И Н С Т И Т У Т
ЯДЕРНОЙ ФИЗИКИ СОАН СССР





ПРЕПРИНТ И Я Ф 76 - 86

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# FERMION REGGEIZATION IN RENORMALIZABLE YANG-MILLS MODELS

Новосибирск

1976

FERMION REGGEIZATION IN RENORMALIZABLE YANG-MILLS MODELS

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We calculate the high energy behavior of vector meson - fermion scattering amplitude near the backward direction in the spontaneously broken Yang-Mills models within the leading logarithmic approximation and find, that the fermion is reggeized. The amplitudes of inelastic fermion-exchange processes in the multiregge kinematics are also calculated. These amplitudes have the multiregge form. The integral equations for the elastic process partial wave amplitudes are obtained.

## Introduction

Recent interest in the particle reggeization problem has been arisen by the results obtained /I,2/ that the vector particles are reggeized in Yang-Mills models with the spontaneously broken vacuum symmetry. In /I,2/ the proof was carried out in the main logarithmic approximation ( $g^2 \ln s \sim 1$ , $g^2 \ll 1$ ) up to the sixth order in coupling constant $g$ . The calculation in the next order of perturbation theory /3,4/ made it possible to propose the general structure of perturbation series. The sum of these series may be presented in the form of the integral equation. Its solution for the negative signature is the reggeon, where as the Pomeranchon is a fixed branch point located at the right hand from $j=1$ /3,4/, which appears, by all means, due to the effective many particles states contribution in t -channel. The results for the sixth and eighth order was then confirmed in works /5/ and /6/ accordingly.

There is the next question naturally arisen, if the fermion is reggeized in such models or not. It's well-known /7/, that in QED electron is reggeized in the main logarithmic approximation, i.e. the backwards Compton amplitude in the own (positive) signature has the form of the reggeon contribution. As for the negative signature the situation is rather complicated even in QED. Already in the sixth order the diagram represented on Fig. I, containing no terms $\sim g^6 \ln^2 s$ and hence, being negligible in the own signature consideration, must be taken into account while the negative signature is concerned, because its imaginary part on is of the normal $\sim i g^6 \ln s$ order. There is just the same situation that in the vector meson reggeization problem in which such diagrams (with the replacement of fermion to boson) don't contribute in the own but remain in the else (for vector meson -positive) signature. Such a contribution be proportional to the three particle loop in the two dimension perpendicular momenta space /I,2/ and mostly is alike as the cut contribution rather than reggeon. In the given paper we deal, as a rule, with the fermion reggeization problem or, saying equally, with the positive signature amplitude. As for the negative signature amplitude, we only present the integral equations. The similar equations may be written in QED and are also presented in our work.

In this paper it is shown that the fermions are reggeized in the main logarithmic approximation. As well as in /2/ the proof is carried out for two Yang-Mills models with spontaneous breakdown of vacuum symmetry in one of which all the vector particles have the same mass, the other involves photon. Both the models contain the fermion doublet $(p,n)$, the fermion masses may differ in second model. If it's a case, two different trajectories $\delta_1(q)$, $\delta_2(q)$ appear with $\delta_1(\hat{q}=M_p)=\frac{1}{2}$; $\delta_2(\hat{q}=M_n)=\frac{1}{2}$. As well as in QED the reggeization in second model occurs in conventional sense only due to the $\gamma$ - quantum presence and infrared catastrophe related to them. Every obtained trajectory is correspondent to two ones /7/ of the opposite parity. We shall not dwell upon this fact further.

The consideration is carried out by two ways. The first way (will be referred as diagram one) is as follows. Similar to /I,2/ we obtain imaginary part of the amplitude JmM from the unitarity condition in s(u) - channel, after that the main logarithmic term is found by means of dispersion relations. This method is, however, simplified to some extent. In the first place, we note that the diagrams of the type represented on Fig.I don't contribute in the main logarithmic approximation and may be neglected at once if only the reggeization problem is solving. The other diagrams may be separated in two types. At first, there are the ladders (Fig.2). Such ladders contain not only the logarithmic but also the double logarithmic terms, being produced due to integration over the range of large transverse components of virtual particle momenta /8/. But the contribution from this range is canceled in the sum with the contribution of the second class of diagrams : i.e. the crossing diagrams associated with the ladders of the type represented on Fig.3. The main problem, therefore, is to check exactly this cancellation of the double logarithmic terms. We propose here a simple rule for the models discussed, how it is possible to make the unambigious separation of the main logarithmic term by means of the term of ladders contribution only. This rule can be checked in the sixth order of perturbation theory, for the vector meson reggeization problem it is verified up to the eighth order. After this the answer is reduced to the sum of "effective" ladders every step of which is correspondent to a transition matrix $T_{ik}$ between



the up and down states, depending only on the transfered moment This results to the reggeons with trajectories depending on the eigenvalue and eigenvectors of $T_{ik}$. For the problem discussed there is a single reggeon with isospin $\frac{1}{2}$ . The state with isospin $3/2$ is appeared to have no contribution in the main logarithmic term (the correspondence eigenvalue $\lambda^{+}_{3/2}=0$). The method stated above is useful only in calculation the main contribution in own signature!

The second way similar to /3,4/ is based on the integral equation and may be used both for the positive and for the negative signatures. As well as in /I,2/ we use a dispersion method, so the amplitudes of inelastic fermion-exchange processes in the multiregge kinematics are also calculated. Up to the eighth order of perturbation theory our calculations give simple multiregge form for the inelastic amplitudes, which can be easily generalized to the arbitrary order. Using this generalization the integral equations may be written for the elastic partial wave amplitude of arbitrary signature /9/ as well as in /3,4/. For the positive signature the result is naturally the same that obtained in the first way; for the negative one, we only present this equation without solving it. The similar equation may be written in QED and will be also presented.

In accordance to the methods of approach the paper consists of two sections: in the first section the problem is solved by the diagram way, in the second - by means of integral equations.

## I. The diagram way

### I.I. Model description

Two simple gauge models based on $SU(2)$ group are considered. The massless Yang-Mills vector particles (B -meson) acquire the mass due to the Higgs-Kibble mechanism /IO/. In model I the vacuum is spontaneously broken by means of the complex isodoublet of the scalar particles. As a result all the B -mesons have the same masses $m$ . The Lagrangian remains invariant under $SU(2)$ transformations. In model 2 breaking is related to the real scalar particles triplet, the neutral vector particle remains massless ( $\gamma$ - quantum), while the charged particle acquires the same mass $m$ .

The presence in this model of interaction between scalar particles and fermions of type: $\mathcal{L}_{int} = -\lambda\bar{\psi}\vec{\tau}\psi\vec{\varphi}/2$ leads to the difference between the fermion masses after spontaneous breaking. There is a simple relation between masses and vertices to be used in calculations :

$$\frac{M_p - M_n}{m} = \frac{\lambda}{q} \qquad (I)$$

Where $q$ is the vector meson charge. These models were described more detailed in /2,II/.

To avoid the complications related to the infrared divergencies and SU(2) non-invariance, the model I would be supposed, as a rule, in the following. The complications, appearing in model 2 will be considered in section I.5.

## I.2. Notations. Born approximation

Consider the process of backward fermion-meson scattering : NB→BN. The Born diagram, corresponding to the main contribution in this process at high energy, is represented on Fig.4. Notations correspond to Fig.4 : p(L') is the moment of initial (final) fermion, L(p') is the moment of initial (final)meson, $s=(L+p)^2, q=p-p'$ We are interested in the range $s \gg m^2, -q^2 \sim m^2$. In this range the Sudakov's technique is useful : the any vector $k_i$ may be represented as follows :

$$k_i = \alpha_i L^* + \beta_i p^* + k_{i\perp} \qquad (2)$$

Where $L^{*2} = p^{*2} = 0$; $L^* \simeq L - \frac{m^2}{s} p$, $p^* \simeq p - \frac{M^2}{s} L$; $2p^* L^* \simeq s$

Further we will omit the stars in notations.

It's a place to present in this notations the main contribution ($\sim q^2\sqrt{s}$), corresponding to the Born term :

$$\mathcal{M} = q^2 \bar{u}(L') \ell_1^\mu \Gamma_{1\mu} \frac{\vec{b}_1\vec{\tau}}{2} \frac{1}{M - \hat{q}_\perp} \frac{\vec{b}_2\vec{\tau}}{2} \ell_2^\nu \Gamma_{2\nu} u(p) \qquad (3)$$

where $\ell_1, \ell_2$ are polarization vectors of initial and final meson; $\vec{b}_1, \vec{b}_2$ are their isotopic vectors,

$$\Gamma_1^\mu = \gamma^\mu - \frac{\hat{L} p^\mu}{(pL)}; \quad \Gamma_2^\nu = \gamma^\nu - \frac{\hat{p}' L^\nu}{(pL)}$$



With the use of the $\Gamma_1^\mu, \Gamma_2^\nu$ instead of $\gamma^\mu, \gamma^\nu$ only the Born diagram on Fig.4 contribute, as well as in QED /7/

## I.3. The Method of Calculations

Let's proceed to the higher orders of perturbation theory. The calculations may be divided into two steps :

I). It is necessary to sum all the diagrams in a given order of perturbation theory, i.e. the ladders (Fig.2) and crossed diagrams associated with the ladders (Fig.3). The result is some "effective" ladders in which the contribution of the range of large transversal momenta is disappeared; the rule being introduced how this disappearance may be obtained by means of the form of the ordinary ladder contribution.

2). The resulting "effective" ladders have to be summarized over the all orders in coupling constant.

### I.3.I. "Effective" ladders.

Let's show, how the main logarithmic contribution from the totality of diagrams may be exactly extracted by the form of the ladders contribution. For simplicity we consider the sixth order of perturbation theory. By this example it will be shown what the term "crossed diagrams associated with the ladder " means, and how the necessary cancellations occur. It seems that in this case the example is more clear than the strong definition, being too complicated to be used.

Two typical ladders are represented on Fig.5(a,b). Consider spin structure of the step contribution in diagram 5a :

$$\langle NB|T|BN\rangle \backsim \frac{2}{s}\hat{p}(M+\hat{q}-\hat{k}_2-\hat{k}_1)L \simeq$$
$$\simeq 2\left[(M-\hat{q}_\perp+\hat{k}_{1\perp}) + (M-\hat{q}_\perp+\hat{k}_{2\perp}) - (M-\hat{q}_\perp)\right] \qquad (4)$$

Two first terms in bracket cancel the fermion propagators and give the double logarithmic contribution in JmM due to the integration over the large $k_\perp$ range. But it is easy to verify for all possible kinds of fermions and mesons that in the models discussed this contribution is exactly canceled with the crossed diagrams 6(a,b) and 6(c,d) respectively. Now consider the step contribution in diagram 5b with Yang-Mills vertex of three meson interaction :

$$\Gamma_{\tau} = -(k_{1\perp} + k_{2\perp})_{\tau} + \alpha_1 L_{\tau} + \beta_2 P_{\tau}$$

The longitudinal part may be neglected and hence :

$$\langle NB|T|NB\rangle \backsim (\hat{k}_{1\perp} + \hat{k}_{2\perp}) =$$

$$= [(M - \hat{q}_{\perp} + \hat{k}_{1\perp}) + (M - \hat{q}_{\perp} + \hat{k}_{2\perp}) - 2(M - q_{\perp})] \quad (5)$$

Two first terms in brackets are exactly cancelled with the diagrams 7(a,b) and 7(c,d) respectively. By generalization we propose the simple rule for the main logarithmic term extraction in the gauge theories.

It is necessary to subtract the sum of inversion propagators of the leg particles adjacent to the vector meson line from the contribution of every step in the ladder. The sum of the rest gives the main logarithmic term. This rule is checked in QED and vector meson reggeization problem up to the eighth order of perturbation theory.

It is necessary to recall that this rule may be used only for the own signature contribution, if the main logarithm occurs in ReM. For imaginary part, having additional $q^2$ ($\mathcal{I}m\, M \ll Re\, M$ , the diagrams of the type represented on FigI must be taken into account except the diagrams 5,6,7.

I.3.2. The sum of the ladders.

The formulae, presented in this paragraph, are mostly some generalization of relations obtained in /I,2/ by means of the S - channel unitarity condition for the case when the two dimension diagrams in the transversal momenta plane have a simple form. So, consider the arbitrary ladders (Fig.8) with the any pair of particles in legs $\psi_i^1, \psi_i^2$ . We are interested in imaginary part in S-channel. Consider the one step in ladder (Fig.9a,b). Two possible types of transition may be responsible for this step :

$$\psi_i^1 \psi_i^2 \to \psi_k^1 \psi_k^2 \,(\text{Fig } 9a), \quad \psi_i^1 \psi_i^2 \to \psi_k^2 \psi_k^1 \,(\text{Fig } 9b)$$



Let's define the matrix of transition $T_{ik}$ in basis $(\psi_i^1 \psi_i^2, \psi_i^2 \psi_i^1)$. As only the main term we are interested in, the matrix $T_{ik}$ corresponds to the nonsense - nonsense transition. In other words, calculating $T_{ik}$, polarizations of B -meson must be choosen as follows: $l_\mu^i = \sqrt{\frac{2}{s}} P_\mu$ ; $l_\nu^k = \sqrt{\frac{2}{s}} L_\nu$. If the T elements depend only on the vector q , then imaginary part of the sum of all ladders is as follows :

$$\mathcal{I}m\, \mathcal{M} = \frac{1}{4s} \langle 1| T^{sn} \chi\, e^{\frac{T}{4\pi} \chi \ln s} T^{ns} |2\rangle \quad (6)$$

where $\langle I|$ and $|2\rangle$ are the external states, $T^{sn}$ is the matrix of transition between the external (sense) and internal (nonsense) states, by symbol $\chi$ the two dimensional Feynman loop is denoted:

$$\chi = \int \frac{d^2 k_\perp}{(2\pi)^2} G_1(k_\perp) G_2(q_\perp - k_\perp) \quad (7)$$

Here $G_1$ , $G_2$ are the propagators of particles $\psi^1, \psi^2$. In general there may be distinction between the particle propagators $\psi_i^1, \psi_i^2$ for different values of index i . If it's a case, the symbol $\chi$ must be treated as a matrix : $\chi^{ik} = \delta_{ik} \chi^i$ . In the next equations we treat $\chi$ as a number, as it is in the model I. In model 2 there is a non essential complication.

The eq.(6) gives a sum of the several reggeons contributions, corresponding to eigenvalues $\lambda$ and eigenvectors $\varphi_\lambda$ of T with the trajectories :

$$\delta_\lambda(q) = n + \frac{\lambda}{4\pi} \chi \quad (8)$$

and residues :

$$\Gamma_\lambda(q) = \frac{1}{4 s^{1+n}} \langle 1| T^{sn} |\varphi_\lambda\rangle \langle \varphi_\lambda| T^{ns} |2\rangle \chi \quad (9)$$

where $n$ is matched from condition $\Gamma_\lambda(q) \xrightarrow[s \to \infty]{} \text{const.}$ (For the fermion reggeization problem $n = \frac{1}{2}$, for vector meson $n = 1$ ).

Owing to transposition symmetry of matrix T all the eigenvectors may be separated on the one of positive and negative parity under index transposition $1 \leftrightarrow 2$ :

$$\varphi^+_\lambda \backsim \frac{1}{\sqrt{2}}\left(\psi^1\psi^2 + \psi^2\psi^1\right) \tag{10}$$

$$\varphi^-_\lambda \backsim \frac{1}{\sqrt{2}}\left(\psi^1\psi^2 - \psi^2\psi^1\right)$$

This vectors are responsible for the contribution of different signature :

$$\sigma_{\varphi^+} = \sigma_1\sigma_2 \tag{11}$$

$$\sigma_{\varphi^-} = -\sigma_1\sigma_2$$

where $\sigma_1$, $\sigma_2$ are signatures corresponding to the particles $\psi^1, \psi^2$. The real part of the amplitude is obtained by means of signature factor.

Although the method of calculation developed here is of no principal hewness in comparison with /1,2/, but it is very convenient for the study of the reggeization problem. At first, it gives the possibility to write the answer at once, and second, it points how the Regge parameters, namely, t -channel quantum numbers and signature appear, in the main logarithmic approximation.

## 1.4. Fermion reggeization in model I.

Consider now immediately the fermion reggeization problem in model I. For definitiveness we study the case of the "proton" exchange in t -channel. Let's choose the basic states as follows: $B^+n, B^0p, nB^+, pB^0$. As it was shown above: $T \backsim -2(M-\hat{q}_\perp)$. So by substituting the number in vertex, it easy to obtain :

$$T = -\frac{(M-\hat{q}_\perp)}{2} g^2 \begin{Vmatrix} -2 & -2\sqrt{2} & 0 & -\sqrt{2} \\ -2\sqrt{2} & 0 & -\sqrt{2} & 1 \\ 0 & -\sqrt{2} & -2 & -2\sqrt{2} \\ -\sqrt{2} & 1 & -2\sqrt{2} & 0 \end{Vmatrix} \tag{12}$$



This matrix has 4 eigenvalues $\lambda^\sigma_T$ and eigenvectors $\varphi^\sigma_T$ in accordance with two possible isospin values 1/2 and 3/2 and two signatures $\sigma = \pm 1$.

$$\lambda^+_{1/2} = \frac{3}{2} g^2 (M-\hat{q}_\perp), \quad \varphi^+_{1/2} = -\frac{1}{\sqrt{6}}(\sqrt{2}, 1, -\sqrt{2}, -1)$$

$$\lambda^+_{3/2} = 0, \quad \varphi^+_{3/2} = \frac{1}{\sqrt{6}}(-1, \sqrt{2}, 1, -\sqrt{2}) \tag{13}$$

$$\lambda^-_{1/2} = \frac{5}{2} g^2 (M-\hat{q}_\perp), \quad \varphi^-_{1/2} = -\frac{1}{\sqrt{6}}(\sqrt{2}, 1, \sqrt{2}, 1)$$

$$\lambda^-_{3/2} = -2\cdot g^2 (M-\hat{q}_\perp), \quad \varphi^-_{3/2} = \frac{1}{\sqrt{6}}(-1, \sqrt{2}, -1, \sqrt{2})$$

(The Clebsh - Gordan coefficients at $B^+n$ have the opposite sign with respect to the current one, since the definition used : $B^\pm = \frac{1}{\sqrt{2}}(B_1 \pm i B_2)$ don't coincide with that of spherical function).

By comparing (13) with (8), it is clear that $JmM^+$ is described by means of the single reggeon contribution with isospin $T = 1/2$ and following trajectory :

$$\delta(\hat{q}) = \frac{1}{2} + \frac{3}{8\pi} g^2 (M-\hat{q}_\perp)\,\chi \tag{14}$$

where symbol $\chi$ is defined by the equality (see eq.(7)):

$$\chi = \int \frac{d^2k_\perp}{(2\pi)^2} \frac{(-1)}{\left[m^2-(q-k)^2_\perp\right]} \frac{1}{\left[M-\hat{k}_\perp\right]} \tag{15}$$

(The additional minus in (15) is a consequence of the definition of vector meson propagator :

$$G_{\mu\nu} = \frac{-g_{\mu\nu}}{m^2-t}, \quad g_{00} = 1)$$

The state $\varphi^+_{3/2}$ has no contribution at all.

To answer the question whether the fermion is reggeized, the residue has to be found. For the elements T we obtain :

$$\langle 1 | T^{sn} | \varphi^+_{1/2} \rangle = \frac{\sqrt{3S}}{4} g^2 \left[\bar{u}(L')\, \ell^M_1\, \Gamma_{1,M}\, b_1\, \vec{c}\,\right]$$

$$\langle \varphi^+_{1/2} | T^{ns} | 2 \rangle = -\frac{\sqrt{3}\, s}{4}\, g^2 \left[ \vec{b}_2 \vec{\tau}\, l_2^{\mu} \Gamma_{2\mu} u(p) \right] \qquad (I6)$$

By substituting (I6) in (6) and reestablishing ReM by signature factor :

$$\mathrm{Re}\, \mathcal{M} = \frac{-2}{\pi\left(\delta(\hat{q}) - \frac{1}{2}\right)}\, \mathrm{Im}\, \mathcal{M} \qquad (I7)$$

the following expression is obtained :

$$\mathrm{Re}\, \mathcal{M} = g^2 \bar{u}(l_1')\, l_1^{\mu} \frac{\vec{b}_1 \vec{\tau}}{2}\, e^{\frac{3g^2}{8\pi}(M - \hat{q}_\perp)\chi \ln s} \cdot \frac{1}{M - \hat{q}_\perp}\, l_2^{\nu} \Gamma_{2\nu} \frac{\vec{b}_2 \vec{\tau}}{2} u(p) \qquad (I8)$$

From comparison (I8) and (3) it is clear that the fermion is reggeized.

### I.5. Fermion reggeization in model 2.

We shall not dwell upon the detail of all the calculations in model 2. Only the complications, appearing in this model will be considered here. In the first place, there is a difference between the masses of $B^\circ$ and $B^\pm$ (isotopic non-invariance), and as a result the symbol $\chi$ must be treated as a matrix. In a strict sense, it is necessary to find the eigenvalues and eigenvectors of matrix $T\chi$ to solve the problem. But taking into account the fact that $T$ is a singular matrix, it is easy to see that, as before, the positive signature amplitude would be given by a single Regge pole, corresponding to the external particle state with the isospin $1/2$ in $t$-channel. The sole modification consists in the replacement in eq. (6),(8),(9),(I4), (I8) symbol $\chi$ by $\left[\frac{2}{3}\chi + \frac{1}{3}\tilde{\chi}\right]$, where for the case of the "proton" exchange $\chi$ is given by (I5) with $M = M_n$,

$$\tilde{\chi} = \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{1}{M_p - \hat{k}_\perp} \frac{(-1)}{æ^2 - (q-k)^2_\perp} \qquad (I9)$$



Symbol $\chi$ corresponds to the two dimension Feynman loop with $B^+ n$, $\tilde{\chi}$ corresponds to one with $B^\circ p$; the coefficients in replacement are the square of Clebsh -Gordan coefficients; $æ$ is a cutting parameter due to infrared catastrophe. Such a replacement conserves the fermion reggeization (in conventional sense similar to QED).

The additional complications appear due to the difference between the fermion masses: $M_p \neq M_n$. What happens in fact is that the two trajectories appear : one for "proton", the other for "neutron". How does it occur? On FigI0 all the elements of T are represented corresponding to the "proton" pole. For transitions $pB^\circ \to B^\circ p$ (Fig.I0a) and $nB^+ \to B^\circ p$ (Fig.I0b) we obtain, using our rule : $T \backsim (M_p - \hat{q}_\perp)$. The transitions $nB^+ \to B^+ n$ and $pB^\circ \to pB^\circ$ have a zero contributions as before. The fact is essential that the contribution of diagram I0c in transition $n B^+ \to n B^+$ has the other form:

$$T^{B^\circ}(nB^+ \to nB^+) = g^2 (M_n - \hat{q}_\perp) \qquad (20)$$

But in model 2 there is one more diagram I0d with the scalar particle $\sigma$, which contribute as follows :

$$T^{\sigma}(nB^+ \to nB^+) = \frac{\lambda}{2} \cdot 2gm = g^2 (M_p - M_n) \qquad (2I)$$

The eq.(I) was used here. Sum of (20) and (2I) give for this transition $T \backsim (M_p - \hat{q}_\perp)$. In the same way it may be shown that the sum of diagrams I0e and I0f, corresponding to the "ghost" $\varphi^+$ exchange gives the value $T \backsim (M_p - \hat{q}_\perp)$. Thus, the matrix T is obtained in model 2 from that in model I (eq.I2) by the replacement $M \to M_p$. This corresponds to "proton" reggeization with a trajectory :

$$\delta_p(\hat{q}) = \frac{1}{2} + \frac{3g^2}{8\pi}(M_p - \hat{q}_\perp)\left(\frac{2}{3}\chi + \frac{1}{3}\tilde{\chi}\right) \qquad (22)$$

The analogous proof may be carried out for "neutron".

## 2. The integral equations.

In this section model I is assumed. We use the method which was developed in /3,4/. It is necessary to know inelastic amplitudes for finding the elastic scattering amplitude by the dispersion method. Main contribution to the $s$ - channel imaginary part of the elastic amplitude arises from the multiregge kinematics : $s_i \gg m^2$, $-t_i \sim m^2$. Our calculations up to eighth order of perturbation theory show, that in the main ligarithmic approximation amplitudes of inelastic processes in multiregge kinematics with fermion exchange in some channels with small momentum transfer $q_i$, as well as amplitudes with only vector meson exchange calculated earlier /3,4/, have simple multiregge form which is easily generalized to the arbitrary order. The result is as follows.

The amplitude of n -particle production in multiregge kinematics can be described by means of contribution of single diagram. Fig.II gives examples of such diagrams for the production of $n+2$ particles in vector meson - fermion collision (in) kinematics :

$$1 \gg -\alpha_1 \gg -\alpha_2 \gg \ldots \gg -\alpha_{n+1} \sim m^2/s$$

$$1 \gg \beta_{n+1} \gg \beta_n \gg \ldots \gg \beta_1 \sim m^2/s$$

$$s\alpha_i \beta_{i+1} = -m_i^2 + (q_i - q_{i+1})_\perp^2 ; \quad q_i^2 = q_{i\perp}^2 , \qquad (23)$$

where :

$$q_i = \alpha_i L + \beta_i P + q_{i\perp} ;$$

$$q_1 = p_0 - L ; \quad q_i = q_{i-1} + p_{i-1} , \; i > 1 ; \; m_i^2 = p_i^2 \qquad (24)$$

Here solid lines are fermions, wave lines are vector mesons; double solid and double wave lines are reggeons.

Production amplitude corresponding to such diagram is found according to the set of rules, presented on Fig.I2. As it is seen, each vector meson-reggeon line with momentum $q_i$ gives the factor (see /3,4/) $(t_i - m^2)^{-1} s_i^{\alpha(t_i)-1}$, where $t_i = q_i^2$, $s_i = (q_{i+1} - q_{i-1})^2 = (p_i + p_{i-1})^2 = -s\alpha_{i-1}\beta_{i+1}$, each fermion - reggeon line with momentum $q_i$ gives the factor $[M - \hat{q}_{i\perp}]^{-1} s_i^{\delta(\hat{q}_i) - \frac{1}{2}}$. Here $\delta(\hat{q})$ is given by eq./I4/; $\alpha(t)$ is the trajectory of vector meson obtained in/I/:



$$\alpha(t) = 1 + \frac{g^2}{(2\pi)^3}(t - m^2) \int \frac{d^2 k_\perp}{(m^2 - k_\perp^2)(m^2 - (q_\perp - k_\perp)^2)} \qquad (25)$$

Some of the vertex functions reggeon - reggeon - particle and particle - particle - reggeon ( $\gamma^d_{cc'}$, $\Gamma^c_{ad}$, $\Gamma^c_{\varphi\varphi'}$ ) were found earlier /I-4/. Indexes a, $d_i$, $c_i$ are isotopic ones for vector meson; $\varphi$, $\varphi'$ - for fermion; $\lambda$, $\lambda'$ are polarization indexes ( $\lambda = 3$ corresponds to a longitudinally polarized vector particle); $\ell$ denotes polarization vector for vector meson. With this set of rules we can calculate with unitary conditions the contributions to the elastic amplitude from any intermediate states. The sum of these contributions can be presented in form of integral equation (see ref./I2/ for detail). Before presenting this equation some transformations are convenient to perform. At first, the amplitude may be rewritten in form :

$$A = g^2 \bar{u}(L') \ell_1^\mu \Gamma_{1\mu} \left\{ \frac{\vec{\tau}\vec{b}_1}{2} \frac{\vec{\tau}\vec{b}_2}{2} (A^+_{1/2} + A^-_{1/2}) + \left( \frac{\vec{b}_1 \vec{b}_2}{2} - \frac{(\vec{\tau}\vec{b}_1)(\vec{\tau}\vec{b}_2)}{6} \right) (A^+_{3/2} + A^-_{3/2}) \right\} \ell_2^\nu \Gamma_{2\nu} u(P) \qquad (26)$$

to separate the contribution with isospin $1/2$ and $3/2$ in t - channel and with the positive and negative signature. Further, we transit to the $j$ - representation ( $\omega = j - \frac{1}{2}$ )

$$A^\pm_T = \frac{1}{4i} \int_{\delta - i\infty}^{\delta + i\infty} d\omega \left( \frac{s}{m^2} \right)^\omega \frac{(e^{-i\pi\omega} \pm 1)}{\sin \omega\pi} F^\pm_T(\omega, q) \qquad (27)$$

Let's present $F^\pm_T(\omega, q)$ in the form :

$$F^\pm_T(\omega, q) = \frac{z^\pm_T}{M - \hat{q}_\perp} + C^\pm_T \frac{g^2}{(2\pi)^3} \int \frac{d^2 k_\perp}{[m^2 - (q-k)^2_\perp](M - \hat{k}_\perp)} f^{T\pm}_\omega(k_\perp, q_\perp) \qquad (28)$$

convenient for obtaining the first term in integral equation with the coefficient I. The values $z^\pm_T$, $C^\pm_T$ are defined as follows:

$$z^+_{1/2} = 1 ; \quad z^-_{1/2} = z^+_{3/2} = z^-_{3/2} = 0 ;$$

$$C^+_{1/2} = -\frac{3}{4} ; \quad C^-_{1/2} = \frac{25}{12} ; \quad C^+_{3/2} = 0 ; \quad C^-_{3/2} = 2 \qquad (29)$$

As $z^{+}_{3/2}, C^{+}_{3/2}$ equals zero, the corresponding state don't contribute in the main logarithmic term as it was seen above. The following equation presented graphically on Fig.I3 is obtained for $f^{T\pm}_{\omega}(k,q)$ /9,I2/

$$
\begin{aligned}
&\left[\omega - \left(\alpha((q-k)^2)-1\right) - \left(\delta(\hat{k}) - \tfrac{1}{2}\right)\right] f^{T\pm}_{\omega}(k,q) = \\
&= 1 + \frac{g^2}{(2\pi)^3} \int \frac{d^2k'}{m^2-(q-k')^2} \Big\{ a^{\pm}_{T}\Big[\hat{q} - M + (M-\hat{k})\frac{1}{M-(\hat{k}+\hat{k}'-\hat{q})} \cdot \\
&\cdot (M-\hat{k}')\Big] + b^{\pm}_{T}\Big[\hat{q} - M + (M-\hat{k})\frac{(m^2-(k'-q)^2)}{(m^2-(k'-k)^2)} + \\
&+ (M-\hat{k}')\frac{(m^2-(k-q)^2)}{(m^2-(k'-k)^2)}\Big]\Big\} \frac{1}{M-\hat{k}'} f^{T\pm}_{\omega}(k',q)
\end{aligned}
\tag{30}
$$

Here all the vectors are treated as the two dimensional ones, $\alpha(q^2)$ is the trajectory of vector meson obtained in /I/ (see /25/). The appearance of the expressions $\left(\delta(\hat{k}) - \frac{1}{2}\right)$ and $\left(\alpha((q-k)^2)-1\right)$ in the left hand part of (30) is a consequence of the fact that the spinors and nonsense polarizations get the additional factors $s^{1/2}$ and $s$ and the reggeons are associated with a terms $s^{\delta-\frac{1}{2}}$, $s^{\alpha-1}$. The coefficients a, b are as follows

$$
a^{\pm}_{1/2} = \mp\frac{1}{4}; \quad b^{\pm}_{1/2} = 1; \quad a^{-}_{3/2} = b^{-}_{3/2} = -\frac{1}{2}
\tag{3I}
$$

The solution for $f^{\frac{1}{2}+}_{\omega}(k,q)$ is :

$$
f^{\frac{1}{2}+}_{\omega}(k,q) = \frac{1}{\omega - \left(\delta(\hat{q}) - \frac{1}{2}\right)}
\tag{32}
$$

where $\delta(\hat{q})$ is given by /I4/. For $F^{+}_{1/2}(\omega,q)$ we obtain:

$$
F^{+}_{1/2}(\omega,q) = \frac{1}{M-\hat{q}} \frac{\omega}{\left[\omega - \left(\delta(\hat{q}) - \frac{1}{2}\right)\right]}
\tag{33}
$$

The main logarithmic term is then as follows :

$$
A^{+}_{1/2} = \frac{1}{M-\hat{q}} \left(\frac{s}{m^2}\right)^{\delta(\hat{q}) - \frac{1}{2}}
\tag{34}
$$



what corresponds to the fermion reggeization.

The analogous equations may be written in QED for the amplitude of the backward Compton - effect $e\gamma \to \gamma e$. Presenting the amplitude in the form :

$$
A^{QED} = g^2 \bar{u}\, \ell_1^{\mu} \Gamma_{1\mu} (A^{+} + A^{-}) \ell_2^{\nu} \Gamma_{2\nu} u
\tag{35}
$$

we have in QED the eq.(27),(28),(30) with the parameters :

$$
b^{\pm} = 0, \quad a^{\pm} = \pm 1, \quad C^{\pm} = \mp 1; \quad z^{+} = 1, \quad z^{-} = 0
\tag{36}
$$

In eq.(30) it is necessary to take into account that $\alpha((k-q)^2) - 1 = 0$ (photon isn't reggeized) and to replace $\delta(\hat{k})$ on

$$
\delta_1(\hat{k}) = \frac{1}{2} + \frac{4}{3}\left(\delta(\hat{k}) - \frac{1}{2}\right)
$$

The solution of this equation for positive signature is :

$$
F^{+}_{\omega}(q) = \frac{1}{(M-\hat{q}_{\perp})} \frac{\omega}{\left[\omega - \left(\delta_1(\hat{q}) - \frac{1}{2}\right)\right]}
\tag{37}
$$

where

$$
\delta_1(\hat{q}) - \frac{1}{2} = -\frac{g^2}{(2\pi)^3}(M-\hat{q}_{\perp}) \int \frac{d^2k_{\perp}}{(M-\hat{k}_{\perp})(m^2-(q_{\perp}-k_{\perp})^2)}
\tag{38}
$$

and $m$ is a photon mass. This is in accordance with the well-known result /6/ on electron reggeization. The integral equation for negative signature coincides with recently obtained results of ref. /I3/.

We are deeply indebted to L.L.Frankfurt, E.A.Kuraev and L.N.Lipatov for the interest to the work and many helpful discussions.

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Figures

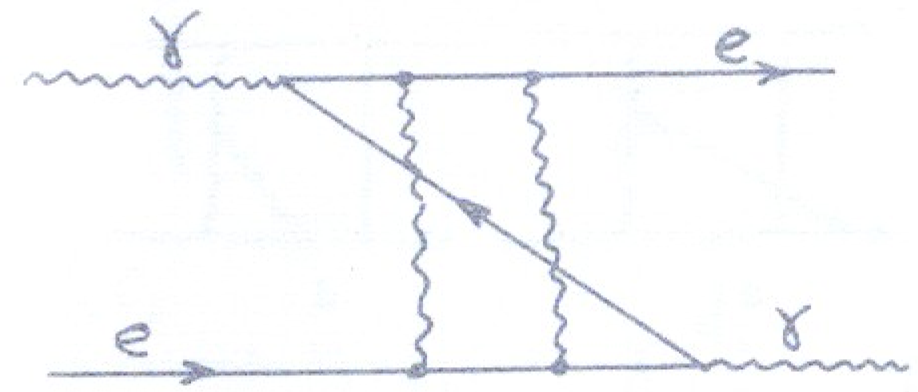



Fig.I.

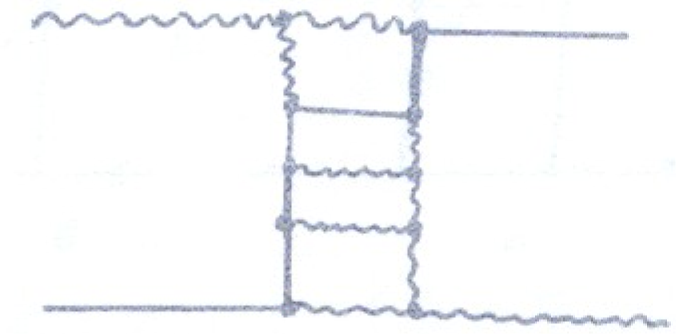


Fig.2.

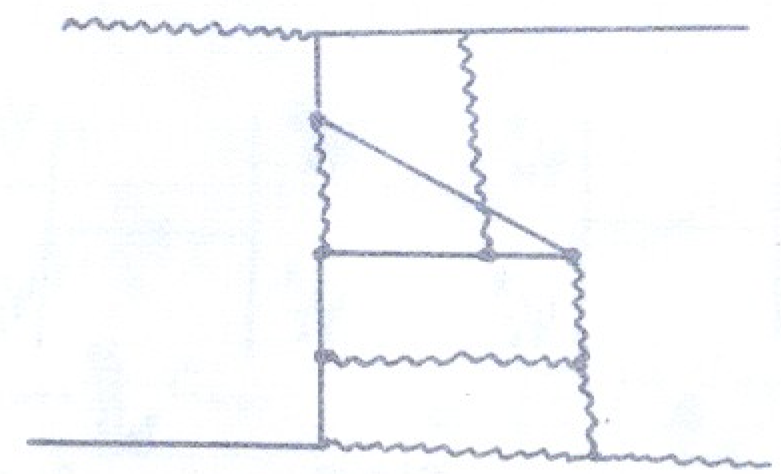


Fig.3.

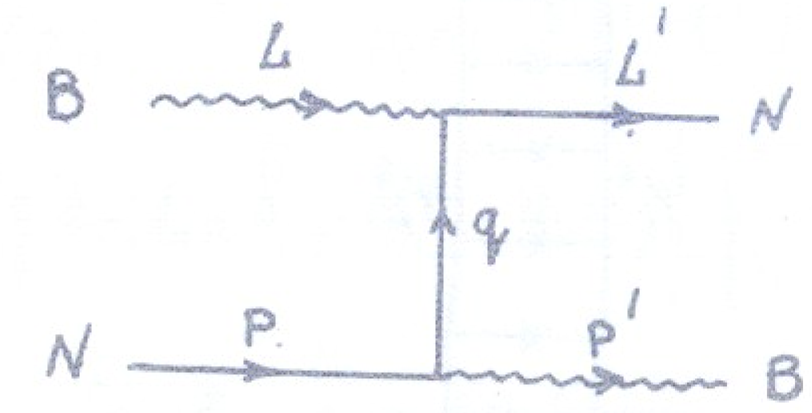
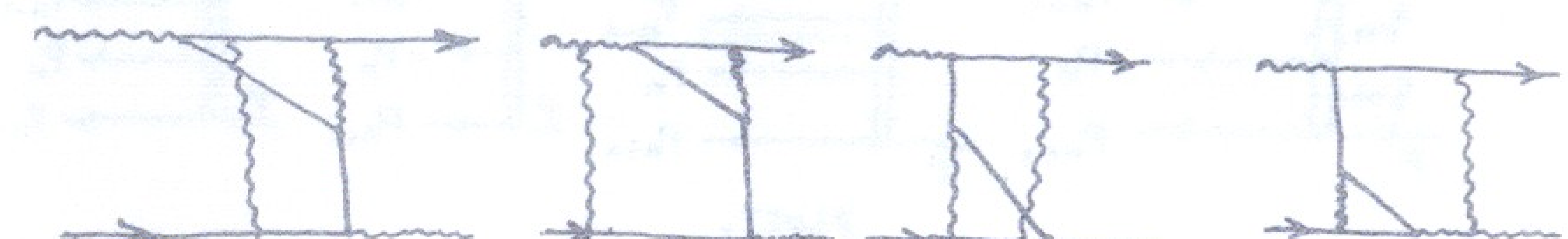



Fig.4.


a). b).

Fig.5.



a b c d

Fig.6.

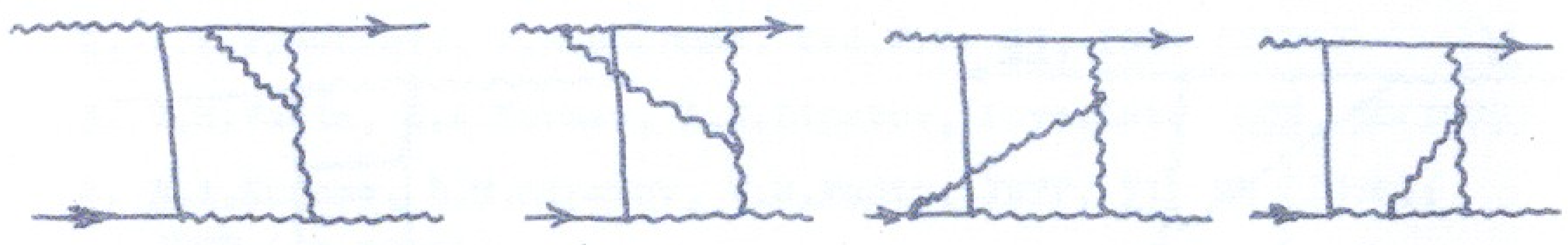



Fig.7.

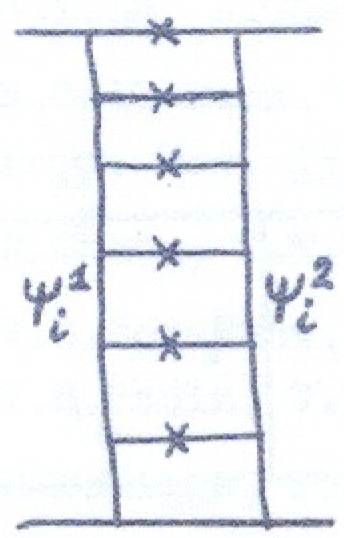



Fig.8.

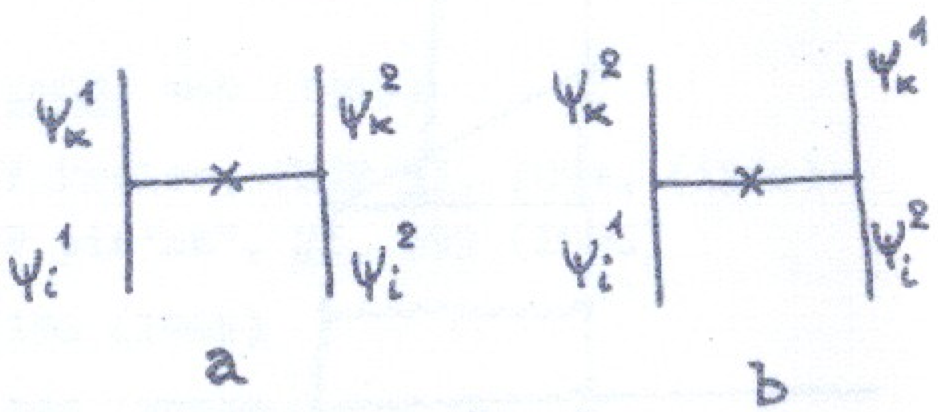



Fig.9.

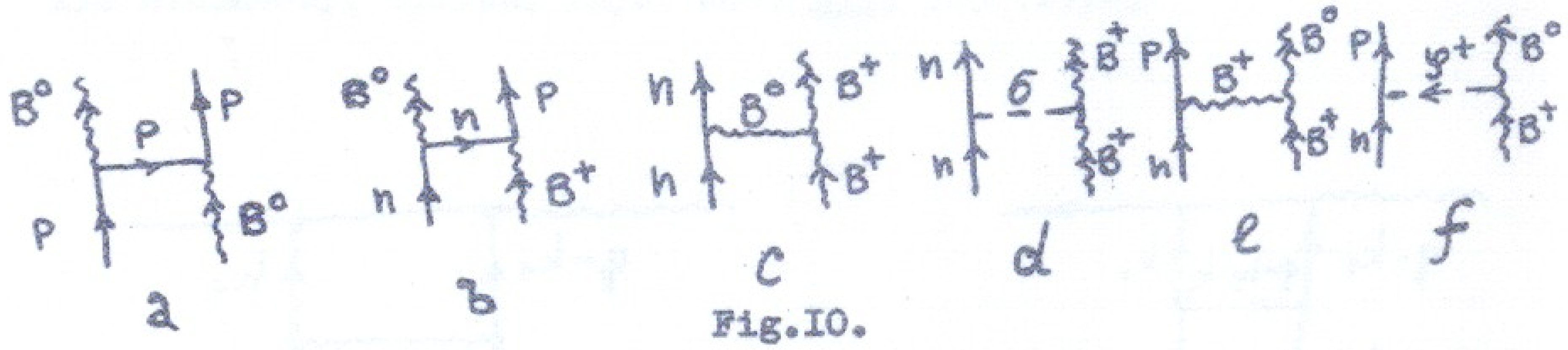
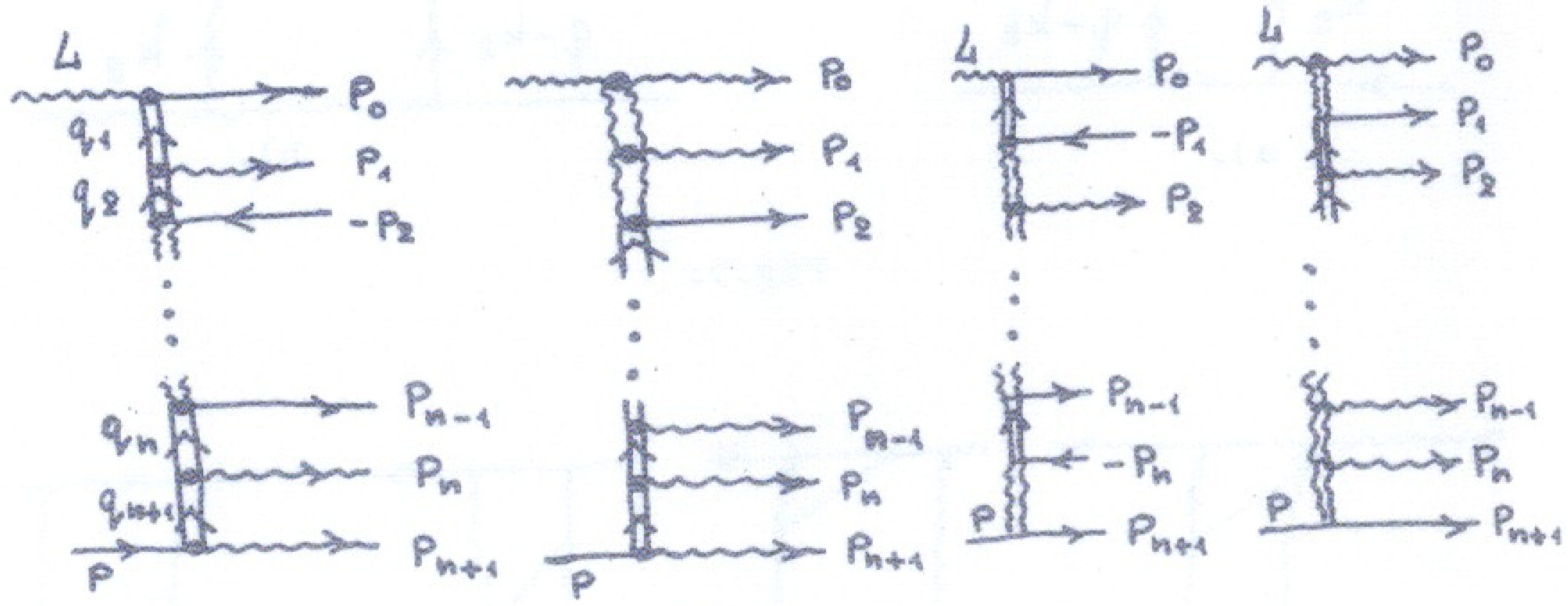
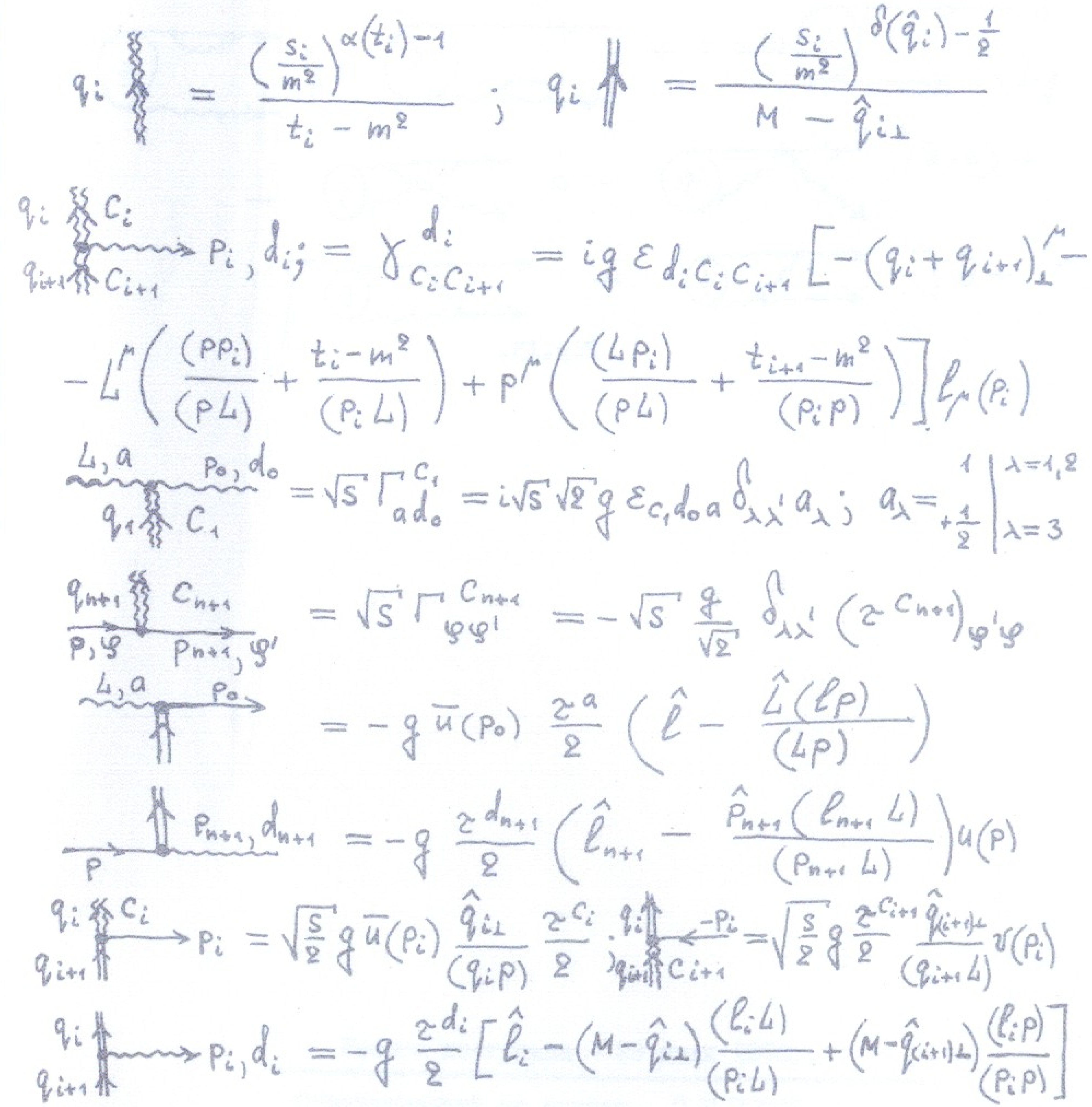



Fig.IO.




FigII.



$$q_i = \frac{\left(\frac{s_i}{m^2}\right)^{\alpha(t_i)-1}}{t_i - m^2} \; ; \quad q_i = \frac{\left(\frac{s_i}{m^2}\right)^{\delta(\hat q_i)-\frac12}}{M - \hat q_{i\perp}}$$

$$p_i, d_i; = \gamma^{d_i}_{c_i c_{i+1}} = ig\,\varepsilon_{d_i c_i c_{i+1}}\Big[-(q_i+q_{i+1})^\mu_\perp - L^\mu\Big(\frac{(pp_i)}{(pL)} + \frac{t_i - m^2}{(p_i L)}\Big) + p^\mu\Big(\frac{(Lp_i)}{(pL)} + \frac{t_{i+1}-m^2}{(p_i p)}\Big)\Big]\ell_\mu(p_i)$$

$$= \sqrt{s}\,\Gamma^{c_1}_{a d_0} = i\sqrt{s}\sqrt{2}\,g\,\varepsilon_{c_1 d_0 a}\,\delta_{\lambda\lambda'}\,a_\lambda \; ; \quad a_\lambda = \begin{cases} 1 & \lambda = 1,2 \\ +\frac12 & \lambda = 3 \end{cases}$$

$$= \sqrt{s}\,\Gamma^{c_{n+1}}_{\varphi\varphi'} = -\sqrt{s}\,\frac{g}{\sqrt2}\,\delta_{\lambda\lambda'}\left(\tau^{c_{n+1}}\right)_{\varphi'\varphi}$$

$$= -g\,\bar u(p_0)\,\frac{\tau^a}{2}\left(\hat\ell - \frac{\hat L(\ell p)}{(Lp)}\right)$$

$$= -g\,\frac{\tau^{d_{n+1}}}{2}\left(\hat\ell_{n+1} - \frac{\hat p_{n+1}(\ell_{n+1}L)}{(p_{n+1}L)}\right)u(p)$$

$$p_i = \sqrt{\frac{s}{2}}\,g\,\bar u(p_i)\,\frac{\hat q_{i\perp}}{(q_i p)}\,\frac{\tau^{c_i}}{2} \; ; \quad -p_i = \sqrt{\frac{s}{2}}\,g\,\frac{\tau^{c_{i+1}}}{2}\,\frac{\hat q_{(i+1)\perp}}{(q_{i+1}L)}\,v(p_i)$$

$$p_i, d_i = -g\,\frac{\tau^{d_i}}{2}\left[\hat\ell_i - (M - \hat q_{i\perp})\frac{(\ell_i L)}{(p_i L)} + (M - \hat q_{(i+1)\perp})\frac{(\ell_i p)}{(p_i p)}\right]$$

Fig.I2.

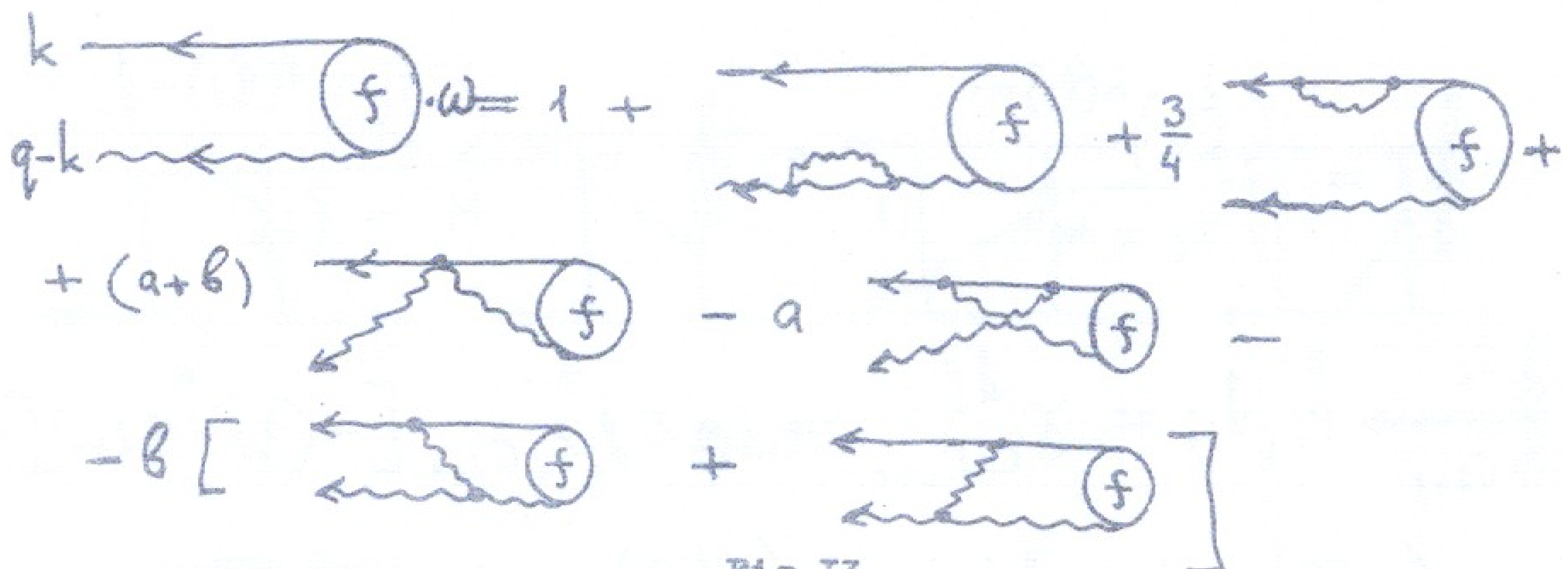



Fig.I3.

Работа поступила - 29 июля I976г.

Ответственный за выпуск - С.Г.ПОПОВ
Подписано к печати 20.IX-I976г. МН 02970
Усл. I,I печ.л., 0,85 учетно-изд.л.
Тираж I50 экз. Бесплатно
Заказ №86.

Отпечатано на ротапринте ИЯФ СО АН СССР