И Н С Т И Т У Т
ЯДЕРНОЙ ФИЗИКИ СОАН СССР

ПРЕПРИНТ И Я Ф 76 - 123

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Новосибирск
1976

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abstract

Faraday effect in the transitions $^4S'_{3/2}-^2D'_{3/2}$ and $^4S'_{3/2}-^2D_{5/2}$ in bismuth is computed taking into account hyperfine splitting of the lines.

1. INTRODUCTION

Recently M1 transitions in bismuth attract attention since they are convenient for the search for hypothetical parity violating weak interaction of electrons with nucleons. This interaction would lead to optical activity of bismuth vapor/1-3/. The experiments are now going on with two M1 transitions from the ground state $^4S'_{3/2}-^2D'_{3/2}$/4,5/ and $^4S'_{3/2}-^2D_{5/2}$/6,7/. Evidently, the optical activity caused by residual external magnetic field is a serious obstacle for these experiments. But from another point of view, the investigation of the Faraday effect in the controlled magnetic field may become a useful tool of lines identification in the presence of band spectrum of molecular bismuth vapor. It is especially important for the second of the transitions discussed. In connection with these circumstances we have carried out the detailed calculation of the Faraday effect for these transitions. Its results are presented in this work.

Due to a large value of the spin of nucleus Bi^{209} (I=9/2) both transitions have sufficiently complicated hyperfine structure (hfs). It is shown at the figures 1 and 2. The height of the line is proportional to its strength. These results are in agreement with the experimental data/8/. The Faraday effect was calculated by us not only for separate hf lines, but also for the conditions of the experiment/4,5/ where the width of the source line does not allow to resolve the hfs.

2. GENERAL FORMULAS FOR THE FARADAY EFFECT

The rotation angle at the length l of the polarization plane of a linearly polarized electromagnetic wave propagating along a magnetic field H in a medium with a refraction index n is

$$\psi = \frac{1}{2}\omega l \, \mathrm{Re}(n_+ - n_-) \qquad (1)$$

($\hbar = c = 1$). Here ω is frequency of light and $n_{\pm}$ are the values of refraction index for the photons of right and left helicities, $\vec{e}_{\pm} = \frac{1}{\sqrt{2}}(\mp\vec{e}_x - i\vec{e}_y)$. Under this definition the angle of rotation is positive if polarization plane rotates clockwise for an observer looking into the source.

Let J be the angular momentum of electronic shell in the ground state. Quantities that refer to the excited state we shall denote by prime. Under our conditions the components of the ground state hfs are equally populated, therefore the density of atoms with a total angular momentum F is

$$\frac{2F+1}{(2J+1)(2I+1)} N$$

where N is the total density of atoms. If the frequency ω is close to the frequency ω_0 of the transition between levels γJF and $\gamma' J' F'$ (γ refers to other quantum number of a level), then

$$n_{\pm} - 1 = \frac{-2\pi N}{(2J+1)(2I+1)} \overline{\sum_{M,M'}} \left| \langle \gamma JFM | M_{\pm} \mp \sqrt{\frac{\alpha\omega^2}{48}} T_{2\pm} | \gamma' J' F' M' \rangle \right|^2 \times$$

$$\times \left\langle \frac{1}{\omega - \omega_0 - \frac{v}{c}\omega_0 + i\Gamma/2} \right\rangle, \quad \overline{\sum_{M,M'}} = \frac{1}{2F+1} \sum_{M,M'} \qquad (2)$$

Here $I = 9/2$ is the nucleus spin, $M_{\pm} = \mp \mu_B (J_{\pm} + S_{\pm})$ is the operator of M1 transition ($j_{\pm} = j_1 \pm i j_2$), $\mu_B = \frac{|e|\hbar}{2mc}$ is Bohr magneton; T_{2q} is the spherical tensor of the second rank connected with the quadrupole momentum operator $\mathcal{D}_{\alpha\beta} = 3r_\alpha r_\beta - \delta_{\alpha\beta} r^2$ by the equality $T_{20} = D_{33}$.

In the last, resonance factor brackets denote averaging over Doppler frequency shift caused by thermal motion. Note that the collisional broadening can be perhaps neglected under reasonable conditions (pressure ~ 10 mm). As a function of the detune $\Delta = \omega - \omega_0$ this factor can be presented in the following way

$$\frac{1}{\Delta_D} \left[g(u) + i f(u) \right] \qquad (3)$$

where $u = \Delta/\Delta_D$; $\Delta_D = \omega_0 \sqrt{2kT/m}$ is the Doppler width; g(u) and f(u) are real functions (see, e.g., /9/). Graphs of the functions g(u) and $\partial g(u)/\partial u$ that are necessary to us below are presented at the figures 3,4. Thus

$$\psi = \frac{-\pi N \omega}{(2J+1)(2I+1)} \left(\Sigma_+ - \Sigma_- \right) \qquad (4)$$

where

$$\Sigma_{\pm} = \overline{\sum_{M,M'}} \left| \langle \gamma JFM | M_{\pm} \mp \sqrt{\frac{\alpha\omega^2}{48}} T_{2\pm} | \gamma' J' F' M' \rangle \right|^2 g(u) \qquad (5)$$

Note that g(u) is an odd function.

The Faraday effect arises mainly due to two mechanisms. One of them is the mixing by magnetic field of different components of hfs of levels. This mixing changes in (5) the matrix elements only, therefore the dependence on the detune is given by the factor g(u). Another mechanism is the Zeeman splitting of levels; here in the first order in a magnetic field H

$$g(u) \to g(u) + \frac{\partial g(u)}{\partial u} \cdot \frac{\delta E}{\Delta_D}, \qquad (6)$$

where $\delta E = \langle JFM | -\mu_z H | JFM \rangle - \langle J'F'M' | -\mu_z H | J'F'M' \rangle$, $\mu_z = -\mu_B (J_z + S_z)$. This part of the Faraday effect is proportional to $\partial g(u)/\partial u$ and is an even function of detune, in distinction from the first one. In the case of weak fields these two effects can be considered independently. Note that the odd Faraday effect has the same characteristic dependence on the detune Δ, as the angle of rotation caused by parity violation.

Summing over M and M' present the sum in (5) in the following symbolical form:

$$\Sigma_{\pm} = \left(M^2 + T^2 \pm 2M^2 H \pm 2T^2 H \pm 2MTH \right) g(u)$$

$$\pm \left(M'^2 H + T'^2 H + 2M'T'H \right) \frac{\partial g(u)}{\partial u} \qquad (7)$$

The origin of the terms in this expression can be understood from the notations. E.g.,

$$2M'T'H = -\sqrt{\frac{\alpha\omega^2}{48}} \overline{\sum_{M,M'}} 2 \langle JFM | M_+ | J'F'M' \rangle^* \langle JFM | T_{2+} | J'F'M' \rangle \frac{\delta E_{MM'}}{\Delta_D}$$

Note that in external magnetic field the interference between M1 and E2 amplitudes in the refraction index does not vanish generally speaking.

Write down explicitly the terms in the formula (7). Denote for brevity $m = \langle \gamma J \| M \| \gamma' J' \rangle$ and $t = \langle \gamma J \| T_2 \| \gamma' J' \rangle \sqrt{\frac{5}{\sqrt{3}}}$. The quantities M^2 and T^2 that determine the absorption are

$$M^2 = m^2 \beta_1 \quad , \qquad T^2 = t^2 \beta_2 \tag{8}$$

where

$$\beta_\alpha = \frac{(2F+1)(2F'+1)}{2\alpha+1} \begin{Bmatrix} I & J & F \\ \alpha & F' & J' \end{Bmatrix}^2 , \qquad \alpha = 1, 2$$

The terms dependent on the magnetic field and responsible for the Faraday effect look as follows

$$\begin{aligned} M^2 H &= m^2 \gamma_1 , & T^2 H &= t^2 \gamma_2 \\ M'^2 H &= m^2 \delta_1 , & T'^2 H &= t^2 \delta_2 \\ TMH &= m t \zeta & T'M'H &= m t \eta \end{aligned} \tag{9}$$

where

$$\gamma_\alpha = \frac{(2F+1)(2F'+1)}{\sqrt{\alpha(\alpha+1)(2\alpha+1)}} \begin{Bmatrix} I & J & F \\ \alpha & F' & J' \end{Bmatrix} \cdot \Bigg[g'(-1)^{I+J'+F+\alpha+1} \cdot \sqrt{J'(J'+1)(2J'+1)}$$

$$\times \Bigg(\frac{\mu_B H}{\Delta E_{F',F'-1}} (2F'-1) \begin{Bmatrix} I & J' & F' \\ 1 & F'-1 & J' \end{Bmatrix} \begin{Bmatrix} I & J' & F'-1 \\ \alpha & F & J \end{Bmatrix} \begin{Bmatrix} F & \alpha & F'-1 \\ 1 & F' & \alpha \end{Bmatrix} \tag{10}$$

$$+ \frac{\mu_B H}{\Delta E_{F'F'+1}} (2F'+3) \begin{Bmatrix} I & J' & F' \\ 1 & F'+1 & J' \end{Bmatrix} \begin{Bmatrix} I & J' & F'+1 \\ \alpha & F & J \end{Bmatrix} \begin{Bmatrix} F & \alpha & F'+1 \\ 1 & F' & \alpha \end{Bmatrix} \Bigg)$$

$$- \begin{pmatrix} J' \leftrightarrow J \\ F' \leftrightarrow F \\ g' \leftrightarrow g \end{pmatrix} \Bigg]$$

$$\delta_\alpha = \frac{(2F+1)(2F'+1)(-1)^{\alpha+1}}{\sqrt{\alpha(\alpha+1)(2\alpha+1)}} \begin{Bmatrix} I & J & F \\ \alpha & F' & J' \end{Bmatrix}^2 \Bigg[(-1)^{I+J'+F} \cdot g' \sqrt{J'(J'+1)(2J'+1)}$$

$$\times (2F'+1) \begin{Bmatrix} I & J' & F' \\ 1 & F' & J' \end{Bmatrix} \begin{Bmatrix} F & F' & \alpha \\ 1 & \alpha & F' \end{Bmatrix} + \begin{pmatrix} J' \leftrightarrow J \\ F' \leftrightarrow F \\ g' \leftrightarrow g \end{pmatrix} \Bigg] \frac{\mu_B H}{\Delta_2} \tag{11}$$

$$\zeta = g' \sqrt{J'(J'+1)(2J'+1)} \frac{(-1)^{I+J'+F}}{\sqrt{10}} \Bigg[\begin{Bmatrix} I & J' & F' \\ 1 & F'-1 & J' \end{Bmatrix} \Bigg(\begin{Bmatrix} F & F' & 2 \\ 1 & 1 & F'-1 \end{Bmatrix}$$

$$\times \begin{Bmatrix} I & J & F \\ 1 & F'-1 & J' \end{Bmatrix} \begin{Bmatrix} I & J & F \\ 2 & F' & J' \end{Bmatrix} + \begin{Bmatrix} F' & F & 1 \\ 2 & 1 & F'-1 \end{Bmatrix} \begin{Bmatrix} I & J & F \\ 2 & F'-1 & J' \end{Bmatrix} \begin{Bmatrix} I & J & F \\ 1 & F' & J' \end{Bmatrix} \Bigg) \tag{12}$$

$$\times \frac{\mu_B H}{\Delta E_{F',F'-1}} (2F+1)(2F'+1)(2F'-1) - (F' \to F'+1) \Bigg] + \begin{pmatrix} J' \leftrightarrow J \\ F' \leftrightarrow F \\ g' \leftrightarrow g \end{pmatrix}$$

$$\eta = \frac{(2F+1)(2F'+1)}{\sqrt{10}} \begin{Bmatrix} I & J & F \\ 1 & F' & J' \end{Bmatrix} \begin{Bmatrix} I & J & F \\ 2 & F' & J' \end{Bmatrix} \Bigg[(-1)^{I+J'+F} g' \sqrt{J'(J'+1)(2J'+1)} \tag{13}$$

$$\times (2F'+1) \begin{Bmatrix} F & F' & 2 \\ 1 & 1 & F' \end{Bmatrix} \begin{Bmatrix} I & J' & F' \\ 1 & F' & J' \end{Bmatrix} + \begin{pmatrix} J' \leftrightarrow J \\ F' \leftrightarrow F \\ g' \leftrightarrow g \end{pmatrix} \Bigg] \frac{\mu_B H}{\Delta_2}$$

In these **formulas** g and g' are the g-factors of the ground state and the excited one correspondingly; $\Delta E_{F,F-1}$ and $\Delta E_{F',F'-1}$ are the distances between the corresponding components of hfs ($\Delta E_{F,F-1} = AF + 4B \cdot F \left[F^2 + \frac{1}{2} - J(J+1) - I(I+1) \right]$) where A and B are the constants of hf splitting.

Some sum rules are useful for the control of the results. Denote by $\psi^{ev}_{FF'}$ and $\psi^{odd}_{FF'}$ the rotation angles caused by the even and odd Faraday effects in the transition JF – J'F' and define the coefficients $R^{ev}_{FF'}$ and $R^{odd}_{FF'}$ in such a way that

$$\psi^{ev}_{FF'} = R^{ev}_{FF'} \frac{\partial g(u)}{\partial u} , \quad \psi^{odd}_{FF'} = R^{odd}_{FF'} g(u)$$

Then, firstly, the following evident sum rule takes place

$$\sum_{FF'} R^{odd}_{FF'} = 0 \tag{14}$$

Indeed, since the odd effect is caused by the mixing of different components of hfs, at the summation over hfs it vanishes.

Secondly, it is clear from simple physical considerations that if the detune is much larger than hf splitting, the Faraday effect does not depend on hfs. Therefore, the equality takes place

$$\sum_{F,F'} \left(\psi^{odd}_{FF'}(u) + \psi^{ev}_{FF'}(u) \right) = \psi^{ev}(I=0) \tag{15}$$

Under the condition mentioned u can be presented as $u = u_o + \delta u_{FF'}$, $u_{FF'} \ll u_o$ where u_o is the distance to the centre of hfs and $u_{FF'}$ is the location of a separate hf line. Hence, with good accuracy

$$\psi^{odd}_{FF'}(u) \approx R^{odd}_{FF'} g(u_o) + R^{odd}_{FF'} \frac{\partial g}{\partial u_o} \delta u_{FF'} \tag{16}$$

The first term here vanishes, according to (14), after the summation over F and F', and the second one depends on u in the same way as the even effect. From (15) and (16) the sum rule follows

$$\sum_{FF'} \left(R^{odd}_{FF'} \frac{E_{FF'}}{\Delta_2} + R^{ev}_{FF'} \right) = R^{ev}(I=0) \tag{17}$$

The sum rule (17) can be deduced directly also, using known properties of 6j-symbols.

3. NUMERICAL RESULTS

Using the formulas (4),(5),(9)-(13), one can obtain the magnitude of the Faraday effect for the transitions of interest between the components of hfs (see the tables 3,4). The values of parameters of these transitions are given in the tables 1,2. For definiteness, the temperature of vapor is taken to be 1200°C. The partial pressure of atomic bismuth constitutes at this temperature 23 mm, according to/10/.

The parameter $\varkappa$ (the fourth column of the table 2) is the relative magnitude of the quadrupole contribution

$$\varkappa = \sqrt{\frac{\alpha \omega^2}{48}} \frac{\langle J \| T_2 \| J' \rangle}{\langle J \| M \| J' \rangle} \tag{18}$$

The reduced matrix elements in this formula are calculated, e.g., in /9/.

We have calculated also the Faraday effect in the first transition in the case when the width of the source line does not allow to resolve the hfs (see /4,5/). For the conditions given in the work/5/ (temperature of vapor 1227°C, the length of the path 1 m, the linewidth of the source 1 Å), the computed value of the Faraday effect in maximum constitutes $1.2 \cdot 10^{-4}$ radian/gauss in a good agreement with the experiment/5/. The calculated value of the maximal absorption 30% is somewhat lower than the experimental one 45%. We assumed the form of the source line to be gaussian.

We have calculated also for a broad source line the ratio of the rotation angle ψ in the maximum of the dispersion curve to the degree of circular polarization P due to parity violation. At the same conditions/5/

$$\psi = 1.0 \cdot P$$

We are sincerely grateful to L.M.Barkov and M.S.Zolotoryov for valuable discussions.

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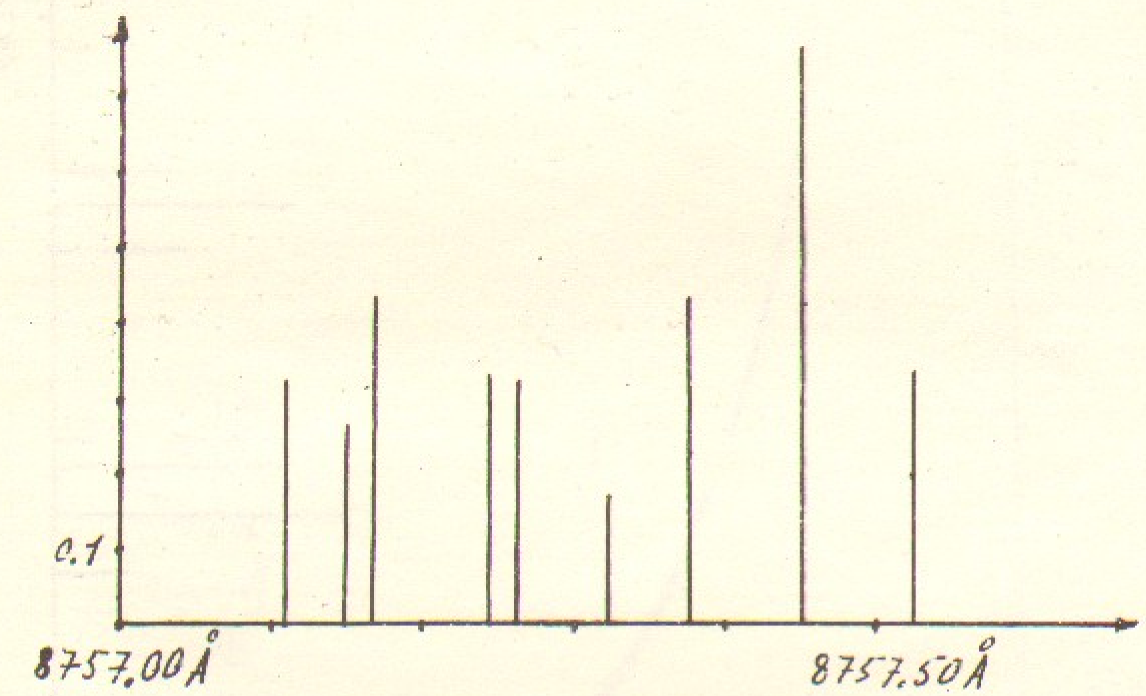

Fig. 1.

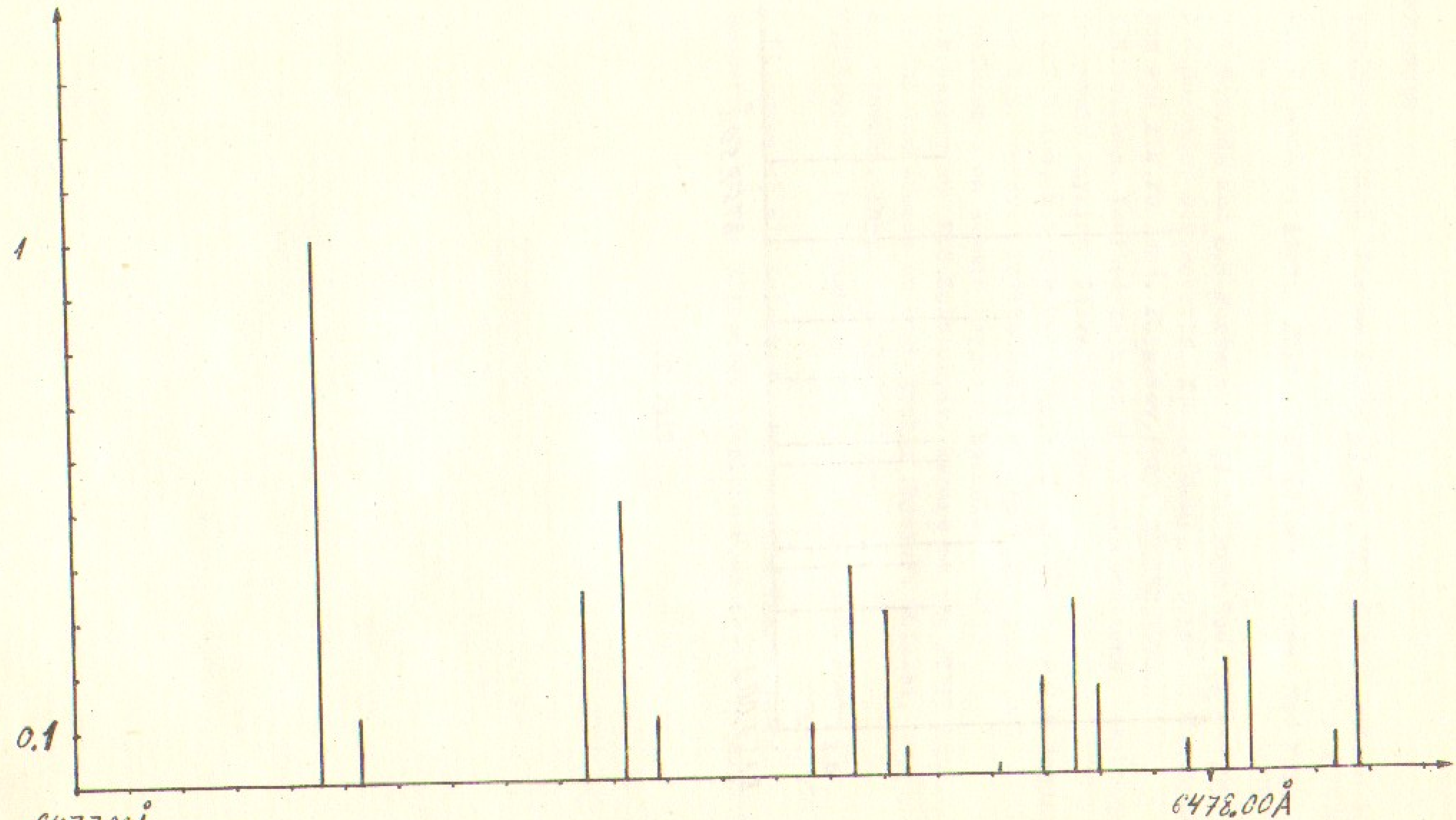
1
0.1
6477.00Å
6478.00Å

Fig. 2.

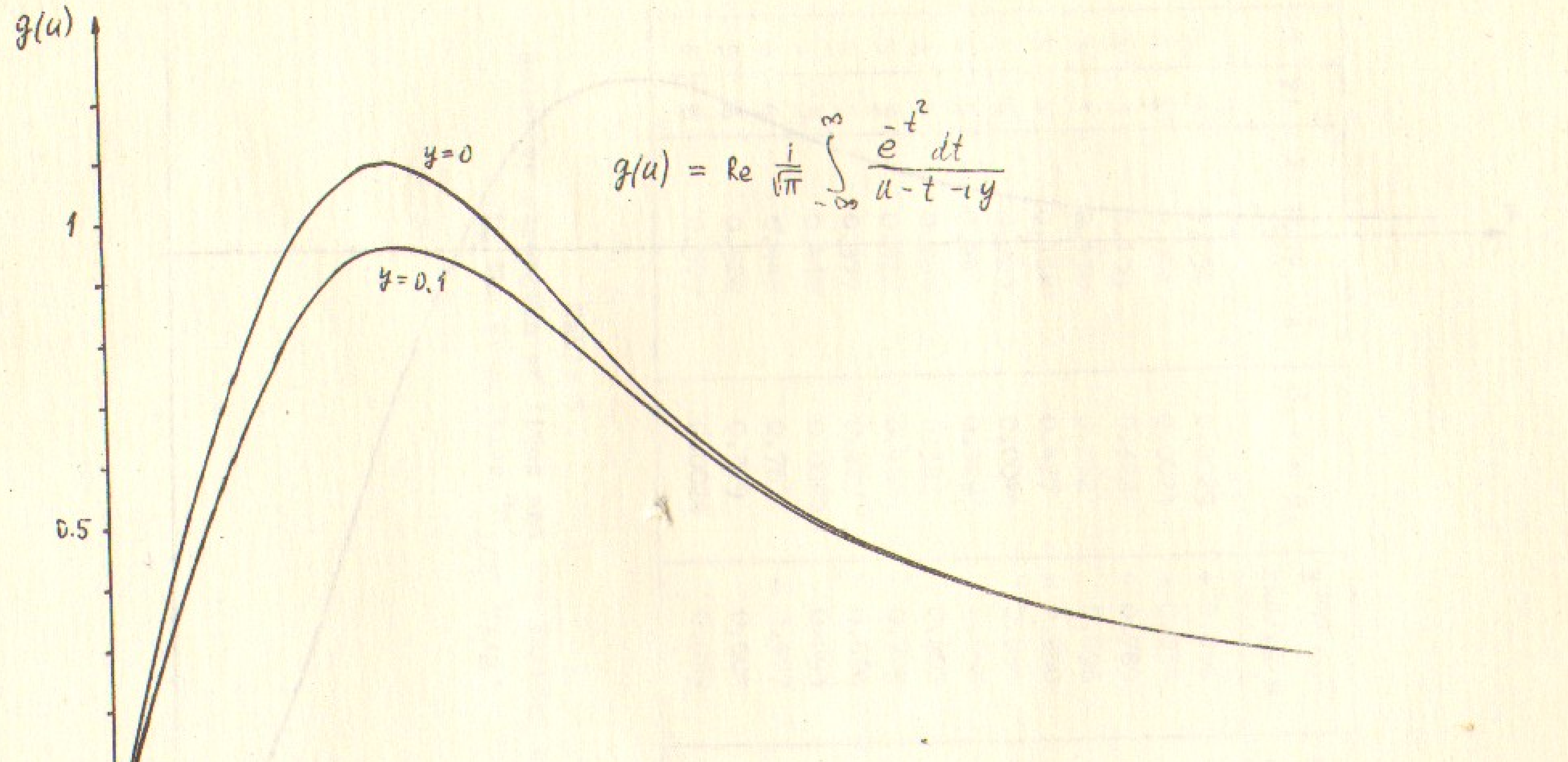
g(u)
y=0
y=0.1
$g(u) = \mathrm{Re}\,\frac{i}{\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{e^{-t^2}\,dt}{u-t-iy}$
1
0.5
1
2
3
4
u

Fig. 3.

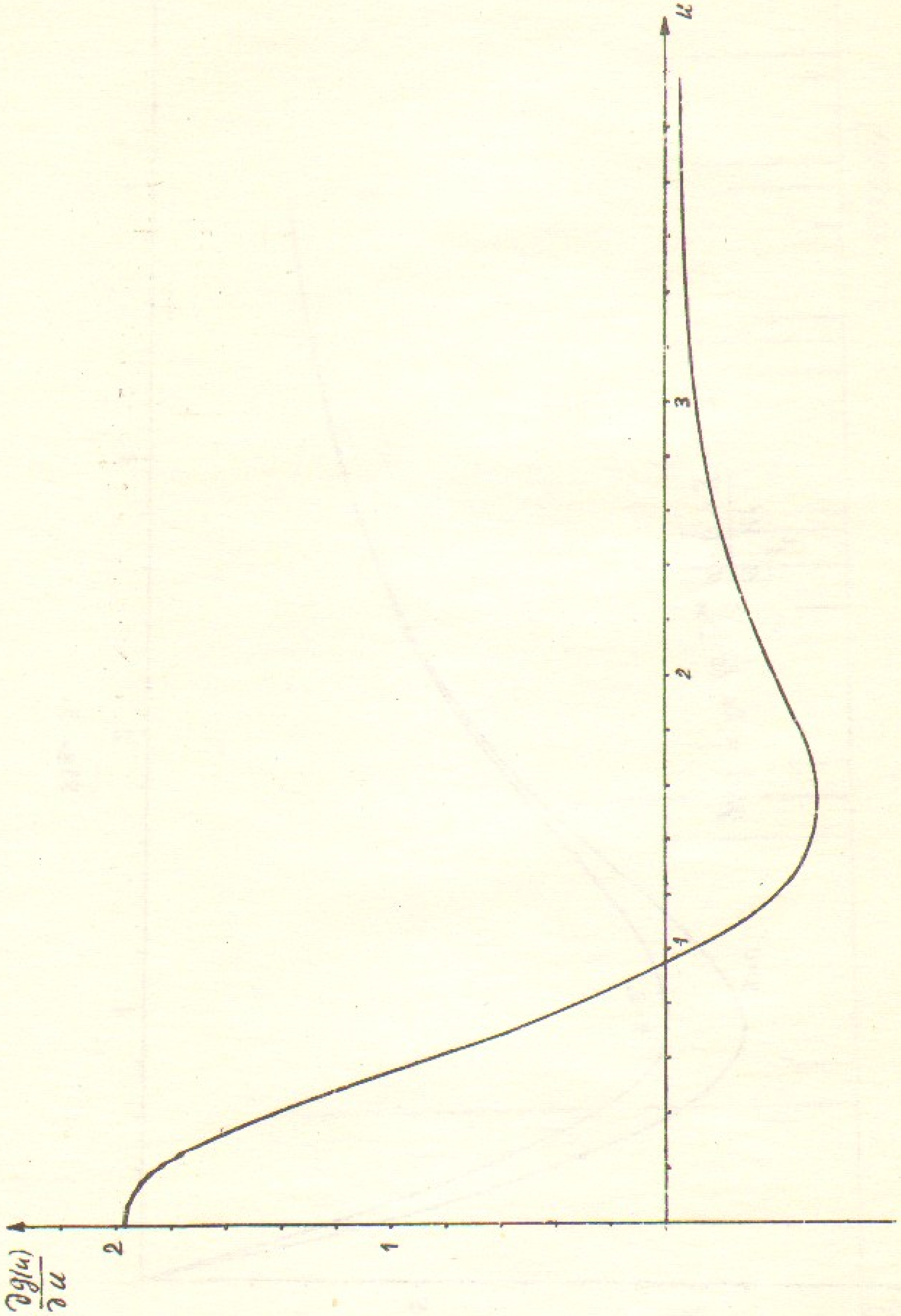

Fig. 4.

	F	F'	$\lambda - 8757$, Å	$\beta_1 + \varkappa^2\beta_2$	$R^{odd}\cdot 10^2$ rad/gs·m	$R^{even}\cdot 10^2$ rad/gs·m
1	5	3	0.054	0.0029	- 0.056	0,016
2	6	4	0.094	0.0024	- 0.034	0.014
3	4	3	0.109	0.322	- 0.789	0.933
4	3	3	0.150	0.262	0.696	- 1.043
5	5	4	0.168	0.427	- 0.250	1.963
6	4	4	0.223	0.0044	0,084	0.003
7	6	5	0.249	0.329	0.058	1.300
8	3	4	0.264	0.322	0.202	1.341
9	5	5	0.322	0.166	- 0.709	0.343
10	4	5	0.377	0.427	0.616	1.590
11	3	5	0.418	0.0029	0.095	0.017
12	6	6	0.448	0.761	- 0.577	0.471
13	5	6	0.522	0.329	0.604	0.391
14	4	6	0.577	0.0024	0.059	0.011

Table 1

Relative strengths of lines and the Faraday effect in the transition $^4S'_{3/2} - {}^2D'_{3/2}$.

	F	F'	$\lambda - 6477$, Å	$\beta_1 + \varkappa^2\beta_2$	$R^{odd}\cdot10^3$ rad/gs·m	$R^{even}\cdot10^3$ rad/gs·m
1	6	7	0.227	0.996	0.327	10.771
2	5	7	0.267	0.122	- 0.497	0.454
3	6	6	0.474	0.347	0.824	4.464
4	5	6	0.514	0.507	- 0.211	2.228
5	4	6	0.544	0.115	- 0.753	0.534
6	6	5	0.684	0.095	0.438	0.987
7	5	5	0.725	0.376	0.753	2.241
8	4	5	0.755	0.295	- 0.702	0.840
9	3	5	0.777	0.054	- 0.589	0.321
10	6	4	0.859	0.016	0.119	0.064
11	5	4	0.899	0.175	0.821	0.959
12	4	4	0.930	0.315	0.141	0.075
13	3	4	0.952	0.164	- 1.123	1.254
14	5	3	1.038	0.049	0.405	0.169
15	4	3	1.068	0.207	0.720	- 0.101
16	3	3	1.091	0.266	- 1.058	0.753
17	4	2	1.172	0.075	0.685	0.226
18	3	2	1.194	0.298	- 0.301	0.968

Table 2

Relative strengths of lines and the Faraday effect in the transition ${}^4S'_{3/2} - {}^2D_{5/2}$

	g	A, cm^{-1}	$B\cdot10^5$, cm^{-1}
${}^4S'_{3/2}$	1.737	- 0.0149	- 3.533
${}^2D'_{3/2}$	1.247	- 0.0410	- 7.609
${}^2D_{5/2}$	1.200	0.0835	1.042

Table 3

g - factors and constants of hf splitting of the levels

	Δ_D, cm^{-1}	$\frac{\langle J\| M \| J'\rangle}{\mu_B}$	$\frac{\langle J\| T_2 \| J'\rangle}{\langle r^2\rangle}$	$\varkappa$
${}^4S'_{3/2} - {}^2D'_{3/2}$	$1.31\cdot10^{-2}$	- 1.723	- 1.240	0.12
${}^4S'_{3/2} - {}^2D_{5/2}$	$1.74\cdot10^{-2}$	- 0.584	1.976	- 0.75

Table 4

Работа поступила - I4 октября 1976г.

Ответственный за выпуск - С.Г.ПОПОВ
Подписано к печати I5.XI-I976г. МН 03027
Усл. I,0 печ.л., 0,8 учетно-изд.л.
Тираж I50 экз. Бесплатно
Заказ № I23.

Отпечатано на ротапринте ИЯФ СО АН СССР