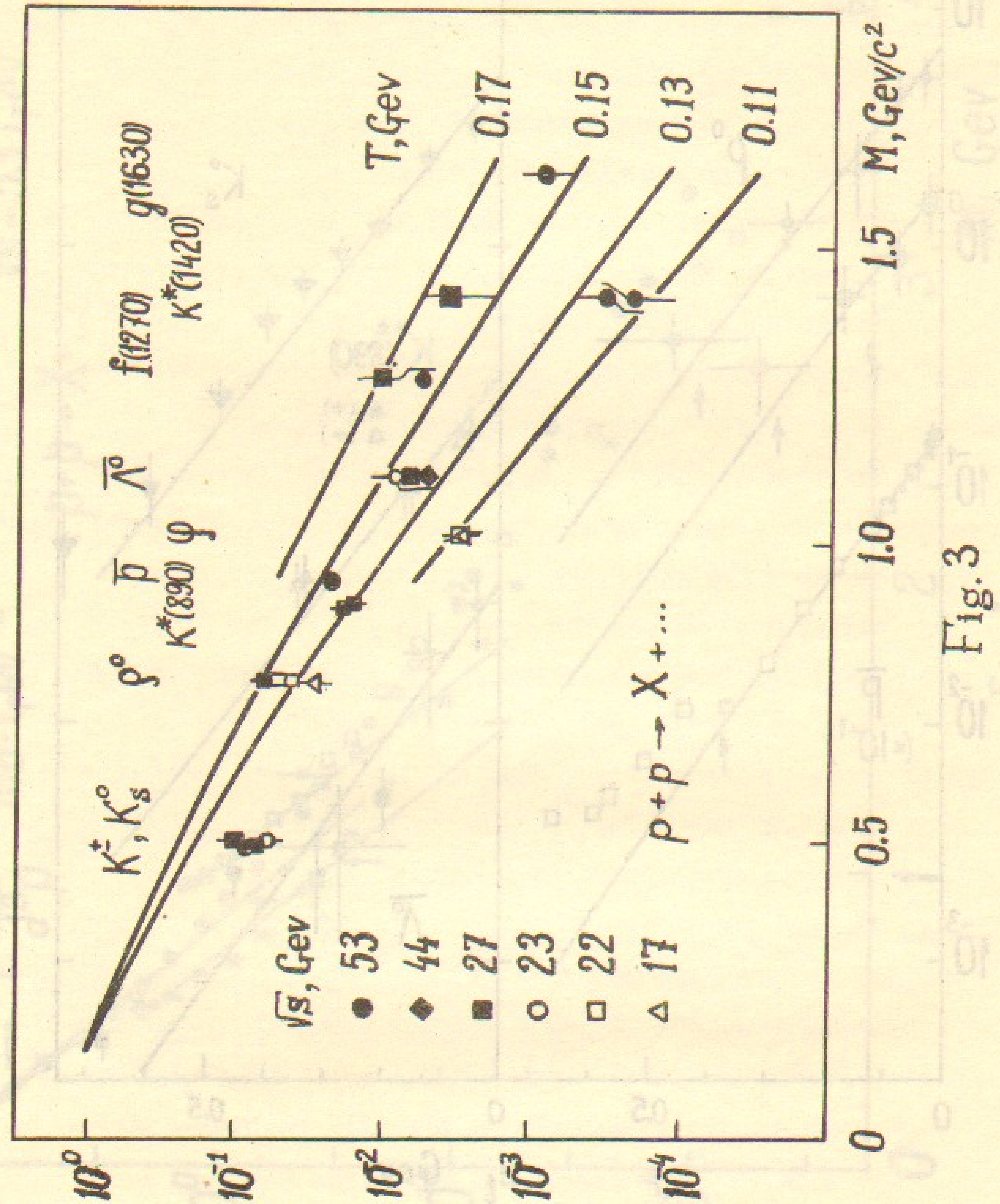
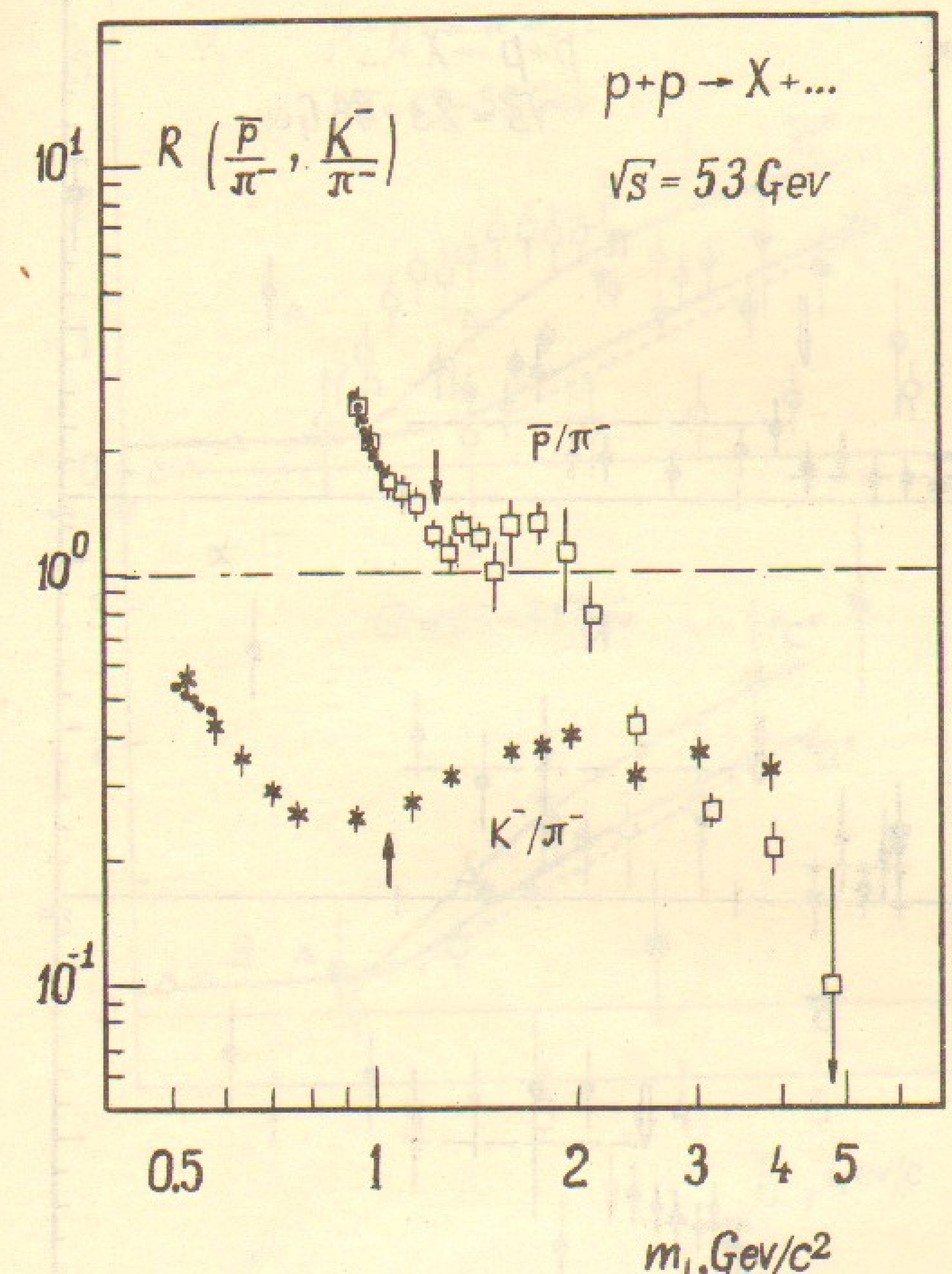

E dσ/d³p mb·Gev⁻²
p+p → X+...
ρ⁰
f
p̄
(×10⁻¹)
K°s
K*±(890)
▲ (+)
△ (−)
Λ̄°
10¹
10⁰
10⁻¹
10⁻²
10⁻³
0
0.5
0
0.5
Et Gev

Fig. 2



$\sqrt{s}$, Gev
53
44
27
23
22
17
T, Gev
0.17
0.15
0.13
0.11
M, Gev/c²
$p+p \to X+...$

Fig. 3



$p+p \to X+...$
$\sqrt{s} = 53$ Gev
$R\left(\frac{\bar{p}}{\pi^-}, \frac{K^-}{\pi^-}\right)$
$\bar{p}/\pi^-$
K^-/π^-
$m_\perp$, Gev/c²

Fig. 4

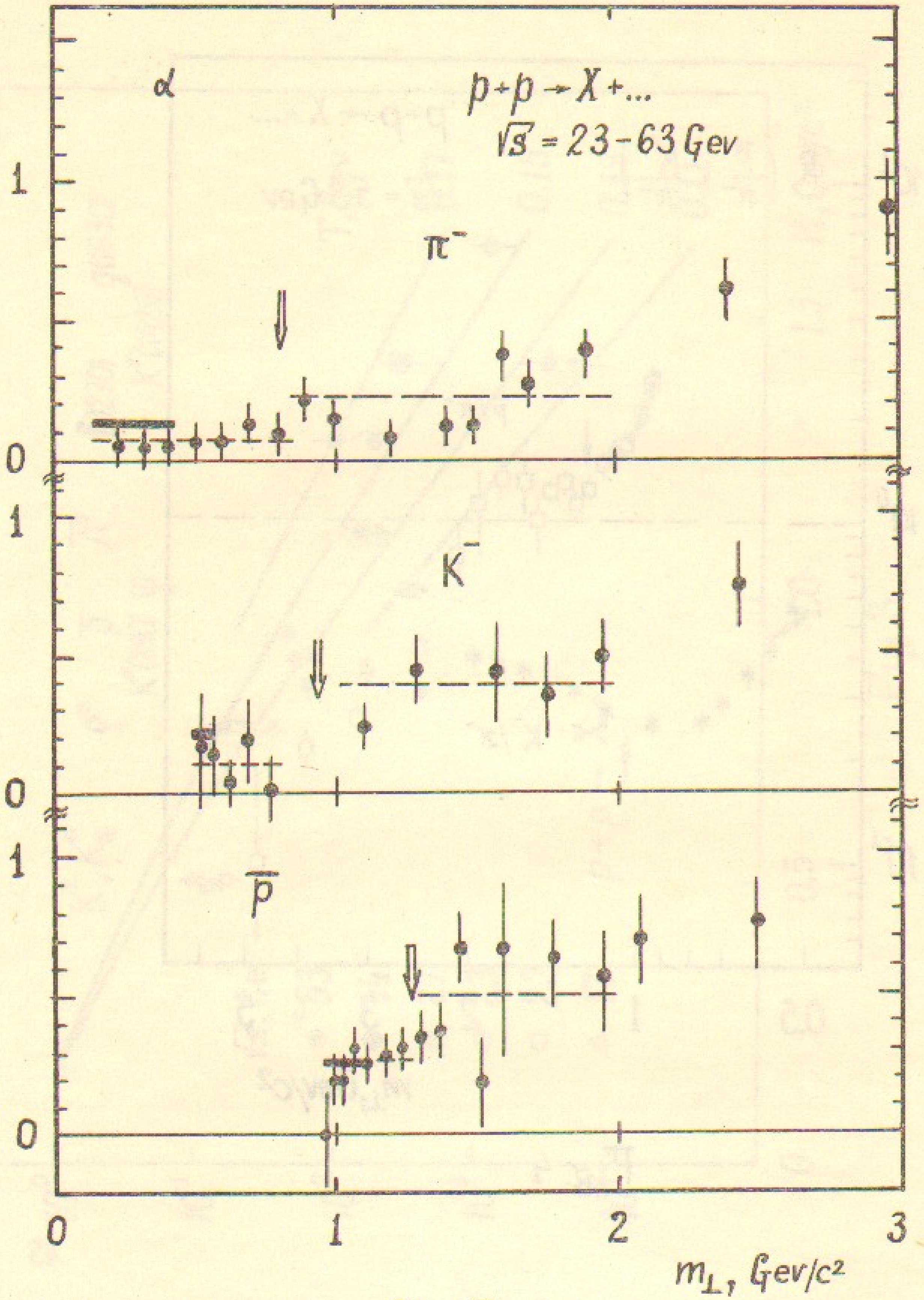

α
p+p → X+...
√s = 23-63 Gev
π⁻
K⁻
p̄
0
1
2
3
m⊥, Gev/c²

Fig.5

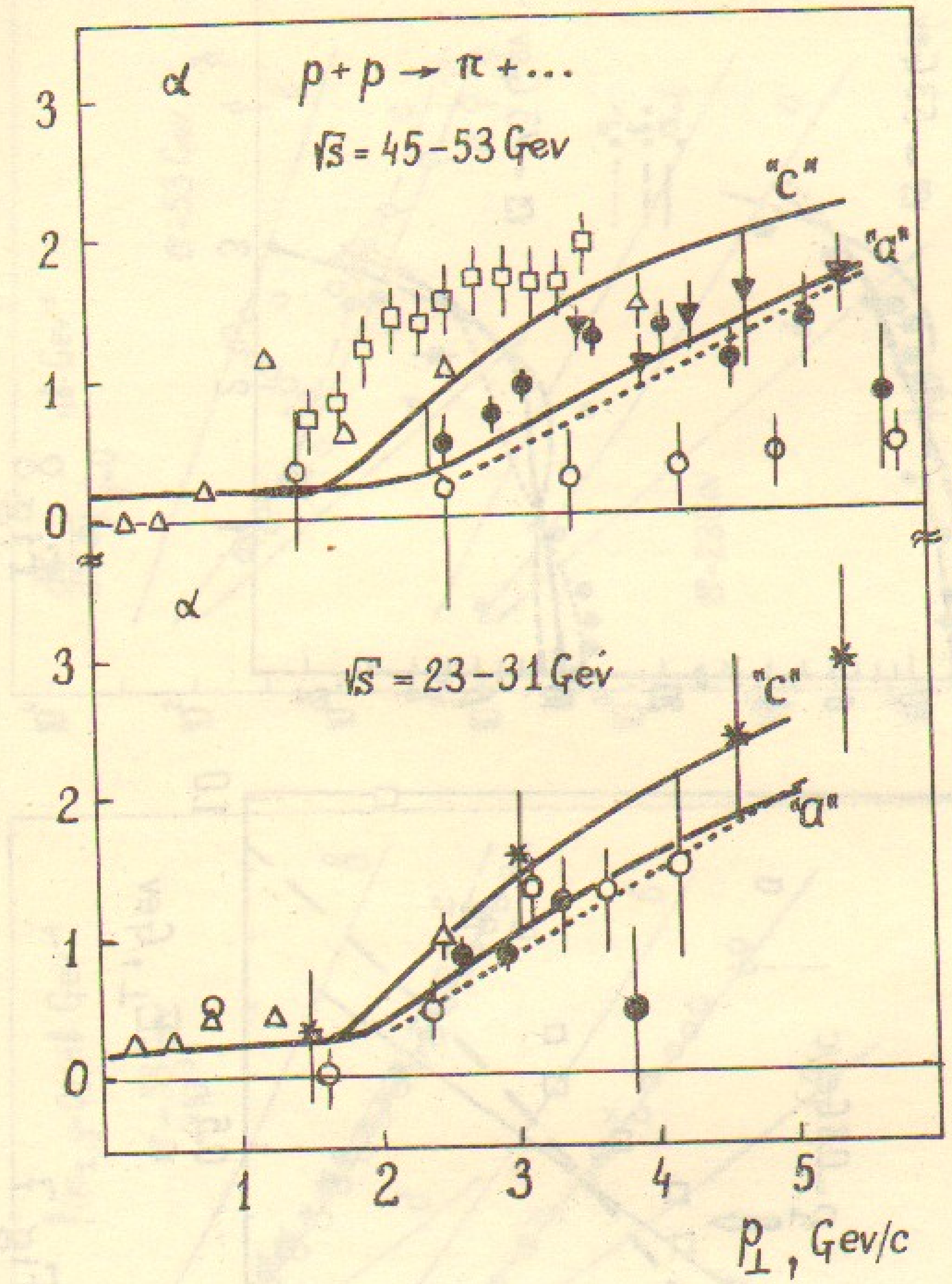

α
p+p → π+...
√s = 45-53 Gev
"c"
"a"
α
√s = 23-31 Gev
"c"
"a"
0
1
2
3
4
5
p⊥, Gev/c

Fig.6

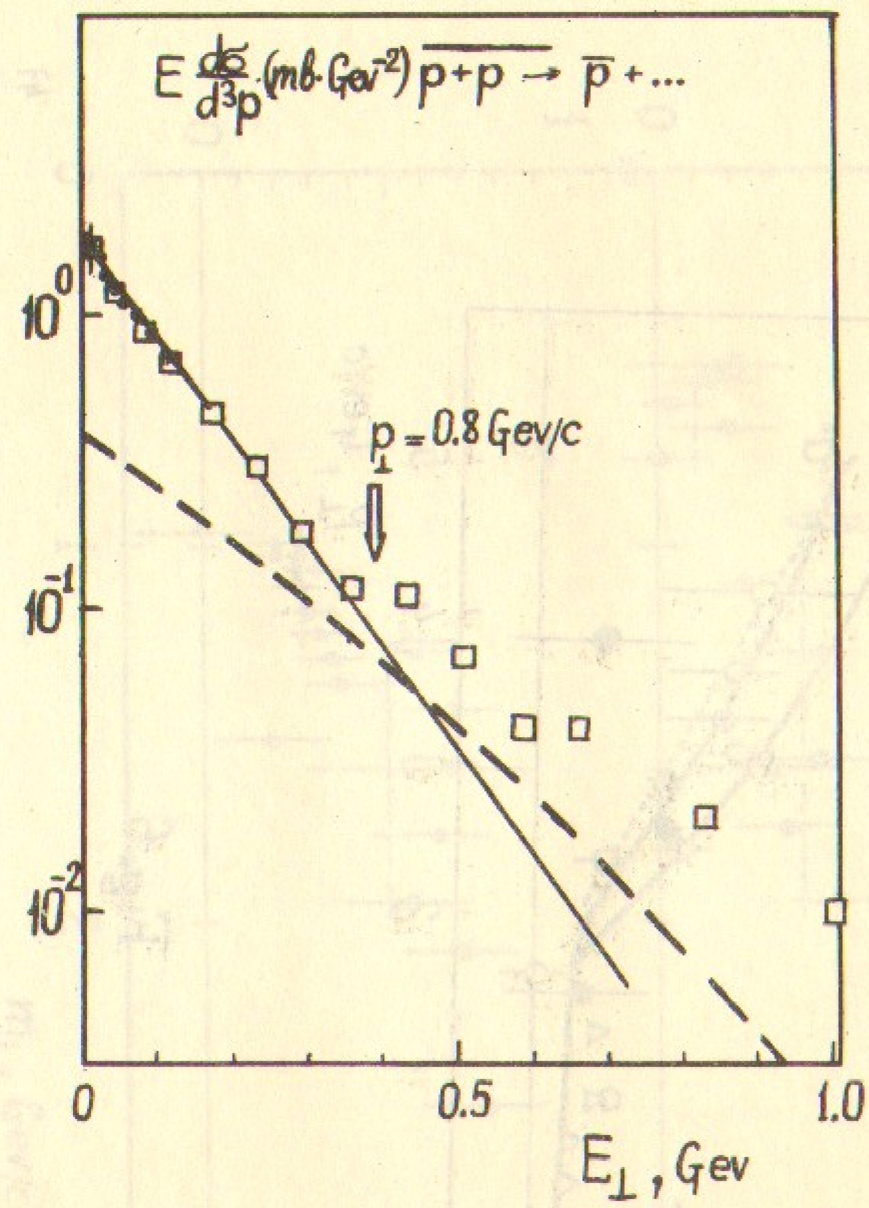


Fig. 7

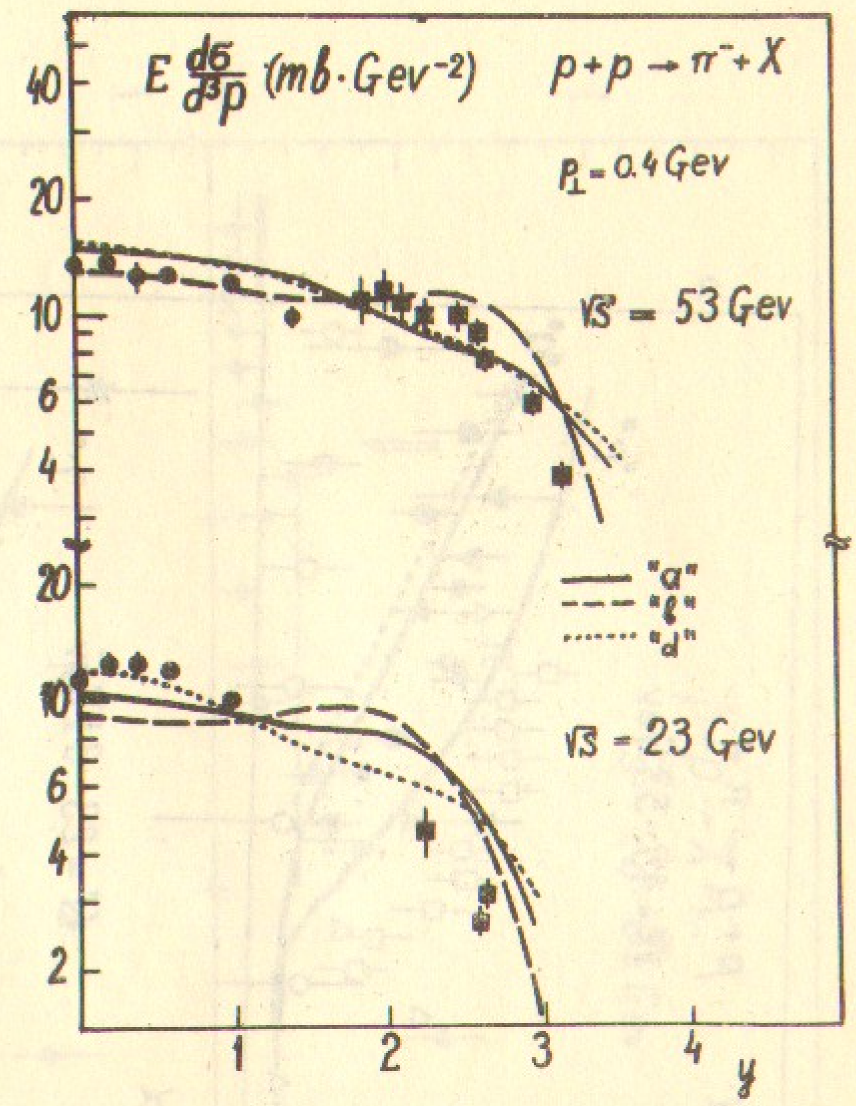


Fig. 8

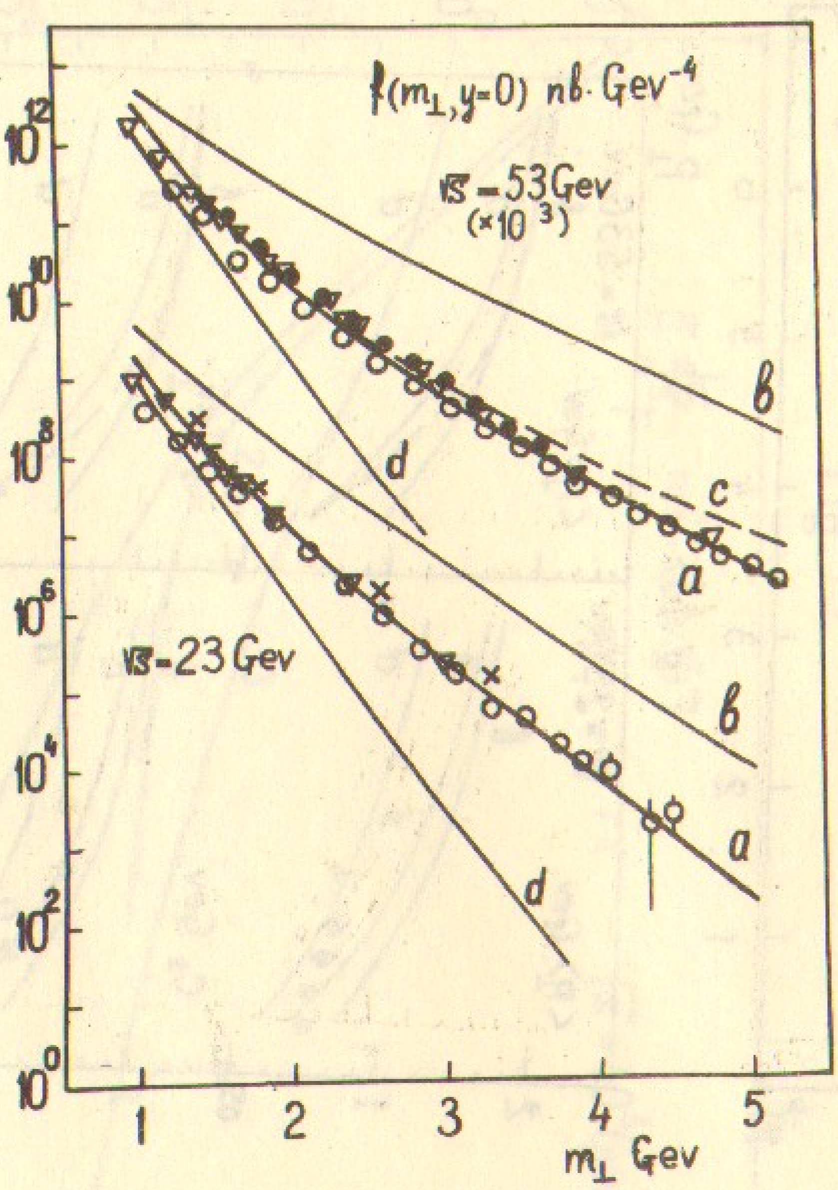


Fig. 9

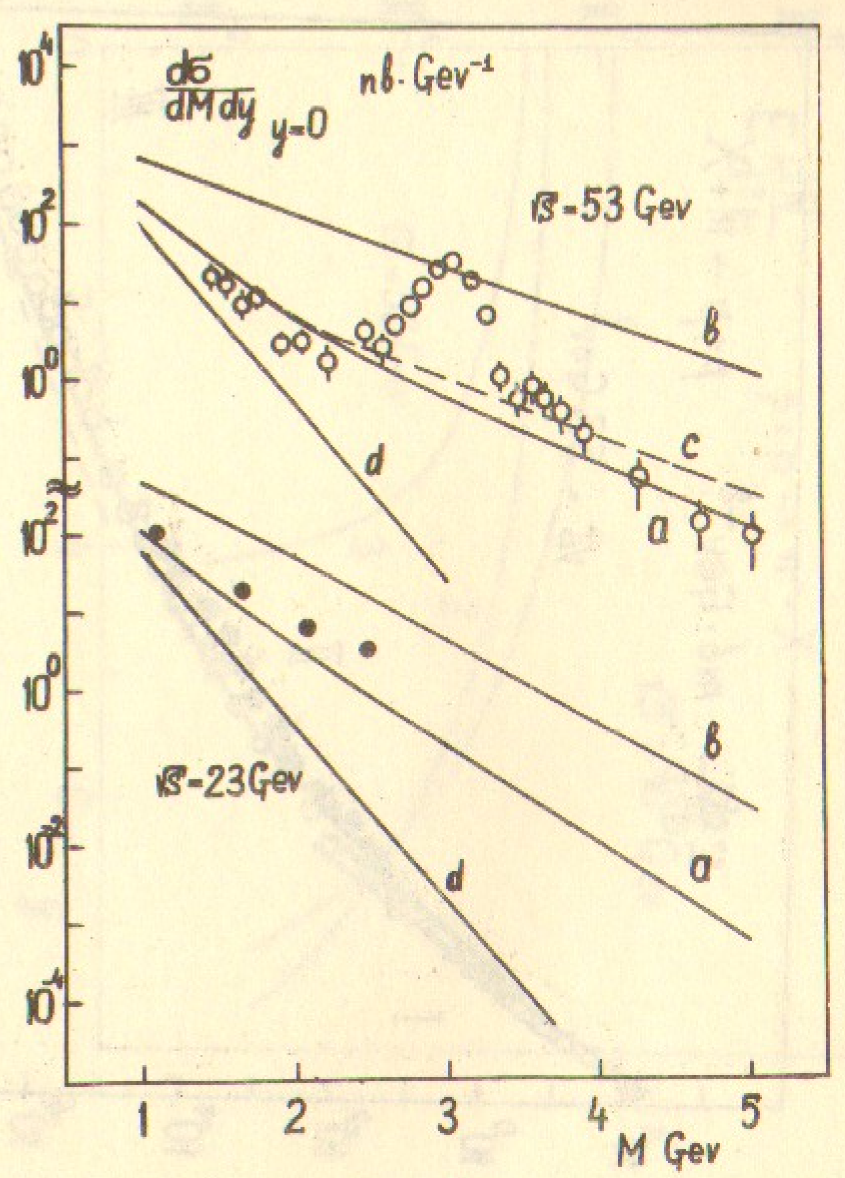


Fig. 10

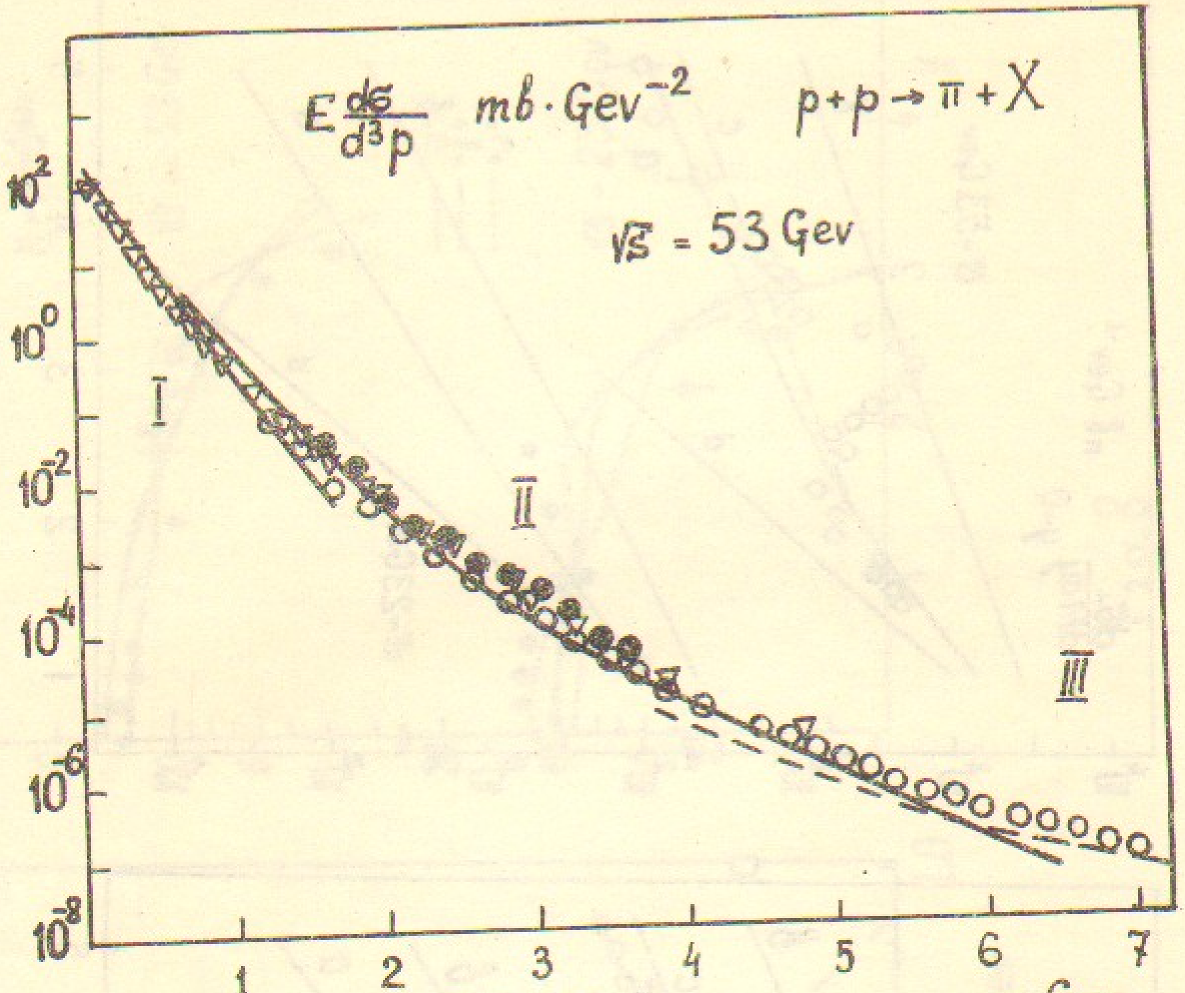


Fig. 11.

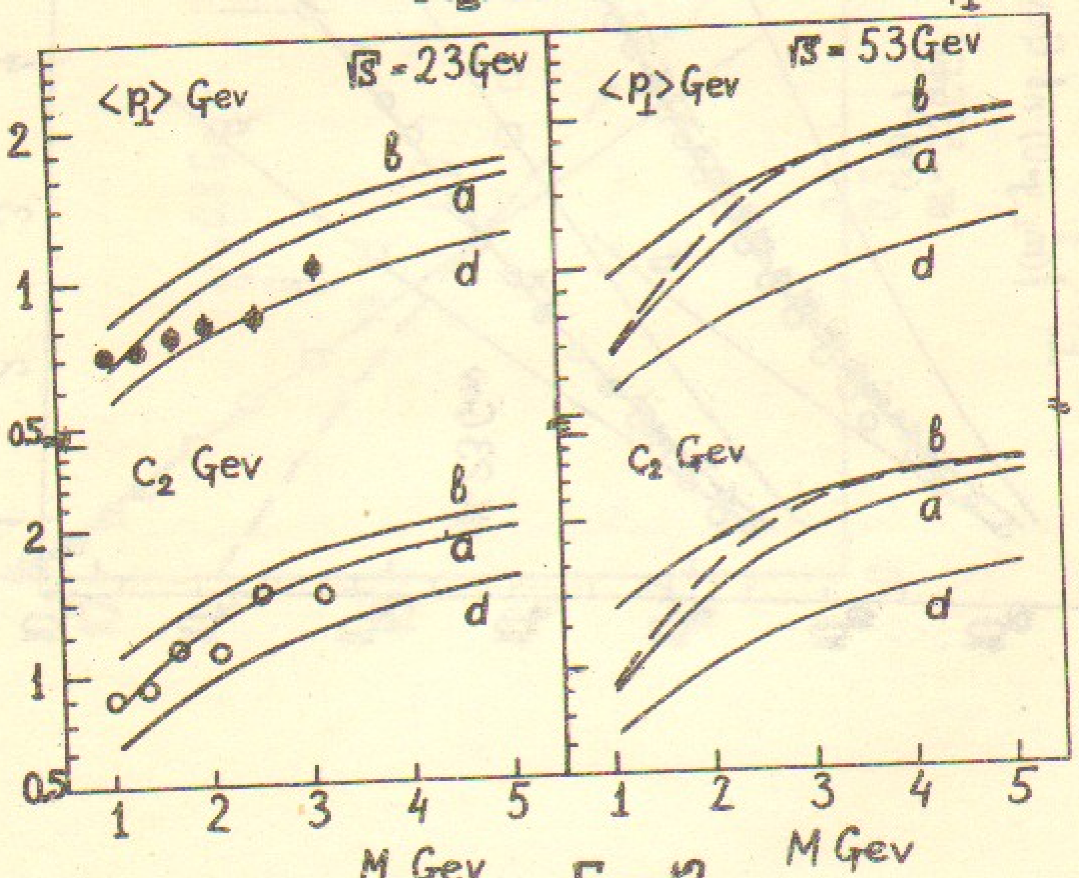


Fig. 12.

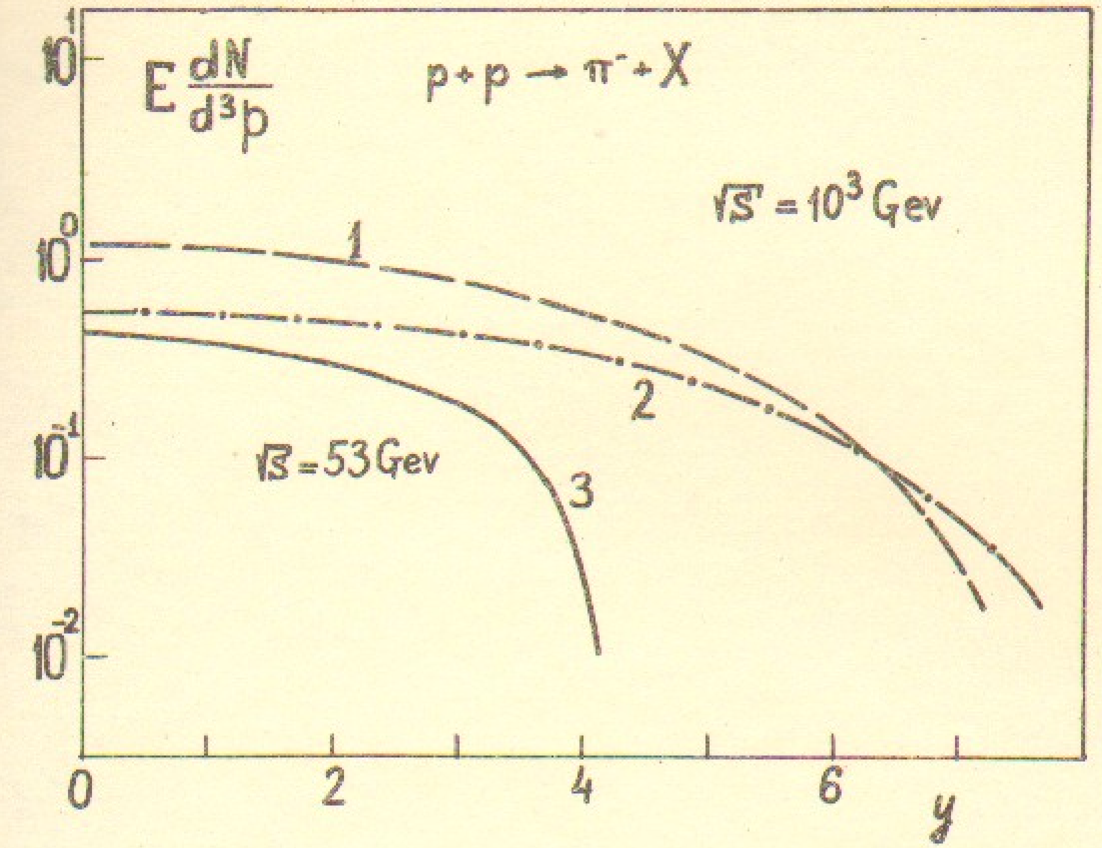


Fig. 13.

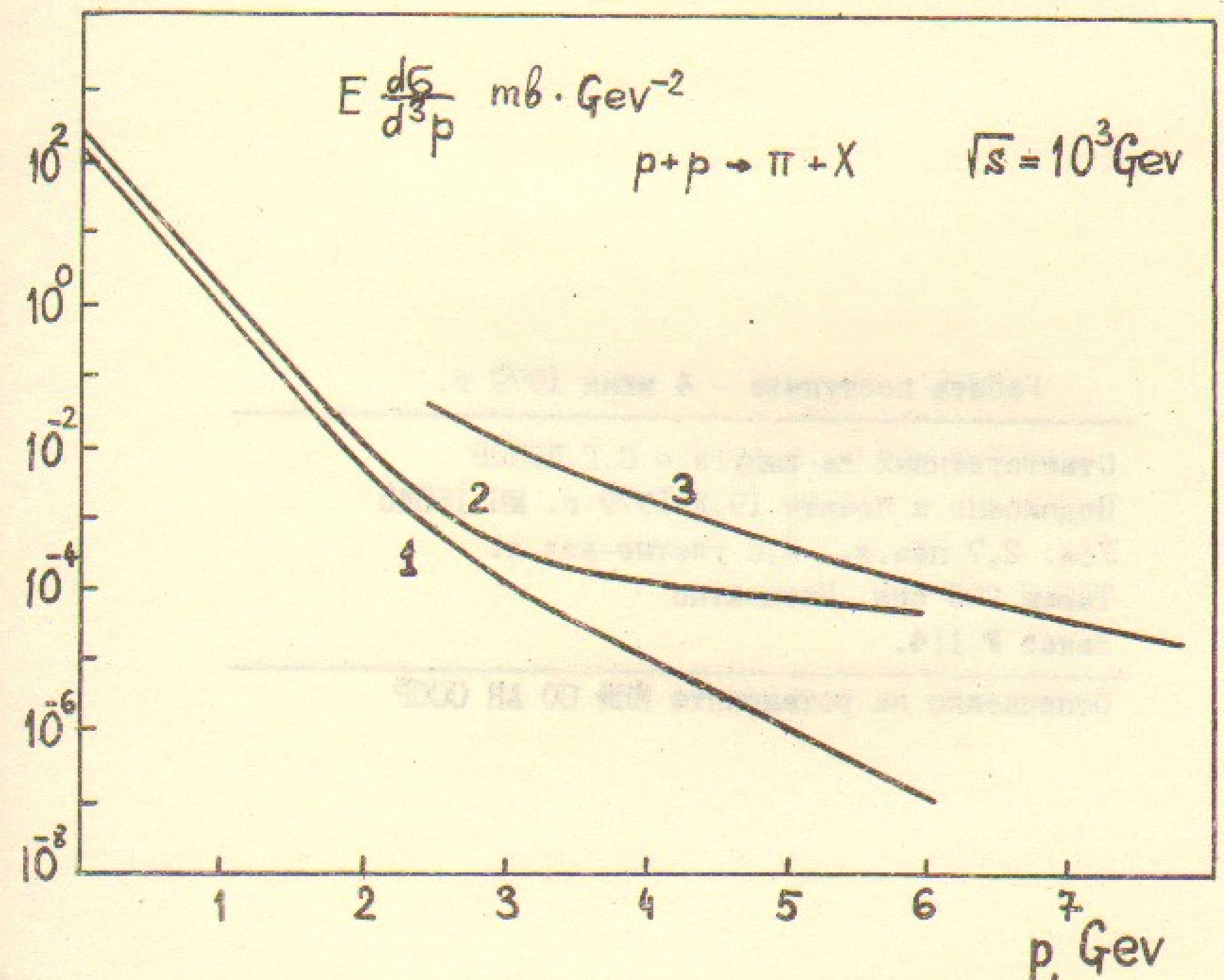


Fig. 14.